EE40
Lecture 21
Prof. Chang-Hasnain

10/22/07

Reading: Chap. 14

Recap: Transfer Function

- Transfer function= H(f) = V_{out}/V_{in}
- It is a complex number and typically a function of frequency.

$$\mathbf{H}(f) = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{V_{out}}{V_{in}} \angle(\theta_{out} - \theta_{in})$$

$$\mathbf{H(f)} = |\mathbf{H(f)}| \angle \theta(f)$$

$|\mathbf{H(f)}|$ is the magnitude

$\theta(f)$ is the phase

Both are functions of frequency f.

Cascaded Transfer Function - 1

$$\mathbf{H}(f) = \mathbf{H}_1(f) \cdot \mathbf{H}_2(f)$$

$$|\mathbf{H(f)}| = |\mathbf{H_1(f)}| \cdot |\mathbf{H_2(f)}|$$

$$\theta(f) = \theta_1(f) + \theta_2(f)$$

$$y = 20\log|\mathbf{H(f)}| = 20\log|\mathbf{H_1(f)}| + 20\log|\mathbf{H_2(f)}| = y_1 + y_2$$

- Bode magnitude plot will be the sum of the two magnitude plots in dB
- Phase will be sum

Cascade Transfer Function - 2

$$\mathbf{H}(f) = \frac{\mathbf{H}_1(f)}{\mathbf{H}_2(f)}$$

$$\theta(f) = \theta_1(f) - \theta_2(f)$$

$$Y = 20\log|\mathbf{H(f)}| = 20\log|\mathbf{H_1(f)}| - 20\log|\mathbf{H_2(f)}|$$

- Bode magnitude plot will be the difference of the two magnitude plots in dB
- Phase will be difference

Example 1

$$H(\omega) = H_1(\omega) \cdot H_2(\omega)$$

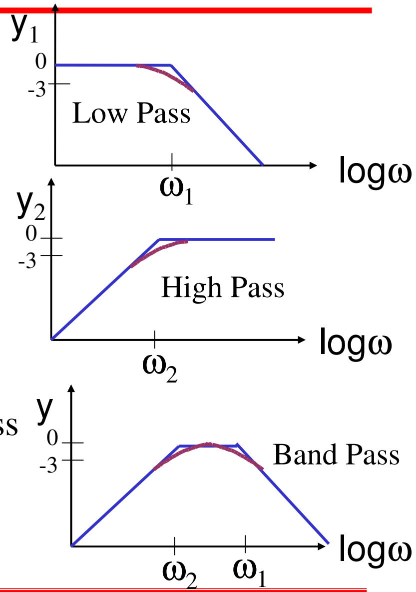
$$H_1(\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_1}} \quad \text{Low pass}$$

$$H_2(\omega) = \frac{1}{1 - j\dfrac{\omega_2}{\omega}} \quad \text{High pass}$$

$$if\ \omega_1 > \omega_2\ ,\ BW = \frac{\omega_2 - \omega_1}{2\pi}$$

The cascaded filter is band pass

$$if\ \omega_1 < \omega_2\ ,\ nothing\ \ passes$$



Example 2

$$H(\omega) = H_1(\omega) \cdot H_2(\omega)$$

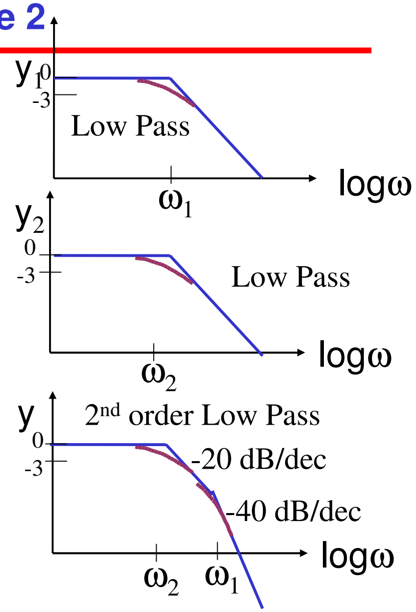
$$H_1(\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_1}} \quad \text{Low pass}$$

$$H_2(\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_2}} \quad \text{Low pass}$$

The cascaded filter is low pass.

