Administrivia

- Computer accounts and newsgroup stuff: Check the EECS 40 homepage!
- HW will cover concepts from Friday, Monday and Wednesday lecture.
- HW 1 corrections:
  - In problem 1 (a), the phrase should read “calculate the power in the devices” instead of “calculate the power in the interconnection”
  - In problem 8, the phrase should read “In figure 8(a) below, the device” instead of “In figure 8(a) below, the box”
  - In problem 6, it should be “the arm, leg and trunk to rise BY 5 degrees Celsius” instead of “the arm, leg and trunk to rise to 5 degrees Celsius”
  - Problems 2, 3, 6 and 7 - add an extra *

Administrivia (contd).

- TA office hours. (location: 140 Cory)
  - Nir: Tu 12 - 2 and Th 2 - 3
  - Jesse: M 2 - 5
  - Jonathan: Tu 10 - 12 and Th 10 - 11
- Changes to lecture schedule
  - Midterms 1 and 2 moved to Wednesday of the week instead of Monday
  - Lecture on Thevenin/Norton swapped with op-amps
- **WTh 5 - 8 lab cancelled!**
- Guest lecture on 07/11!

Last Time...

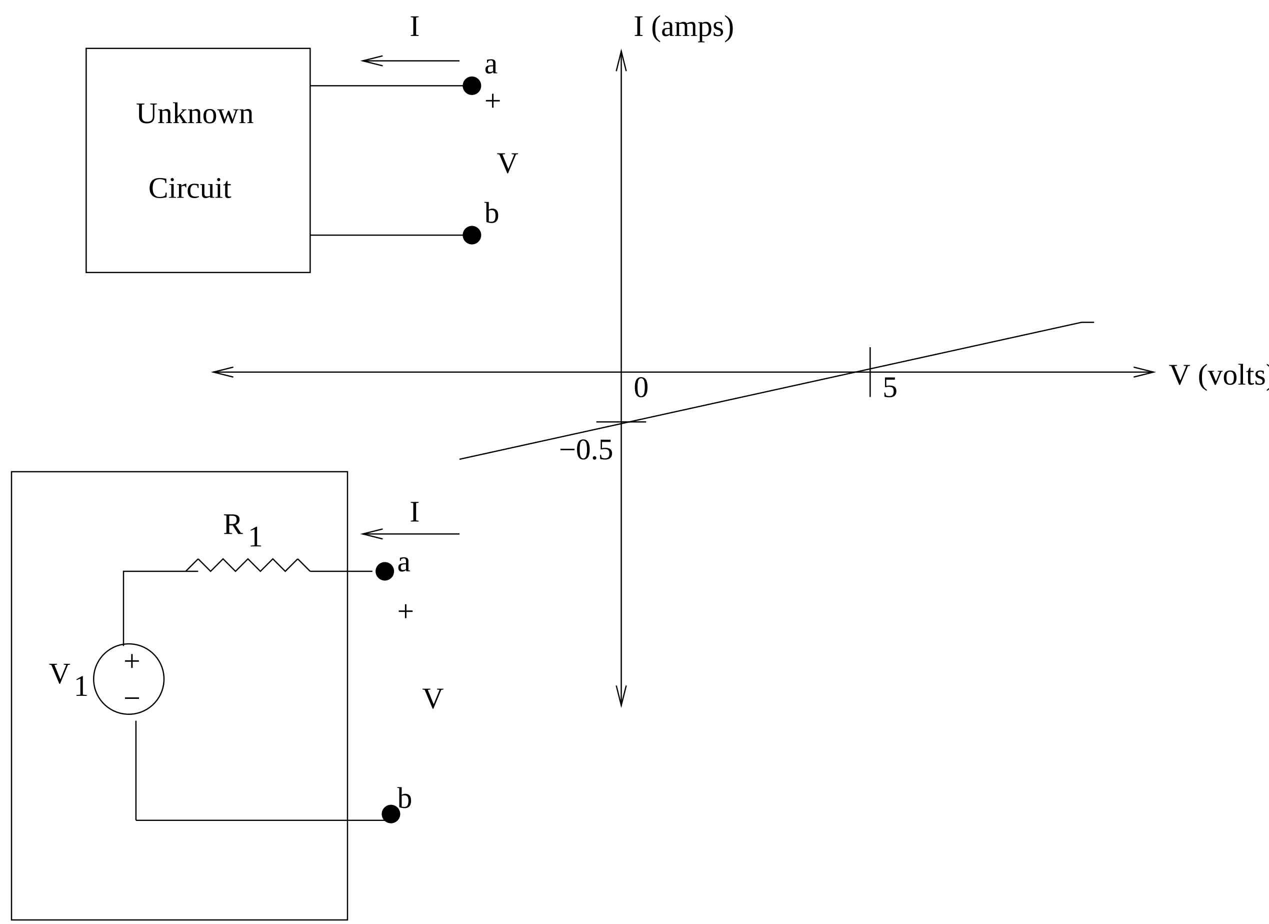
- Circuit analysis tools: KCL, KVL, IV, Series and parallel circuits
- Devices: Dependent sources and resistors
- **Do you have questions on any of these concepts?**

This Time...

