EECS 40, Fall 2007
Prof. Chang-Hasnain

Homework #5 - Solution

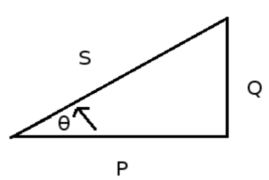
1. (Power in AC circuits) P5.62, P5.66, and P5.67 (11 points)

P 5.62 (3 points), 1 each

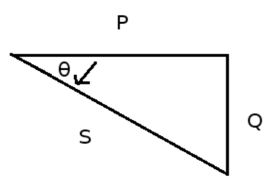
The units for real power are watts (W). For reactive power, the units are volt-amperes reactive (VAR). For apparent power, the units are volt-amperes (VA).

P 5.66 (4 points)

a)



b)



Note that the angle is positive in the inductive case and negative in the capacitive case

P 5.67 (4 points)

Real power represents a net flow, over time, of energy from the source to the load. This energy must be supplied to the system from regenerative, fossil-fuel, or nuclear sources.
Reactive power represents energy that flows back and forth from the source to the load. Aside from losses in the transmission system (lines and transformers), no net energy must be supplied to the system to create the reactive power. Reactive power is important mainly because of the increased system losses associated with it.

2. (Power in AC circuits) P5.70 (12 points)

   This is an inductive load because the reactance is positive.

   $Z = 30 + j40 = 50\angle 53.13^\circ \quad I = \frac{\mathbf{V}}{Z} = \frac{1500\sqrt{2}\angle 30^\circ}{50\angle 53.13^\circ} = 30\sqrt{2}\angle -23.13^\circ$

   $P = I_{rms}^2 R = (30)^2 30 = 27 \text{ kW}$

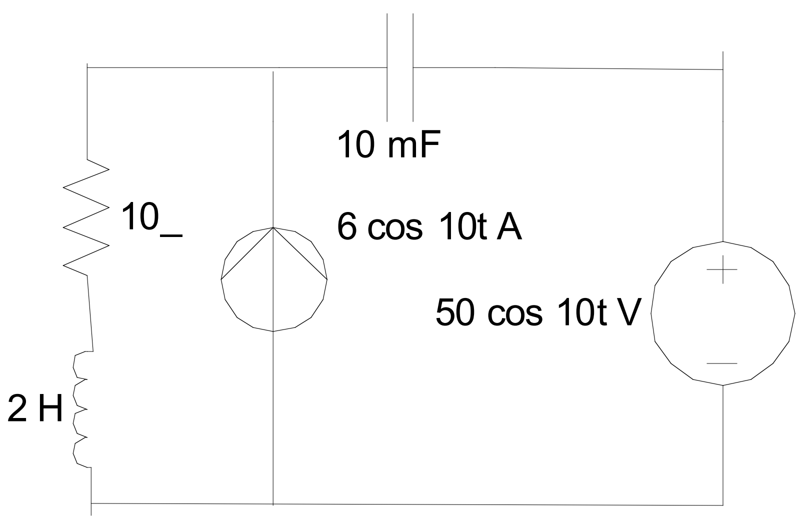
   $Q = I_{rms}^2 X = (30)^2 (40) = 36 \text{ kVAR}$

   $\theta = 53.13^\circ$

   power factor $= \cos(\theta) = 60\%$

   apparent power $= V_{rms} I_{rms} = 1500 \times 30 = 45 \text{ KVA}$

3. (Power in AC circuits) Determine the complex power of the R, L, and C elements, and show that the complex power delivered by the sources is equal to the complex power absorbed by the R, L, and C elements.
   (17 points)



Z_L = j20□, Z_C= -j10□ (2 points, one each)

Calculate currents with superposition:

(i) Voltage source switched off:

I_L = 6 ((10□+j20□) II (-j10□)) / (10□+j20□) A = 4.23□-135 A

I_C = 6 ((10□+j20□) II (-j10□)) / (-j10□) A = 9.48□18.43 A

(ii) Current source switched off:

I_L= 50 V / (10□ + j20□ – j10□) = 3.54□-45 A

I_C= 3.54□135 A

(iii) Combined results:

I_L= 5.52□-95.19 A

I_C= 8.51□40.24 A

$Q_C = I_C V_C$= -j724 VAR

$Q_L = I_L V_L$ = j609 VAR
$P_R = I_L V_R$ = 304 W

Calculate voltage at current source
V_{SI} = 5.52□-95.19 A (10□ + j20□) = 123.43□-31.75 V

Calculate delivered power of sources
PI + QI + PV + QV = 629 W - j389.69 VAR - 324.8 W + j274.87 VAR

The total generated real and reactive power equals the total absorbed real and reactive power.

4. (Transfer functions) (6 points)
   a. Find the sum of the transfer function given in eq 6.9 (p 280) and that in eq 6.21 (p 292), and explain.
      $1/(1+j(f/f_B)) + j(f/f_B)/(1+j(f/f_B) = (1+j(f/f_B)) / (1+j(f/f_B)) = 1$
      We are adding a low-passand a high-pass with the same break frequency. Therefore, all frequencies can pass.
   b. Find the sum of 6.10 and 6.23 and explain why this is different from your answer to part (a).
      We get $1 - f/f_B \,/\, 1 - f/f_B^2$. We get a different result, because we are adding the magnitudes instead of the complete complex functions. In general, adding to complex numbers and calculating sum's magnitude is not equal to adding the magnitudes of each complex number. As an example, consider the numbers j and -j.