$$if\ \omega_1 = \omega_2\ ,\ \ y\ is\ 6dB\ down$$

$$Corresponding\ to\ Q = \frac{1}{2}\ , \zeta = 1$$



Chapter 14

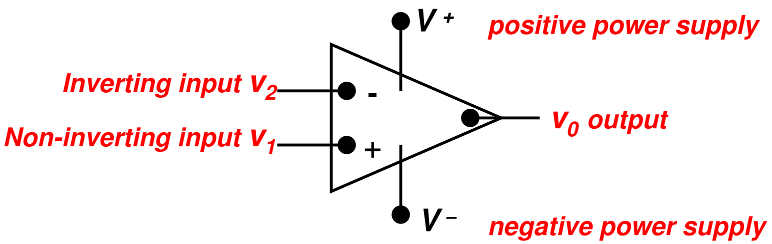
- OUTLINE
  - Op-Amp from 2-Port Blocks
  - Op-Amp Model and its Idealization
  - Negative Feedback for Stability
  - Components around Op-Amp define the Circuit Function
- Reading
  - Chap 14

The Operational Amplifier

- The ***operational amplifier*** (“***op amp***”) is a basic building block used in analog circuits.
  - Its behavior is modeled using a dependent source.
  - When combined with resistors, capacitors, and inductors, it can perform various useful functions:
    - **amplification/scaling** of an input signal
    - **sign changing** (inversion) of an input signal
    - **addition** of multiple input signals
    - **subtraction** of one input signal from another
    - **integration** (over time) of an input signal
    - **differentiation** (with respect to time) of an input signal
    - **analog filtering**
    - **nonlinear functions** like exponential, log, sqrt, etc
  - Isolate input from output; allow cascading

Op Amp Terminals

- 3 signal terminals: 2 inputs and 1 output
- IC op amps have 2 additional terminals for DC power supplies
- Common-mode signal= $(v_1+v_2)/2$
- Differential signal = v_1-v_2



Model for Internal Operation

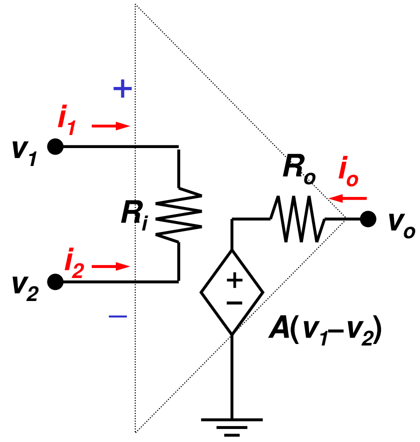
- A is differential gain or open loop gain
- Ideal op amp

$$A \to \infty$$
$$R_i \to \infty$$
$$R_o = 0$$

  – Common mode gain = 0

$$v_{cm} = \frac{(v_1 + v_2)}{2} \quad , v_d = v_1 - v_2$$
$$v_o = A_{cm} v_{cm} + A_d v_d$$
$$Since \quad v_o = A(v_1 - v_2) \quad , A_{cm} = 0$$

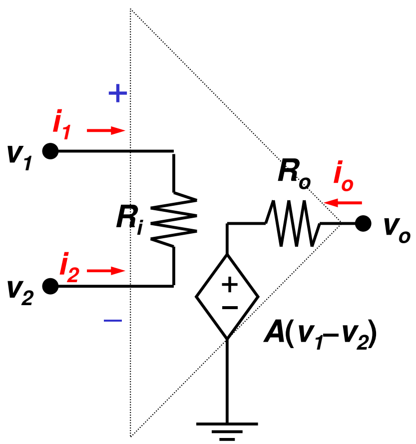
- Circuit Model



Model and Feedback

- Negative feedback
  – connecting the output port to the negative input (port 2)
- Positive feedback
  – connecting the output port to the positive input (port 1)
- Input impedance: R looking into the input terminals
- Output impedance: Impedance in series with the output terminals

- Circuit Model



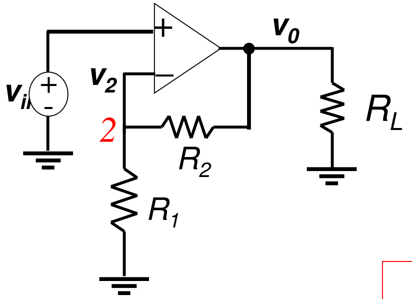
Summing-Point Constraint

- Check if under negative feedback
  – Small v_i result in large v_o
  – Output v_o is connected to the inverting input to reduce v_i
  – Resulting in v_i=0
- Summing-point constraint
  – $v_1 = v_2$
  – $i_1 = i_2 = 0$
- Virtual short circuit
  – Not only voltage drop is 0 (which is short circuit), input current is 0
  – This is different from short circuit, hence called “virtual” short circuit.

Non-Inverting Amplifier

- Ideal voltage amplifier

$$Closed\ \ loop\ \ gain = A_v = \frac{v_o}{v_{in}}$$



$$v_1 = v_2 = v_{in}\ ,i_1 = i_2 = 0$$

$Use\ \ KCL\ \ At\ \ Node\ \ 2.$

$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1}$$

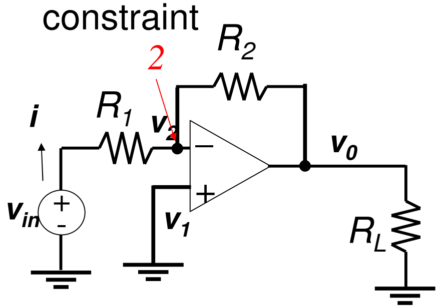
$$Input\ \ impedance = \frac{v_{in}}{i} \rightarrow \infty$$

Ideal voltage source – independent of load resistor

Since V_0 is independent of R_L, Output impedance is 0.

Inverting Amplifier

- Negative feedback → checked
- Use summing-point constraint



$$Closed\ \ loop\ \ gain = A_v = \frac{v_o}{v_{in}}$$

$$v_1 = v_2 = 0\ ,i_1 = i_2 = 0$$

$Use\ \ KCL\ \ At\ \ Node\ \ 2.$

$$i = \frac{(v_{in} - v_2)}{R_1} = \frac{(v_{out} - v_2)}{R_2}$$

$$v_o = -\frac{R_2 v_o}{R_1}$$

$$Input\ \ impedance = \frac{v_{in}}{i} = R_1$$

Ideal voltage source – independent of load resistor

Since V_0 is independent of R_L, Output impedance is 0.

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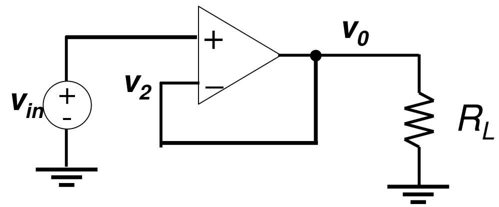
Voltage Follower



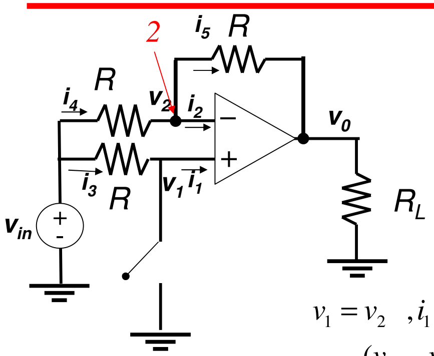
$$R_2 = 0$$

$$R_1 \rightarrow \infty$$

$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1} = 1 + \frac{R_2}{R_1} = 1$$

Example 1



- Switch is open

$$v_1 = v_2 \quad , i_1 = 0 \rightarrow i_3 = 0$$

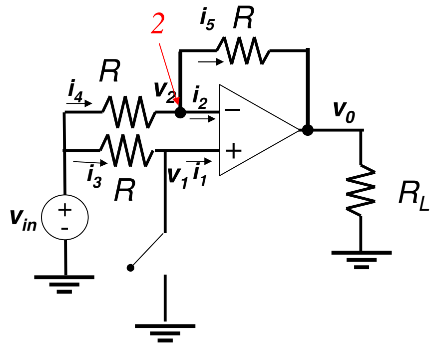
$$i_3 = \frac{(v_{in} - v_1)}{R} \rightarrow v_1 = v_2 = v_{in} \rightarrow i_4 = 0 \rightarrow i_5 = 0$$

$$i_5 = \frac{(v_0 - v_2)}{R} \rightarrow v_0 = v_2 = v_{in}$$

$$A = \frac{v_o}{v_{in}} = 1 \quad , R_{in} \rightarrow \infty$$

Example 1

- Switch is closed



$$v_1 = v_2 = 0 \quad , i_1 = 0 \rightarrow i_3 = 0$$

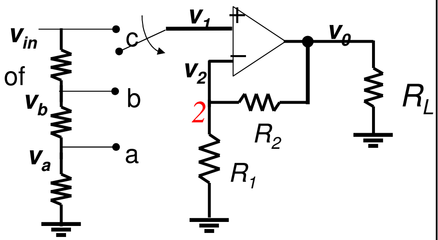
$$i_4 = \frac{(v_{in} - v_2)}{R} = i_5 = -\frac{(v_0 - v_2)}{R}$$

$$v_0 = -v_{in}$$

$$A = \frac{v_o}{v_{in}} = -1 \quad , R_{in} = R/2$$

Example 2

- Design an analog front end circuit to an instrument system
  - Requires to work with 3 full-scale of input signals (by manual switch): $0 \sim \pm 1, 0 \sim \pm 10, 0 \sim \pm 100$ V
  - For each input range, the output needs to be $0 \sim \pm 10$ V
  - The input resistance is 1MΩ



$$v_o = (1 + \frac{R_2}{R_1})v_1$$

$$v_1 = v_{in} \quad Switch \ \ at \ \ c$$

$$v_1 = \frac{R_a + R_b}{R_a + R_b + R_c} v_{in} \quad Switch \ \ at \ \ b$$

$$v_1 = \frac{R_a}{R_a + R_b + R_c} v_{in} \quad Switch \ \ at \ \ a$$

Example 2 (cont'd)

$$R_{in} = R_a + R_b + R_c = 1M\Omega$$

$$Max \ A_v = 10 = (1 + \frac{R_2}{R_1}) \quad Switch \ \ at \ \ c$$

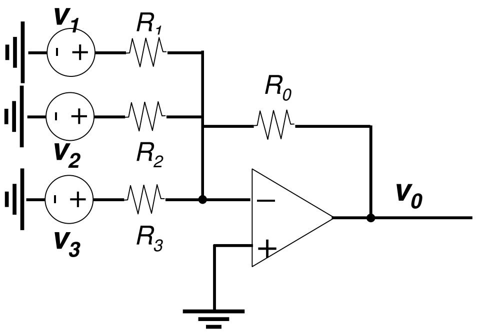
$$A_v = 1 = \frac{R_a + R_b}{R_a + R_b + R_c}(1 + \frac{R_2}{R_1}) \quad Switch \ \ at \ \ b \therefore \frac{R_a + R_b}{R_a + R_b + R_c} = 0.1$$

$$A_v = 0.1 = \frac{R_a}{R_a + R_b + R_c}(1 + \frac{R_2}{R_1}) \quad Switch \ \ at \ \ a \therefore \frac{R_a}{R_a + R_b + R_c} = 0.01$$

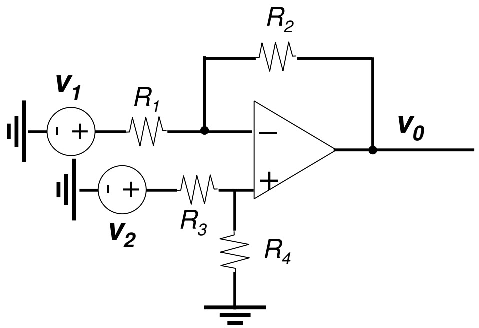
$$\therefore R_a = 10k\Omega, R_b = 90k\Omega, R_c = 900k\Omega$$

$$R_2 = 9R_1$$

Summing Amplifier

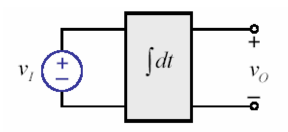


Difference Amplifier

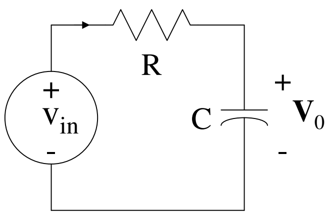


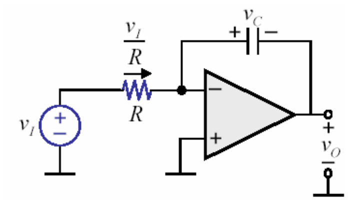
Integrator

- Want $v_o = K\int v_{in}dt$



- What is the difference between:



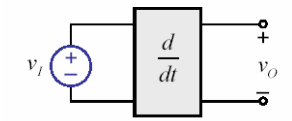


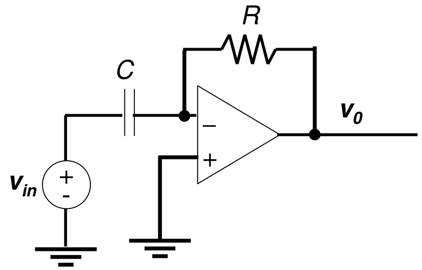
$$v_O \approx \frac{1}{RC}\int_{-\infty}^{t} v_I\, dt$$

$$v_O = -\frac{1}{C}\int_{-\infty}^{t} \frac{v_I}{R} dt$$

Differentiator

- Want





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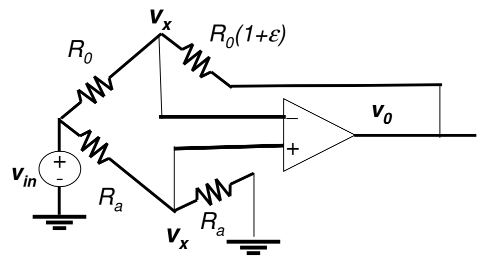
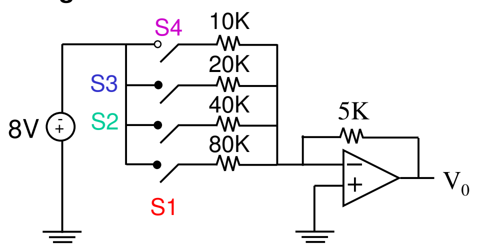
Bridge Amplifier



Application: Digital-to-Analog Conversion

A DAC can be used to convert the digital representation of an audio signal into an analog voltage that is then used to drive speakers -- so that you can hear it!

"Weighted-adder D/A converter"



4-Bit D/A

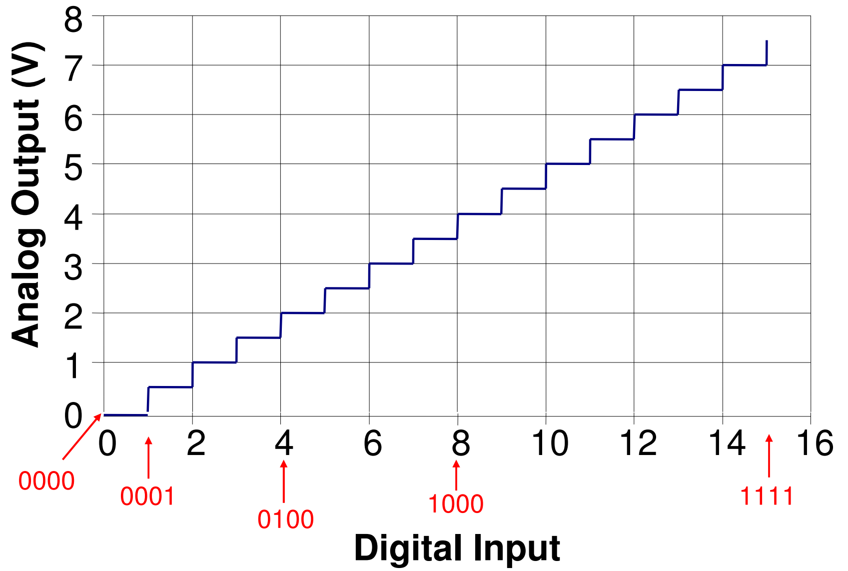
(Transistors are used as electronic switches)

S1 closed if LSB =1
S2 " if next bit = 1
S3 " if " " = 1
S4 " if MSB = 1

Binary number	Analog output (volts)
0 0 0 0	0
0 0 0 1	.5
0 0 1 0	1
0 0 1 1	1.5
0 1 0 0	2
0 1 0 1	2.5
0 1 1 0	3
0 1 1 1	3.5
1 0 0 0	4
1 0 0 1	4.5
1 0 1 0	5
1 0 1 1	5.5
1 1 0 0	6
1 1 0 1	6.5
1 1 1 0	7
1 1 1 1	7.5

MSB LSB

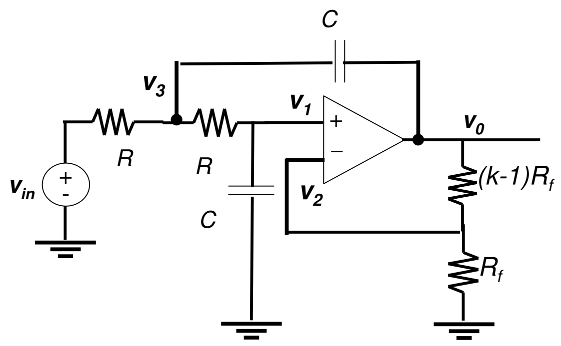
Characteristic of 4-Bit DAC



Active Filter

- Contain few components
- Transfer function that is insensitive to component tolerance
- Easily adjusted
- Require a small spread of components values
- Allow a wide range of useful transfer functions

Active Filter Example



Active Filter Solution

$$v_1 = v_2 = \frac{v_o}{k}$$

$$Use\ \ KCL\ \ At\ \ Node\ \ A \Rightarrow \frac{(v_3 - v_1)}{R} = j\omega C v_1$$

$$Use\ \ KCL\ \ At\ \ Node\ \ B \Rightarrow \frac{(v_{in} - v_3)}{R} = j\omega C(v_3 - v_o) + \frac{(v_3 - v_1)}{R}$$

$$\frac{v_o}{v_{in}} = \frac{k}{1 - \omega^2 R^2 C^2 + j\omega RC(3-k)}$$

$$Let\ \omega_B = 1/RC$$

$$|H(\omega)| = \left|\frac{v_o}{v_{in}}\right| = \frac{k}{\sqrt{\left(1 - \frac{\omega^2}{\omega_B^2}\right)^2 + \frac{\omega^2}{\omega_B^2}(3-k)^2}}$$

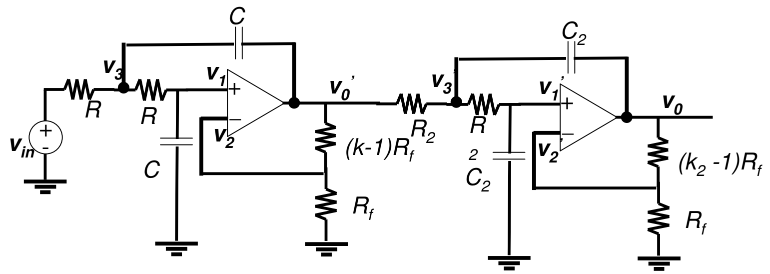
$$\omega = 0, |H(\omega)| = k \quad DC\ \ gain$$

$$\omega = \omega_B, |H(\omega)| = \frac{k}{3-k}$$

$$\omega >> \omega_B, |H(\omega)| = \frac{k}{\left(\frac{\omega^2}{\omega_B^2}\right)} \sim \omega^{-2}$$

$$20\log|H(\omega)|\ \ decays\ at\ a\ rate\ of\ 40dB/decade$$

Cascaded Active Filter Example



Cascaded Active Filter Solution

$$\frac{v_o}{v_{in}} = \frac{k_2}{1-\omega^2 R_2^{\,2} C_2^{\,2} + j\omega R_2 C_2 (3-k_2)} \frac{k}{1-\omega^2 R^2 C^2 + j\omega RC(3-k)}$$

$Let\ \omega_B = 1/RC,\ \omega_{B2} = 1/R_2 C_2$

$$|H(\omega)| = \left|\frac{v_o}{v_{in}}\right| = \frac{k_2}{\sqrt{\left(1-\frac{\omega^2}{\omega_{B2}^{\,2}}\right)^2 + \frac{\omega^2}{\omega_{B2}^{\,2}}(3-k_2)^2}} \frac{k}{\sqrt{\left(1-\frac{\omega^2}{\omega_B^{\,2}}\right)^2 + \frac{\omega^2}{\omega_B^{\,2}}(3-k)^2}}$$

$\omega = 0, |H(\omega)| = k_2 k \quad DC \quad gain$

$$\omega = \omega_B, |H(\omega)| = \frac{k_2}{3-k_2}\frac{k}{3-k}$$

$$\omega >> \omega_B, |H(\omega)| = \frac{k_2 k}{\left(\frac{\omega^4}{\omega_{B2}^{\,2}\omega_B^{\,2}}\right)} \sim \omega^{-4}$$

$20\log|H(\omega)|$ *decays at a rate of* $80 dB / decade$