- IV example
- Voltage Divider and Current Divider
- Voltmeters and Ammeters
- Nodal analysis: Steps
- Nodal analysis: Illustrate with examples
  - Only independent sources
  - Dependent and independent sources
  - Floating voltage source
- Overview of mesh analysis

IV example

- Give a circuit model (assume a voltage source in series with the resistor) for the I-V graph below:



IV example

- Solution: There are number of ways to solve this circuit. The easiest method is to consider the x-intercept and y-intercept:

  x-intercept: V = 5 V, I = 0. This corresponds to an open-circuit across terminals a and b. Therefore, applying KVL for the circuit model:

  $$-V_1 + V = 0$$

  Notice that R_1 has NO VOLTAGE DROP ACROSS IT, SINCE CURRENT FLOWING THROUGH IT IS ZERO BECAUSE OF THE OPEN CIRCUIT AT TERMINALS ab. Thus, $V_1 = 5$ volts

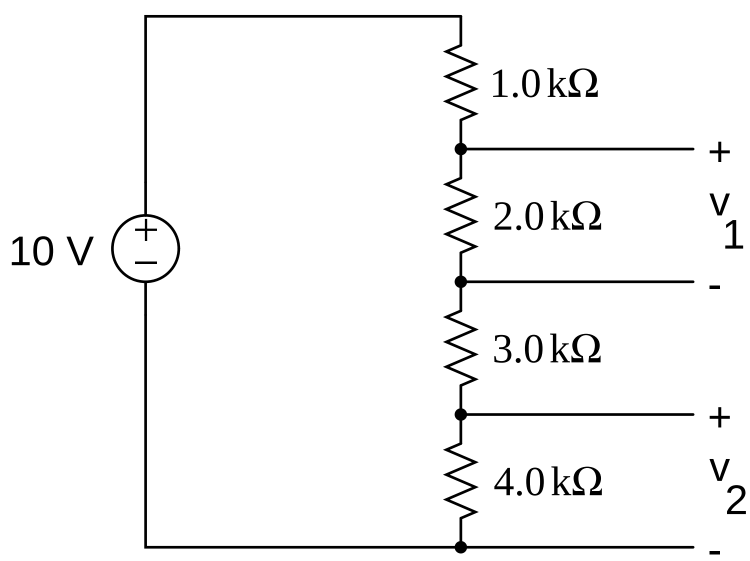
  y-intercept: V = 0, I = -0.5 A. This corresponds to a short-circuit across terminals ab. Therefore, we can apply ohm's law to find R_1 (remember the passive sign convention!):

  $$I = -\frac{V_1}{R_1}$$

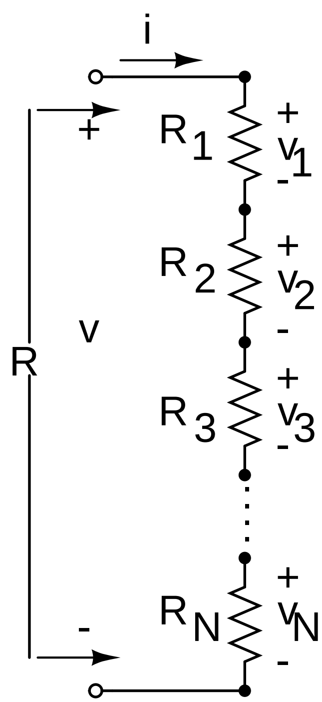
  We know I = -0.5 A and $V_1 = 5$ volts. Therefore, R = 5 Ω

Voltage and Current Divider

- Voltage Divider: Find v_1 and v_2 in the circuit below.



Voltage Divider Concept



- Find v_M (the Mth resistor) in terms of R_1, R_2,...,R_N and v.

Voltage Divider Concept

- Idea is current through elements in series is the same and the equivalent resistance (R) of resistors in series is $R_1 + R_2 + \cdots + R_N$

  $v_M = iR_M$

  $v_M = \frac{V}{R} R_M$

  $v_M = \frac{R_M}{R_1+R_2+\cdots+R_N} V$

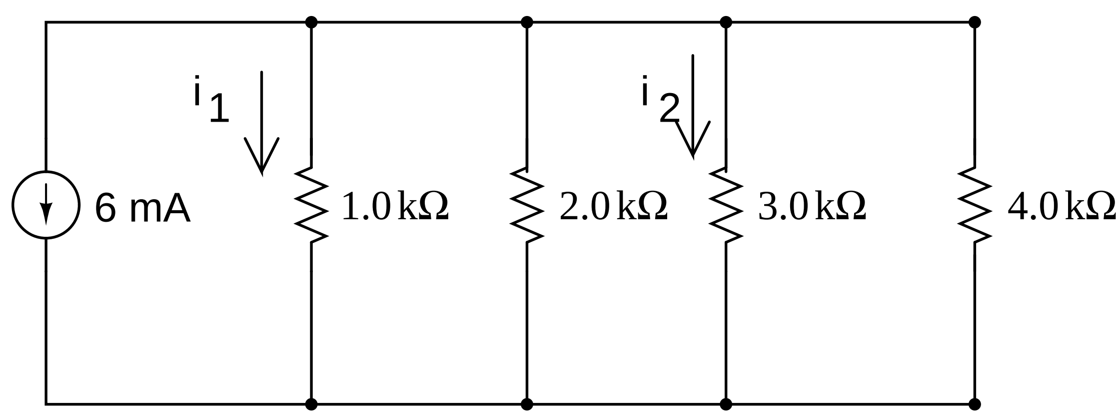
- Thus, the voltage across R_M is:

$$v_M = \frac{R_M}{R_1+R_2+\cdots+R_N} V$$

- Hence, in the circuit given earlier, v_1 = 2 V and v_2 = 4 V.

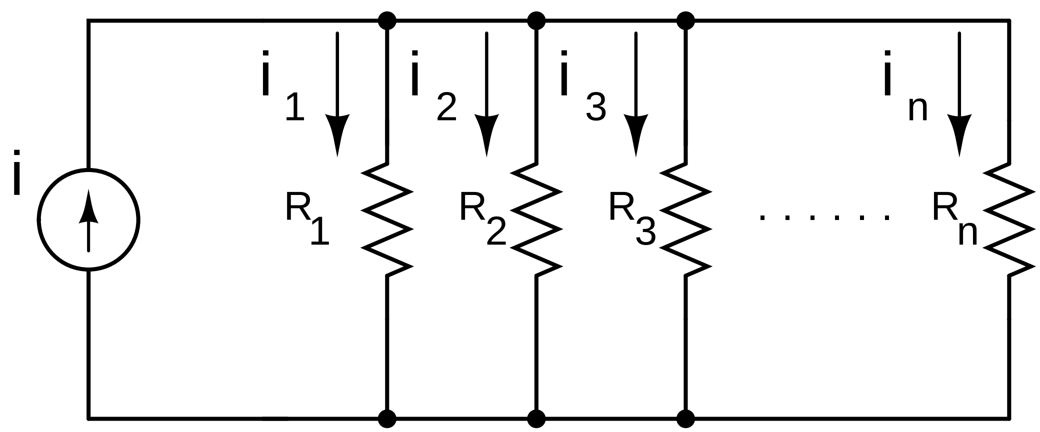
Current Divider

- Current Divider. Find i_1 and i_2 in the circuit below.



Current Divider Concept

- Find i_M in terms of R_1, R_2,...,R_N and i.



Current Divider Concept

- Idea is voltage across elements in parallel is the same and the equivalent resistance (R) of elements is ($R_1 \parallel R_2 \parallel \cdots \parallel R_N$)

  $i_M = \frac{V}{R_M}$

  $i_M = \frac{iR}{R_M}$

  $i_M = \frac{i(R_1 \| R_2 \| \cdots \| R_N)}{R_M}$

- Thus, current throughR_N is:

$$i_M = \frac{i(R_1 \| R_2 \| \cdots \| R_N)}{R_M}$$

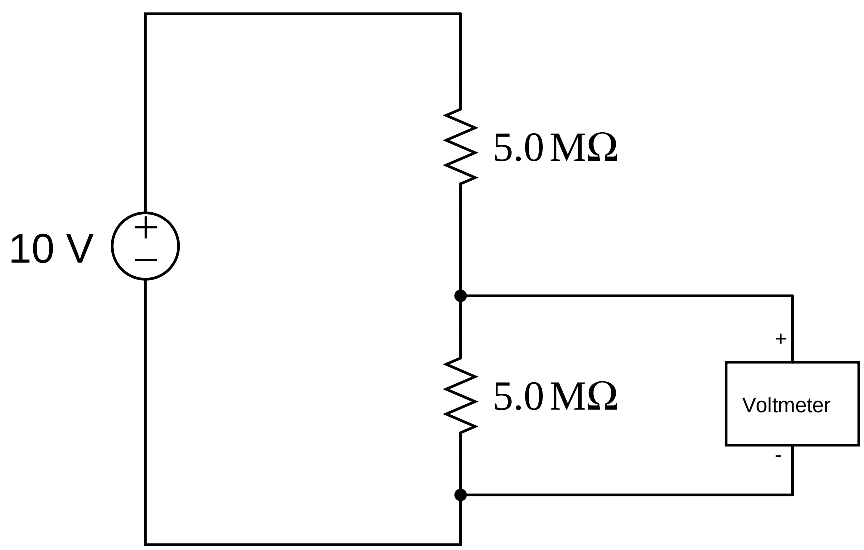
- Hence, in the circuit given earlier, i_1 = 2.88 mA and i_2 = 0.96 mA.

Voltmeters and Ammeters

- Voltmeter: measures voltage across a device.
  - Should connect in parallel with the device of interest
  - Works on the principle of Ohm's law.
  - Therefore, internal resistance of a voltmeter should be VERY HIGH compared to the resistance of the device across which the voltage is to be measured.

Voltmeters and Ammeters

- Example: What voltage does the voltmeter measure? Assume internal resistance is 1 MΩ



Voltmeters and Ammeters

- Solution: First, we combine the 5M and 1M resistors in parallel:

$$\frac{(5M)(1M)}{5M+1M} = \frac{5}{6}\ \text{M}\Omega$$

By using the voltage divider formula:

$$V_{voltmeter} = \left(\frac{\frac{5}{6}}{\frac{5}{6}+5}\right) 10\ \text{V}$$
$$= 1.43\ \text{V}$$

Obviously, this is not even close to the expected value of 5 V.

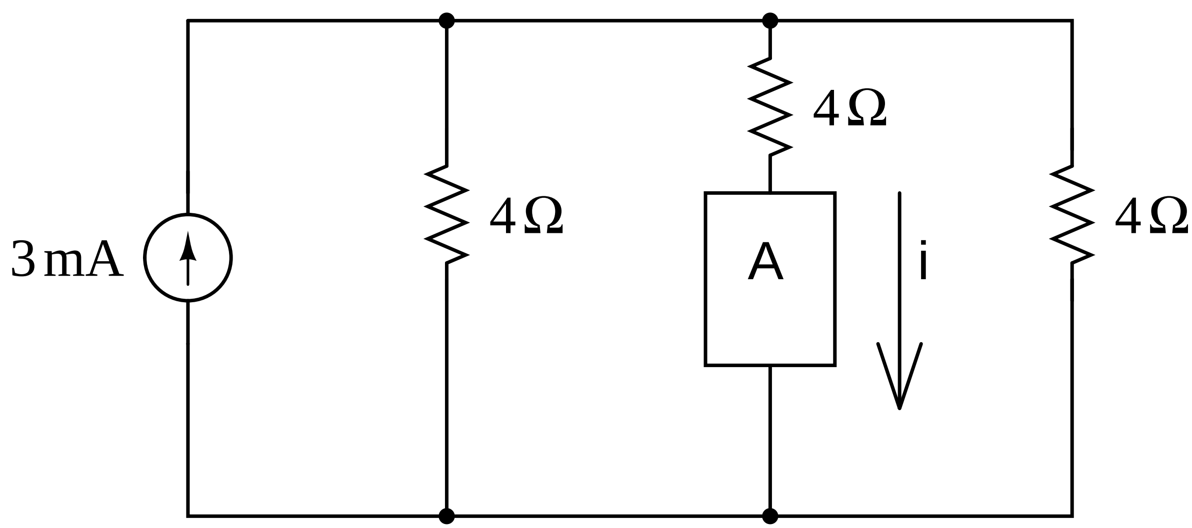
Voltmeters and Ammeters

- Ammeters: measures current through a device.
  - Should connect in series with the device through which the current of interest is flowing.
  - Also works on the principle of Ohm's law.
  - Internal resistance of ammeter is very small. Why: An ammeter works by measuring the voltage across its known internal resistance and then using Ohm's law to calculate the current. If the internal resistance is not small enough, then the equivalent resistance of the circuit would change, which would cause the current flowing in the circuit to vary.

Voltmeters and Ammeters

- Example: What is the current measured by the ammeter?

  Assume internal resistance of the meter is 2 Ωs



Voltmeters and Ammeters

- Solution: Using the current divider rule:

$$i = \left(\frac{4\|4}{4\|4+(4+2)}\right) 1 \text{ mA} = \left(\frac{2}{2+6}\right) 1 \text{ mA} = 0.75 \text{ mA}$$

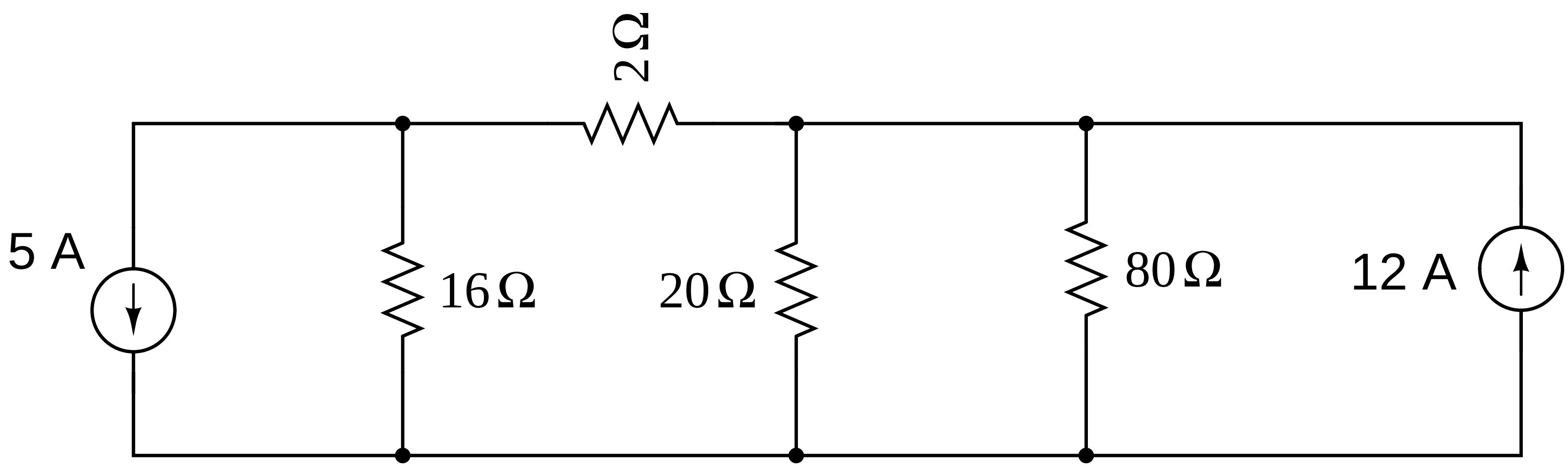
- This is not the expected value of 1 mA. The 2 Ω internal resistance of the ammeter is comparable to the 4 Ω resistance. Hence, the current source is forced to drive (4 $\|$ 6 $\|$ 4) Ω instead of (4 $\|$ 4 $\|$ 4) Ω.

Nodal Analysis: Steps

- An **algorithm** to find **unknown node voltages** in any electric circuit (assumption: circuit has an unique solution).
- The algorithm (a little different from the reader):
  1. Select a reference node.
  2. Label the unknown node voltages.
  3. Apply KCL at each unknown node voltages.
  4. Substitute voltages for currents using device IV relationships (example, use Ohm's law for resistors)
- With circuits that have more than 3 unknown node voltages, we will ask you to set up equations only.

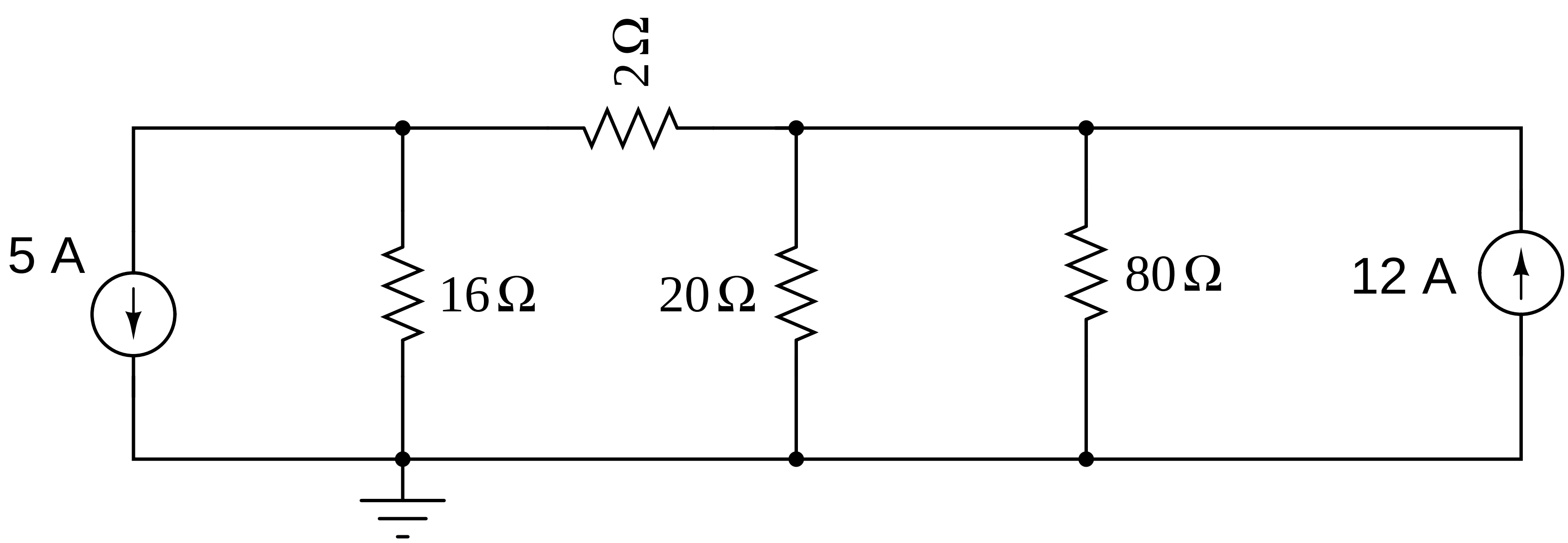
Nodal Analysis: Example 1

- Let us walk through the algorithm for the following circuit (only independent sources, drill exercise 4.5 (a)):



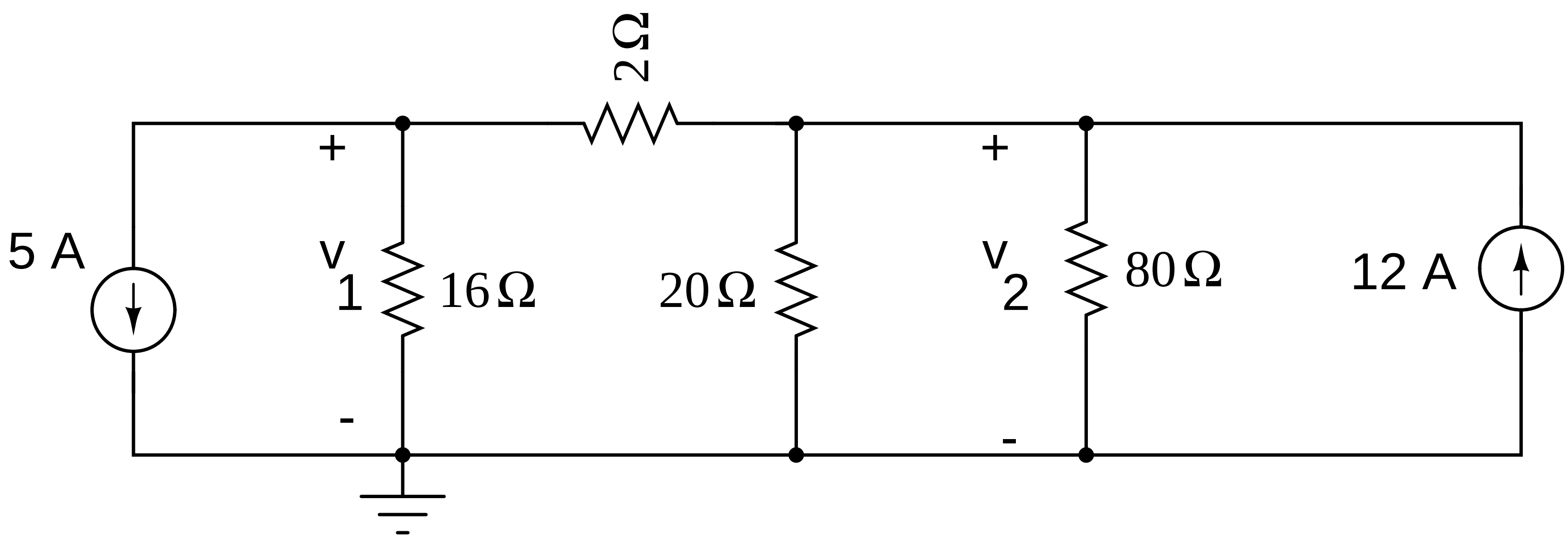
Nodal Analysis: Example 1, Step 1

- Select a reference node



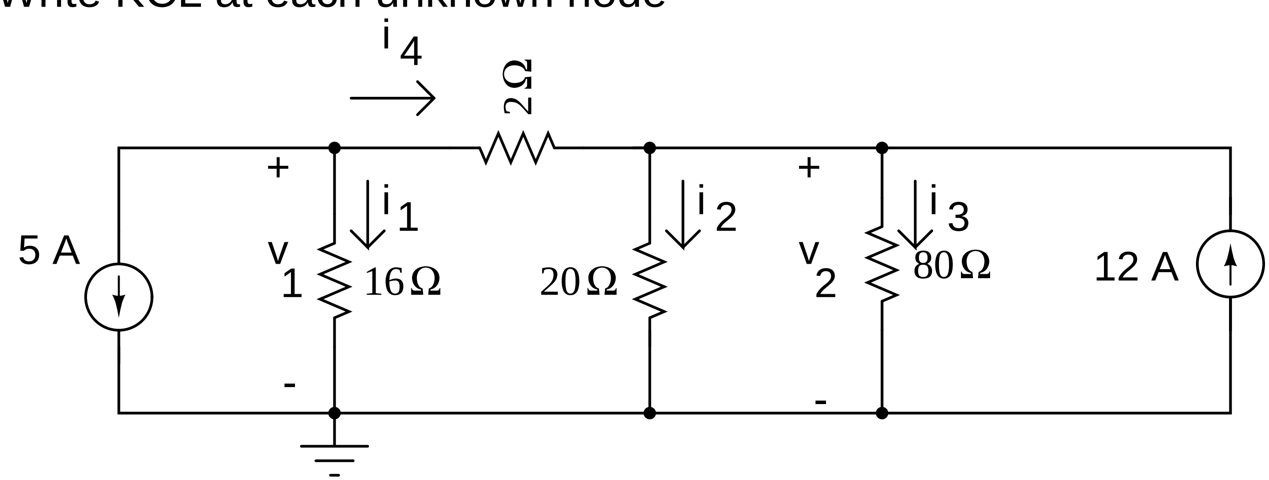
Nodal Analysis: Example 1, Step 2

- Label the unknown node voltages



Nodal Analysis: Example 1, Step 3

- Write KCL at each unknown node



- The KCL equations are (assume current leaving a node is negative):

At v_1: $-5 - i_1 - i_4 = 0$

At v_2: $i_4 + 12 - i_2 - i_3 = 0$

Nodal Analysis: Example 1, Step 4

- Substitute voltages for currents using device IV relationships (example, use Ohm's law for resistors)

  At v_1: $-5 - \frac{v_1}{16} - \frac{v_1 - v_2}{2} = 0$

  At v_2: $\frac{v_1 - v_2}{2} + 12 - \frac{v_2}{20} - \frac{v_2}{80} = 0$

- Solving, we get:

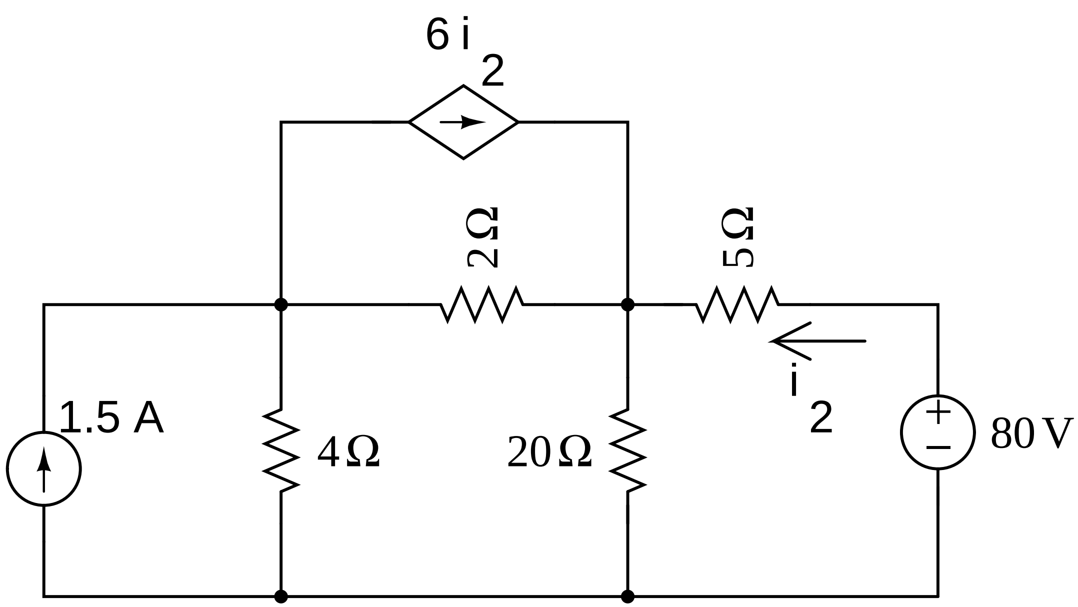
  At v_1: $v_1(\frac{-1}{16} + \frac{-1}{2}) + v_2(\frac{-1}{2}) = 5$

  At v_2: $v_1(\frac{1}{2}) + v_2(\frac{-1}{2} + \frac{-1}{20} + \frac{-1}{80}) = -12$

- Now, we have two equations in two unknowns. They can be solved (you can use matrices or your calculator on the test) to give: v_1 = -15.5 V and v_2 = 7.5 V.

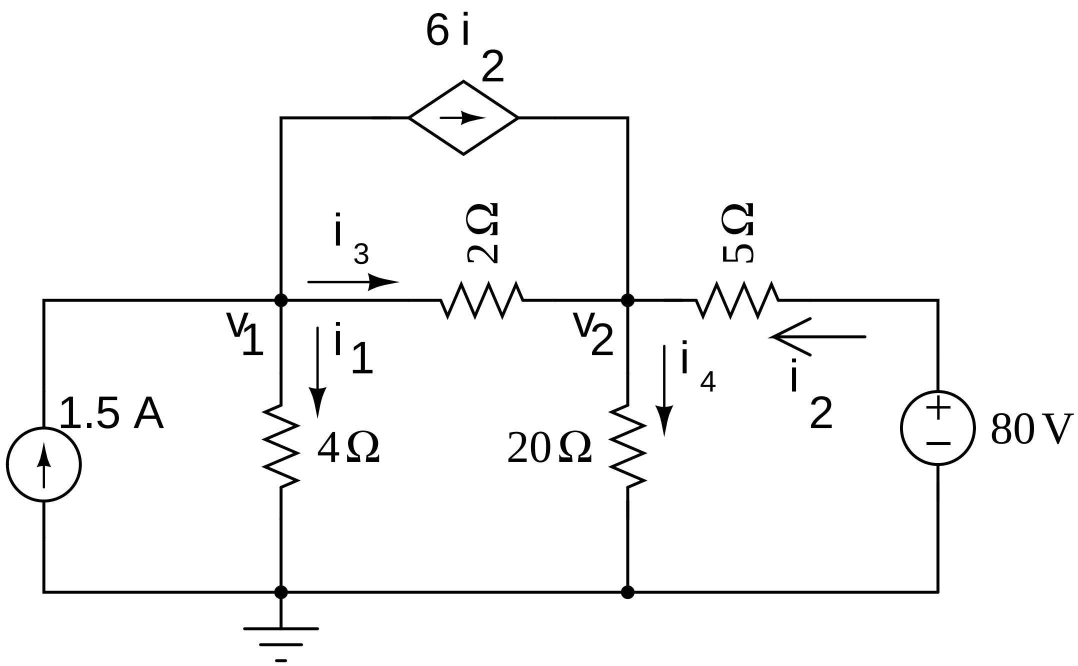
Example 2 - Dependent Sources

- Consider the following example (Drill 4.7):



- Now, you can use constraint equations. You can write the solutions on the next page.

Example 2 - Dependent Sources



- Solution: Select a reference node, label the unknown node voltages and set up the currents to write KCL (let us use the current entering = current leaving form of KCL).

  At v_1: $1.5 = 6i_2 + i_1 + i_3$

  At v_2: $6i_2 + i_3 + i_2 = i_4$

Example 2 - Dependent Sources

- Now, we are ready to use ohm's law and setup the equations to find v_1 and v_2:

  At v_1: $1.5 = 6i_2 + \frac{v_1}{4} + \frac{v_1 - v_2}{2}$

  At v_2: $6i_2 + \frac{v_1 - v_2}{2} + i_2 = \frac{v_2}{20}$

- We have two equations, but three unknowns. But, we know $i_2 = \frac{80 - v_2}{5}$. This is the constraint equation. Substituting for i_2 and rearranging:

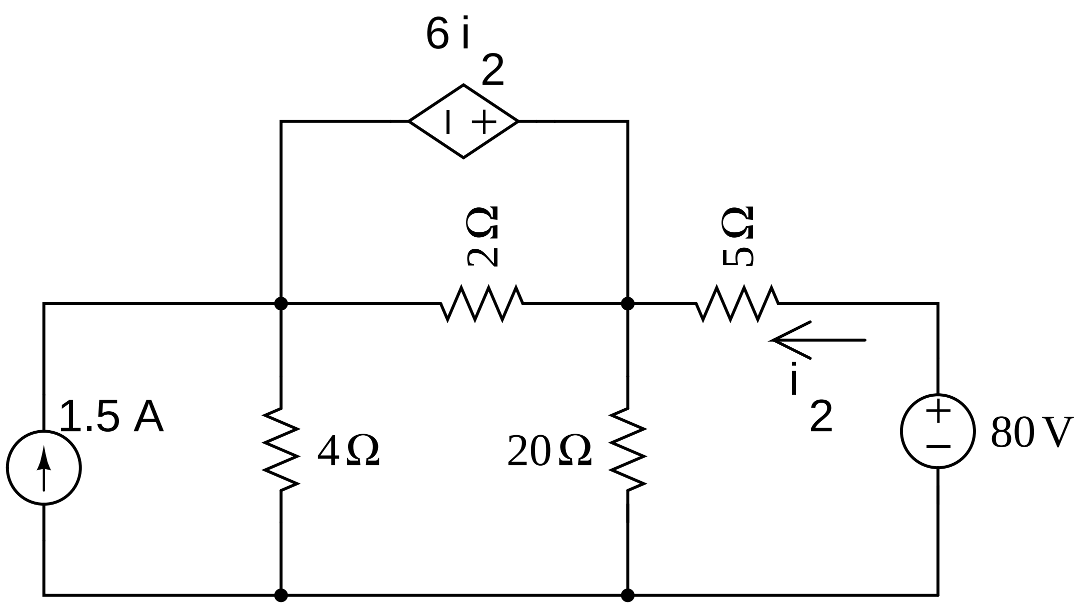
  At v_1: $v_1(\frac{-1}{4} + \frac{-1}{2}) + v_2(\frac{6}{5} + \frac{1}{2}) = 94.5$

  At v_2: $v_1(\frac{-1}{2}) + v_2(\frac{1}{20} + \frac{6}{5} + \frac{1}{2} + \frac{1}{5}) = 112$

- Solving: v_1 = 10 V and v_2 = 60 V.

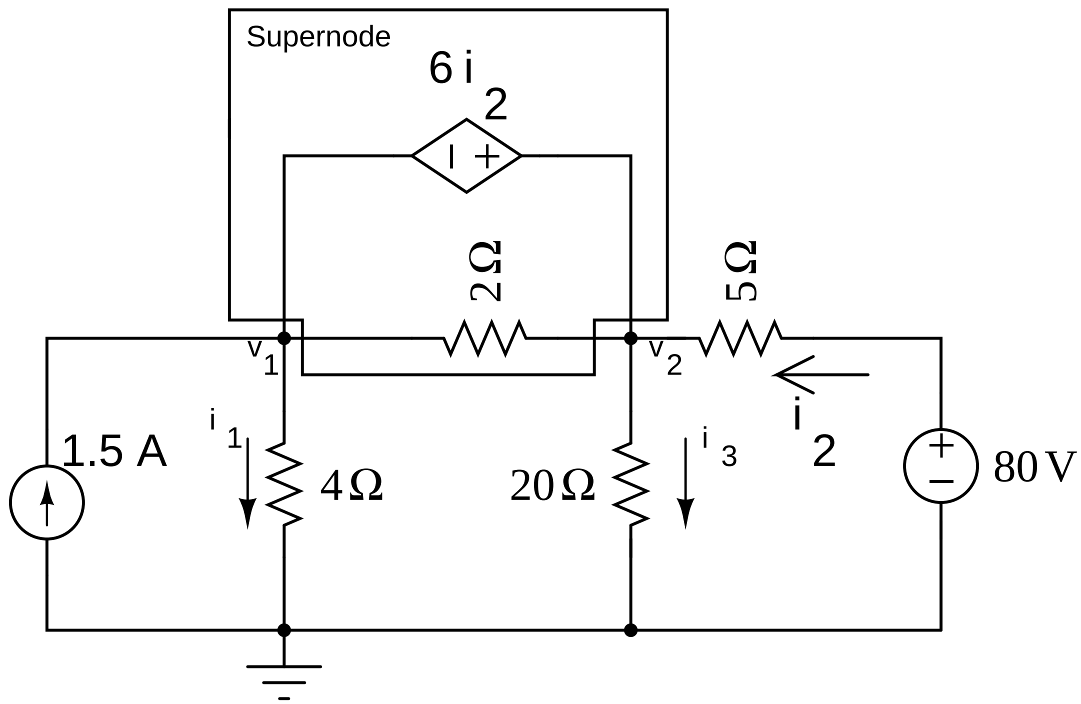
Example 3 - Floating voltage sources

- Floating voltage source: A voltage source (dependent or independent) whose terminals are not connected to ground. For example, consider example 2, but with the dependent current source replaced by a dependent voltage source :



- To solve such circuits, you can again use constraint equations. However, I will introduce a new technique, the supernode.

Example 3 - Floating voltage sources



- Notice how I have enclosed the dependent source with a box labelled "supernode". I am going to apply KCL (current entering = current leaving) to this "node":

  $1.5 + i_2 = i_1 + i_3$

Example 3 - Floating voltage sources

- The other steps are as usual, we use ohm's law to eliminate the currents:

  $1.5 + \frac{80 - v_2}{5} = \frac{v_1}{4} + \frac{v_2}{20}$

- Notice how we still have two unknowns in the above equation. Therefore, we need the constraint equation. The point about supernodes is this: it helps you identify where you need the constraint equations, since you enclose the dependent source in a box and label it a supernode. The constraint equation is:

  $v_1 - v_2 = 6i_2$

  $v_1 - v_2 = 6(\frac{80 - v_2}{5})$

- Solving: v_1 = 89.5 V and v_2 = 32.5 V.

An overview of mesh analysis

- "The KVL analog" of nodal analysis
- The algorithm (refer to reader):
  - Identify unknown loop currents
  - Apply KVL to each mesh (or loop).
  - Substitute currents for voltages using device IV relationships (example, use Ohm's law for resistors)

Nodal versus mesh analysis

- Which is better? Nodal is preferred over mesh.
- Nodal is used more often because:
  - Handles supernodes more conveniently
  - Mesh analysis only works for planar circuits (circuits which can be drawn on a plane with no crossing branches), it does not work for non planar circuits.
    * Proof: Requires mathematical topology. Beyond the scope of this class. If interested, please refer to appendix C in Nilsson and Riedel.
- Look over mesh analysis in the reader.
- Good news: I won't quiz you on mesh analysis. No homework problems as well!

Summary

- We added more tools to our circuit analysis toolbox:
  - Voltage and Current divider techniques
  - Nodal analysis
- We also looked at two instruments: voltmeters and ammeters

In Conclusion...

- Next time: Capacitors, RC circuits, propogation delay, look at inductors
- Reading: will be online at the EECS 40 webpage under "Additional Reader Notes" by Sunday.
- Lab 3 next week should help you understand RC circuits
- Questions?

Have a great weekend!