5. P6.23 (First order lowpass filter) (4 points)
   The time constant is given by □ = RC and the half-power frequency is f_B = 1/(2□RC). Thus, we have f_B = 1/(2□□)

6. P6.25 (First-order lowpass filter) (12 points)

The transfer function is given by Equation 6.9 in the text:

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

The given input signal is

$$v_{in}(t) = 4 + 2\sin(1000\pi t + 30^\circ) + 5\cos(30 \times 10^3 \pi t)$$

which has components with frequencies of 0, 500, and 15,000 Hz.

Evaluating the transfer function for these frequencies yields:

$$H(0) = \frac{1}{1 + j(0/500)} = 1$$

$$H(500) = 0.7071\angle -45^\circ$$

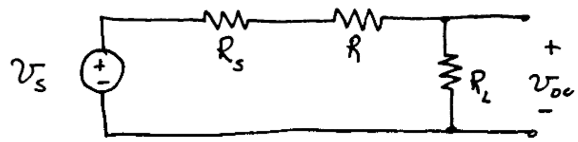
$$H(10^4) = 0.03331\angle -88.09^\circ$$

Applying the appropriate value of the transfer function to each component of the input signal yields the output:

$$v_{out}(t) = 4 + 1.414\cos(1000\pi t - 105^\circ) + 0.1666\cos(30 \times 10^3 \pi t - 88.09^\circ)$$

7. P6.31 (a) (First order lowpass filter) (10 points)

(a) First, we find the Thévenin equivalent for the source and resistances.



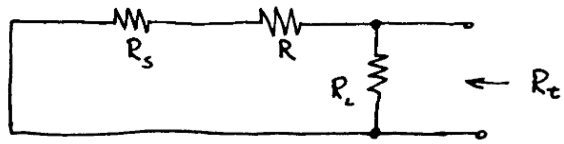
The open-circuit voltage is given by

$$v_t(t) = v_{oc}(t) = v_s(t)\frac{R_L}{R_s + R + R_L}$$

In terms of phasors, this becomes:

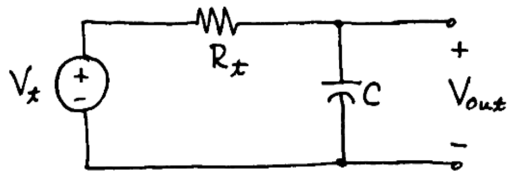
$$\mathbf{V}_t = \mathbf{V}_s \frac{R_L}{R_s + R + R_L} \quad (1)$$

Zeroing the source, we find the Thévenin resistance:



$$R_t = \frac{1}{1/R_L + 1/(R + R_s)}$$

Thus, the original circuit has the equivalent:



The transfer function for this circuit is:

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)} \quad (2)$$

where, $f_B = \dfrac{1}{2\pi R_t C}$

Using Equation (1) to substitute for $\mathbf{V}_t$ in Equation (2) and rearranging, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} = \frac{R_L}{R_s + R + R_L} \times \frac{1}{1 + j(f/f_B)} \quad (3)$$

8. (Decibels, Logarithmic Frequency Scales) (10 points)

a. P6.42 (4 points)

To convert to decibels, we take 20 times the common logarithm of the transfer function. Thus, we have:

$20\log(0.5) = -6.021$ dB

$20\log(2) = +6.021$ dB

$20\log(1/\sqrt{2}) = -3.010\,\text{dB}$

$20\log(\sqrt{2}) = 3.010\,\text{dB}$

b. P6.44 (a) (2 points)

We have $10^{N_d} \times 100 = 3500$.

Taking the common logarithm of both sides, we have:

$N_d + 2 = 3.544$

$N_d = 1.544$ decades

c. P6.46 (4 points)

On a linear scale, the frequencies are 1, 3.25, 5.5, 7.75, 10 Hz.
(We add 9/4 = 2.25 Hz to each value to obtain the next value.)
On a logarithmic scale, the frequencies are 1, 1.778, 3.162, 5.623, 10 Hz.
(We multiply each value by $\sqrt[4]{10} = 1.778$ to obtain the next value.)

9. (Bode plots) (8 points) Consider H(f) = jfA/(B+jfC). Find

(a) the corner frequency

Transform H(f) into normal form --> f_B = B/C

(b) the slope of the asymptotic magnitude Bode plot for f above the corner frequency in decibels per decade

Above the corner frequency, the 1 in the denominator of the normal form is negligible. Therefore, you can cancel out f and the magnitude is constant --> slope = 0

(c) the slope of the magnitude Bode plot below the corner frequency

Here, the (f/fB) part of the denominator is negligible. Therefore, H(f) = j(f/fB)(A/C). Therefore, the slope is 20 dB per decade.

(d) the gain for f above the corner frequency in decibels.

H(f) = A/C for large frequencies.

10. P6.55 (Bode plots) (10 points)

$$H(f) = \frac{1 - j(f/100)}{1 + j(f/100)} \qquad |H(f)| = \frac{\sqrt{1 + (f/100)^2}}{\sqrt{1 + (f/100)^2}} = 1$$

Thus, $|H(f)|_{dB} = 20\log(1) = 0$.

The phase is $-2 \times \arctan(f/100)$.

The asymptotic Bode plots are: