UNIVERSITY OF CALIFORNIA AT BERKELEY EECS Department

EECS 40, Fall 2007
Prof. Chang-Hasnain

Homework #6

Due at 5 pm in 240 Cory on Thursday, 10/25/07
Total Points: 100

- Put (1) your name and (2) discussion section number on your homework.
- You need to put down all the derivation steps to obtain full credits of the problems. Numerical answers alone will at best receive low percentage partial credits.
- No late submission will be accepted except those with prior approval from Prof. Chang-Hasnain.
- Problems of this HW are from Hambley 4th Edition

Series Resonance

1. P6.73

The impedance of a series RLC circuit is minimum in magnitude at the resonant frequency and is equal to the resistance. The resonant frequency is the frequency at which the impedance is purely resistive. The quality factor is defined as Q = □0 L/R.

2 **P6.74** A bandpass filter is a filter that passes components in a band of frequencies, rejecting components with higher and lower frequencies. Bandwidth is the span of frequencies for which the transfer function magnitude is higher than its maximum value divided by the square root of two.

3 P6.80

P6.80 (a) The impedance of the circuit is given by

$$Z(j\omega) = \frac{1}{1/R - j/(\omega L)} - j\frac{1}{\omega C} = \frac{1/R + j/(\omega L)}{1/R^2 + 1/(\omega^2 L^2)} - j\frac{1}{\omega C}$$

At the resonant frequency, we set the imaginary part equal to zero.

$$\frac{1/(\omega_0 L)}{1/R^2 + 1/(\omega_0^2 L^2)} - \frac{1}{\omega_0 C} = 0$$

from which we obtain

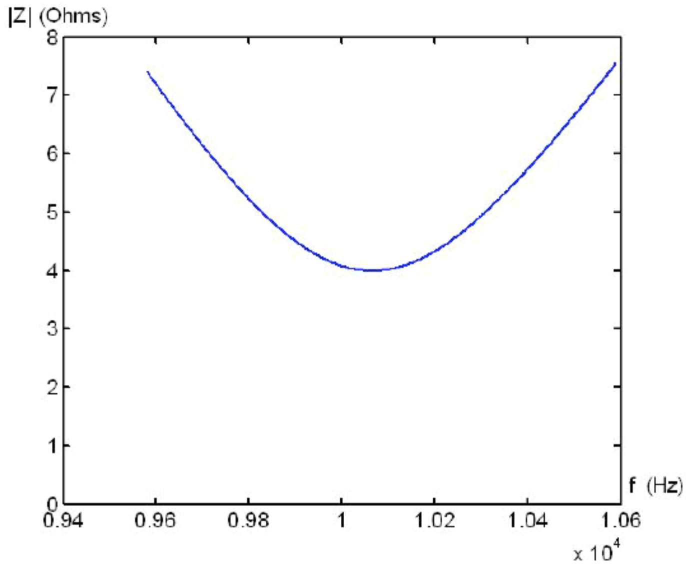
$$\omega_0 = \sqrt{\frac{1}{LC - L^2/R^2}} \quad \text{or} \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC - L^2/R^2}}$$

(b) $f_0 = 10.09$ kHz

(c) A MATLAB program to plot the impedance magnitude is

```
R = 1000;
L = 1e-3;
C = 0.25e-6;
f0 = (1/(2*pi))*sqrt(1./(L*C - L^2/R^2));
f = 0.95*f0:1:1.05*f0;
Z = (1./(1/R - i./(2*pi*f*L)) - i./(2*pi*f*C));
plot(f,abs(Z))
```

The resulting plot is:



Parallel Resonance

4. 6.83

The impedance of a parallel RLC circuit is maximum in magnitude at the resonant frequency and is equal to the resistance. The resonant frequency is the frequency at which the impedance is purely resistive. The quality factor is defined as $Q = R/\omega_0 L$ which is the reciprocal of the definition for the series resonant circuit.

5. 6.87

$|Zp|_{max} = R = 10K\Omega$

$Q_p = f_0/B$

$L = R/(2\pi f_o Q_p) = 3.183\ \mu H$

$C = Qp/(2\pi f_o R) = 79.58pF$

Second Order Filter

6. 6.89

Four types of ideal filters are lowpass, highpass, bandpass, and band reject (or notch) filters. Their transfer functions are shown in Figure 6.34 in the book.

7. 6.92

An AM radio signal having a carrier frequency of 980 kHz has components ranging in frequency from 970 kHz to 990 kHz. A bandpass filter is needed to pass this signal and reject the signals from other AM radio transmitters. The cutoff frequencies should be 970 and 990 kHz.

8. **6.97** You may sketch the Bode plot by hand. What kind of filter is that?

(a) Applying the voltage-division principle, we have

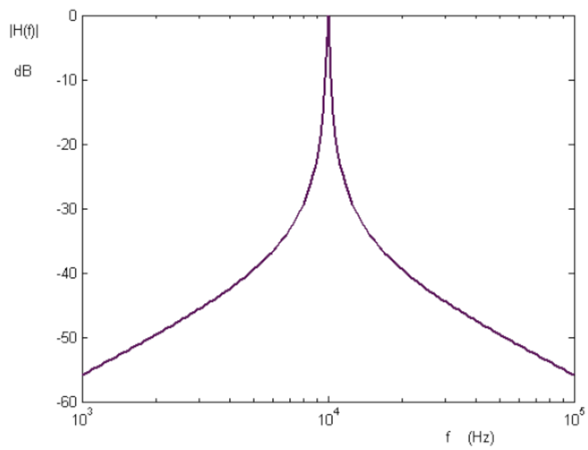
$$H(f) = \frac{R}{R + j2\pi fL - j/(2\pi fC)}$$

(b) A MATLAB program to produce the desired plot is

```
R = 10
L = 0.01
C = 2.533e-8

    logf = 3:0.01:5;
    f = 10.^logf;
    w = 2*pi*f;
    H = R./(R+j*w*L + 1./(j*w*C ));
    semilogx(f,20*log10(abs(H)))
```

The resulting plot is

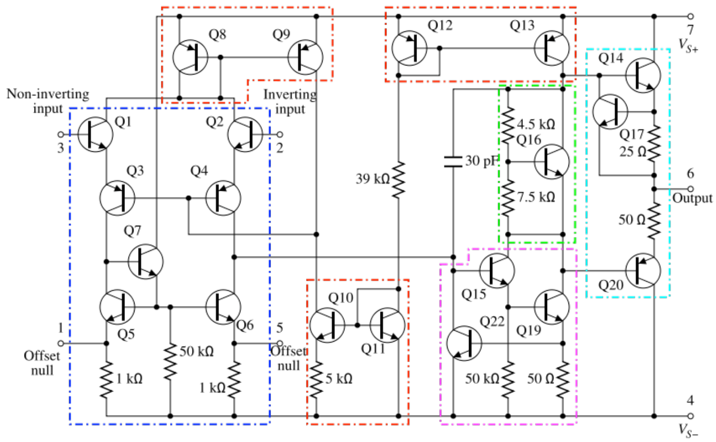


(c) At very low frequencies, with the capacitance considered to be an open circuit, no current flows and H(f) becomes very small in magnitude as shown in the plot. |H(f)| = log(2 pi f C R) so slope 20dB/dec.
(d) At very high frequencies with the inductance considered as an open circuit, no current flows and H(f) becomes very small in magnitude as shown in the plot. This time |H(f) = -log(2 pi f L/ R) so slope of -20dB/dec.

Operational Amplifiers

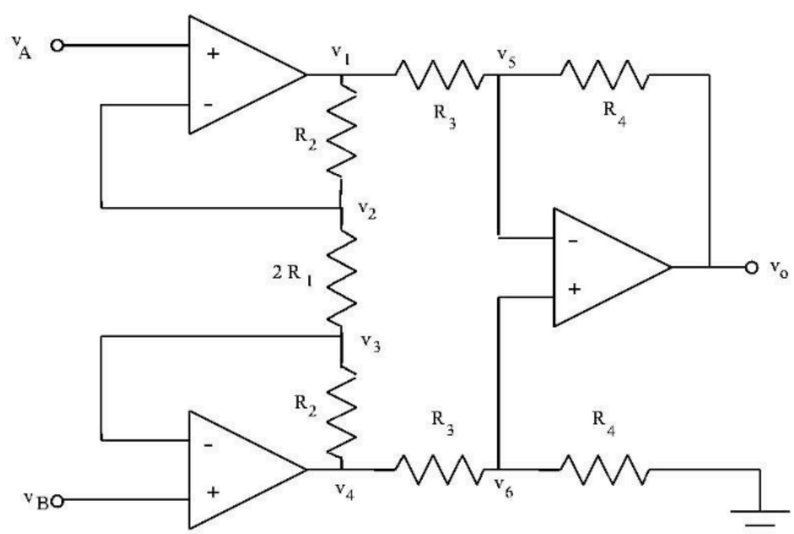
9. List characteristics of an ideal operational amplifier. Search a schematic of a real operational amplifier. How many primitive elements (transistor, resistors) does it contain? Why do we use such a complicated device for often quite trivial tasks?

Ideal op amp has infinite input impedances (so zero input currents), infinite gain for the differential input signal, zero gain for the common-mode input signal, zero output impedance, and infinite bandwidth.
The schematic on Wikipedia contains 33 elements by my count:



Op amps enjoy wide usage because they can be manufactured as integrated circuits rather than individual component-by-component, and this makes them relatively cheap, particularly because they are quite versatile and therefore mass-produced.

10. (Operational Amplifier)



Calculate v_o as a function of v_A and v_B.

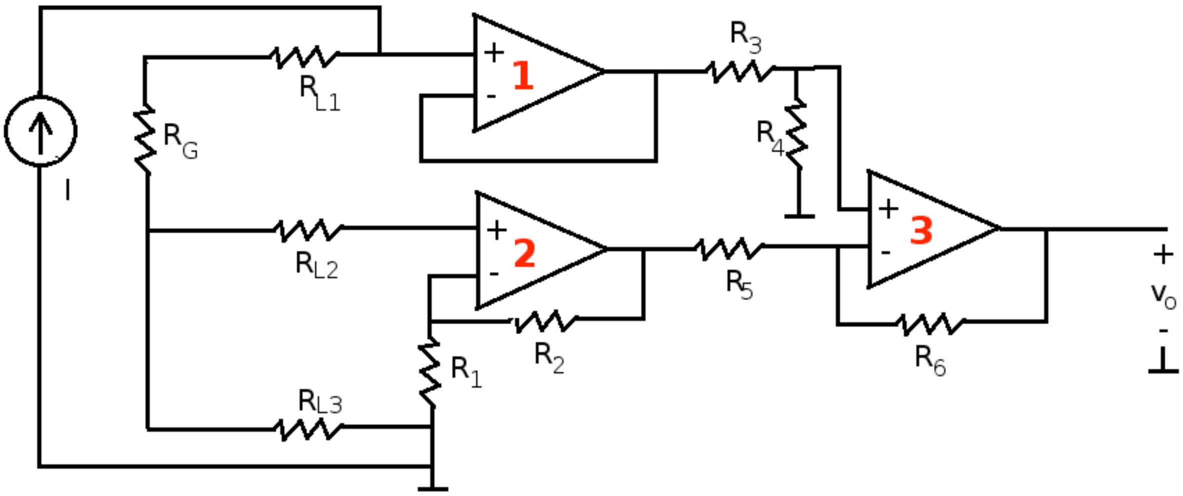
First, observe that all three op amps satisfy negative feedback. Therefore we can use the summing point constraints.
Using voltage divider, we see that **$V6 = V_4*R_4/(R_3+R_4)$.**
Summing point constraint dictates that **V5 = V6**.
Current to the right through R4 on top will be $(V_1-V_5)/R_3$ due to the condition that current into the opamp input terminals be 0. So **$V0 = V5 – R_4*(V_1-V_5)/R_3$** = $V_5*(1+R_4/R_3) – V_1 R_4/R_3 = V_5*(R_3+R_4)/R_3 – V_1 R_4/R_3$.
Plugging in from V5 our previous two equations, we get V0 = $V_4*R_4/(R_3+R_4)*(R_3+R_4)/R_3 – V_1 R_4/R_3 = (V_4-V_1)*R_4/R_3$.

From the summing point constraints on the left two op-amps we have **$V_3=V_B$** and **$V_2 = V_A$**. Our desired quantity (V_4-V_1) can be written as **$V_4-V_1 = I_2*(R_2 + 2*R_1 +R_2)$** from noticing that the current is constant up branch attaching V4 and V1, and calling this I_2. We also know that $I2*2*R_1 = V_3-V_2 = V_B-V_A$. So **$I_2 = (V_B-V_A)/(2*R_1)$.** We substitute this back into our equation for V_4-V_1 to find $V_4-V_1 = (V_B-V_A) * (R_2+R_1)/ R_1$.

Putting this into the equation for V0 finishes off our work, with **$V_0 = (V_B-V_A)*R_4(R_2+R_1) / (R_1R_3)$**.

2. (Operational Amplifier)

To measure the temperature in a furnace, a temperature sensitive resistor R_G is used. The evaluation electronics need to be remote from the furnace due to the high temperatures. The lines are modeled by the ohmic resistances R_{Li}. The following circuit shall be used to compensate the impact of the lines. All operational amplifiers may be assumed to be ideal.



a) Identify the basic configurations in which operational amplifiers 1, 2, and 3 are used. What is the purpose of operational amplifier 1?

*Op2 is a non-inverting amplifier like that shown on p676 (the + input is a voltage source, and the other a resistor). Op amp1 is a voltage follower (also p676) which essentially functions as a dependent source with voltage equal to that of its (+) terminal. Op3 has as its (+) a voltage source and its (-) a voltage source followed by a resistor. Can replace the voltage source at the (-) terminal with ground, and just add the voltage source to the (+) terminal (ground is just a matter of reference) to see that Op3 is just an inverting amplifier like p668, offset by a constant voltage. (**3 points each op amp description)***

b) Calculate v_o in dependence on I and all resistances.

All the current I flows through R_{L1} then R_G then R_{L3}.
Output of Op1 we said was a voltage follower, so ***Vout1= I*(R_{L1}+R_G+R_{L3}).***
Output of Op2 we said has form Vin(1+R2/R1) (using eq on p676, and noticing that even the resistor labels match that diagram). Vin = R_{L3}*I, so* ***Vout2= I*R_{L3}(1+R2/R1)=I*R_{L3}(R1+R2)/R1.***

*If the (+) terminal to Op3 were ground, then we'd expect Vo = -Vout2*R6/R5. Because the (+) terminal has a non-zero value V_{3in+}, we find* ***Vo = -(V_{out2}-V_{3in+})*R6/R5 + V_{3in+} = V_{3in+} (1+R6/R5) - V_{out2}.***

Finally, ***V_{3in+} =V1out*R4/(R4+R3) = I*(R_{L1}+R_G+R_{L3})* R4/(R4+R3).***

Now we plug in to get
V_{3in+} = I(R_{L1}+R_G+R_{L3})* R4/(R4+R3) and so Vo = I*(R_{L1}+R_G+R_{L3})* R4/(R4+R3) **
(1+R6/R5) - *I*R_{L3}(R1+R2)/R1.*

Bonus Problem: The negative slope represents a delay because a decreasing angle for higher frequencies means a lag. For example, a phasor with angle of 90 is ahead of a phasor with a higher frequency and a 45 degree angle in the time domain. Because of the negative slope (- higher frequencies will always lag (or at best be even with) lower frequencies, thus will be delayed in time, so d(phi)/d(omega) must be negative.