For the design problems, many solutions are possible!

Problem 1b Work backwards.

V_{out} looks like the "tips" of a triangle wave. Clippers cut off the tips of wave forms. So if I could get a triangle wave input V_{out}', then putting this through a clipper would work.

assuming ideal diode model—adjust voltage sources for V_F ≠ 0.
How to get $V_{out}^1$? Integrate square wave. Suppose I use an integrator with $RC = 1$. Then to get $V_{out}^1$, send through $V_{out}^{II}$:

$V_{out}^1$ increases at a rate of $8 \text{ V/s}$ for $\frac{1}{4}$ s, decreases at $8 \text{ V/s}$ for $\frac{1}{2}$ s, increases for $\frac{1}{2}$ s, etc. Integrator adds a sign.

How to get $V_{out}^{II}$? If we put $-\cos(2\pi t)$ through comparator, that would work.

To get $V_{out}^{III} = -\cos 2\pi t$, use the differentiator (which adds a sign). The $2\pi$ scaling factor that comes out can either be cancelled by setting $RC = \frac{V_{out}^{III}}{2\pi}$ or ignored since it is lost after the comparator.
Put all the circuits together for final design.

**Problem 26.**

\[ V_{out} = \max \{ V_{in1}, V_{in2} \} \]

\[ V_{out} = V_a + 0 \quad \text{if } V_a > V_b \]
\[ V_b + 0 \quad \text{if } V_a \leq V_b \]
Problem 3.\textsuperscript{o} (one of many solutions!)

In Homework 7, we found that the following circuit produces $V_{\text{out}} = V_{\text{in}}^2$: $-\left(\frac{\omega}{2}\frac{C}{L}\ln(C_{\text{ox}})\right)^{-1}$.

Let's now abbreviate the above by $V_{\text{in}}^2$. $V_{\text{out}}$.
Problem 4°: Replace transistor with a small-signal model:

\[ I_{DSAT_{DC}} = \frac{1}{2}(1mA) (3V-1V)^2 = 2mA \]

\[ g_m = \frac{w}{L} \mu N C_{ox} (V_{GS_{DC}} - V_{TH}) = 1mA \frac{1}{V^2} (3V-1V) = 2mA/V \]

Leaving \( \Delta V_{GS} \) in equations so

\[ \frac{1}{\lambda I_{DSAT_{DC}}} = \frac{1}{0.1 \times 2mA} = 5k\Omega \]
Problem 5. In class, we found that for

\[ V_{\text{out}} = -RI_0 \left( e^{\frac{V_{\text{in}}}{V_T}} - 1 \right) \]

So we can pre-multiply \( V_{\text{in}} \) by \( V_T \), then scale by \( \frac{1}{RI_0} \), and then add 1 V.

At room temperature, \( V_T = 0.026 \) V.

\[ V_{\text{in}} = 9.74 \text{ k}\Omega \]

\[ R = \frac{1}{I_0} \]
Problem 6:

- $I_0$ can be extremely small ($10^{-15}$ A). To make $R = V/I_0$, need "exa-ohm" resistance—not possible. Other gain elements with $10^{15}$ gain also impractical.

- $V_T$ changes with temperature, proportionally. Changes in temperature that normally occur in circuits would throw off results.

Other issues may also exist.

Problem 7:

$V_{GS} = 3V - 2kT_D$  
$12V = 1kT_D + V_{DS} + 2kT_D$

Hope for saturation!

$I_D = \frac{1}{2} \frac{W}{L} m n C_D (V_{GS} - V_{TH})^2$

$= 100 \text{mA} \sqrt{2} (3V - 2kT_D - 1V)^2$  

$solve: \quad I_D = \{ 9.4 \text{mA}, 114 \text{mA} \}$
Problem 8: If MN increases due to heat, then $I_D$ will increase. Then the voltage over the 2kΩ resistor will increase, lowering $V_{ES}$. This counteracts the increase in $I_D$ and results in a current supply which varies less due to temp changes.

Problem 9

$t = 0:\ V_{out} = 1\ V$

\[ + \frac{145\text{pF}}{10\text{V}} \quad 20\text{kΩ} \]

Stays until $V_{C2} = 5\ V$.

It takes $0.69\ \tau$ to discharge halfway.

$\tau = 0.691\cdot \frac{145\text{pF}}{20\text{kΩ}} = 2\text{µs}$

Switch flips.

$t = 2\text{µs} \ V_{out} = -1\ V$

\[ + \frac{V_{C1}}{145\text{pF}} \quad 20\text{kΩ} \]

$C_1$ has been charging for 2µs in circuit with 10V & 10kΩ.

\[ V_{C1}(t=2\text{µs}) = 10\text{V}(1-e^{-\frac{t}{10\text{kΩ}\cdot 145\text{pF}}}) \]

$= 7.5\ V$

How long to discharge to 5V?

$5\ V = 7.5\ e^{-\frac{t}{20\text{kΩ}\cdot 145\text{pF}}}$

$\Delta t = 1.176\ \text{µs}$

Switch flips at $t = 3.176\ \text{µs}$.

$t = 3.176\ \text{µs} \ V_{out} = 1\ V$

$C_2$ has been charging, starting at 5V, for 1.176 µs.

\[ V_{C2}(t=1.176\ \text{µs}) = 10(1-e^{-\frac{(t-2\text{µs})}{10\text{kΩ}\cdot 145\text{pF}}}) + 5\ e^{-\frac{(t-2\text{µs})}{10\text{kΩ}\cdot 145\text{pF}}} \]

$= 7.77\ V$

How long to discharge to 5V?

$\Delta t = (t-3.176\ \text{µs}) = 1.28\ \text{µs}$

Switch flips at 4.45µs $V_{out} = -1\ V$

Ok, we get the idea.

How about a general formula?
Suppose we know $\Delta t_k$, the time between the last two switches. Let's find $\Delta t_{k+1}$, the time to the next switch.

The capacitor of interest charged for $\Delta t_k$ in the charging circuit. It charged to:

$$V_{C_0} = 10(1 - e^{-\frac{\Delta t_k}{T_c}}) + 5 e^{-\frac{\Delta t_k}{T_c}}$$

where $T_c = 100 \text{ k}\Omega \cdot 145 \text{ pF} = 1.45 \text{ ms}$

Rewrite more simply as

$$V_{C_0} = 10 - 5 e^{-\frac{\Delta t_k}{1.45 \text{ ms}}}$$

How long to discharge capacitor down to 5V? $\Delta t_{k+1}$.

$$5 \text{ V} = V_{C_0} e^{-\frac{\Delta t_{k+1}}{T_d}}$$

where $T_d = 20 \text{ k}\Omega \cdot 145 \text{ pF} = 2.9 \text{ ms}$

Solve:

$$\Delta t_{k+1} = -2.9 \text{ ms} \cdot \ln \left( \frac{5 \text{ V}}{V_{C_0}} \right)$$

$$= -2.9 \text{ ms} \cdot \ln \left( \frac{5 \text{ V}}{10 - 5 e^{-\frac{\Delta t_k}{1.45 \text{ ms}}}} \right)$$

This is our update formula.
The time between switches reaches a limit of about 1.4 microseconds.

So the output keeps switching back and forth between 1 V and -1 V: an oscillator!

The capacitor voltage, when fully charged, reaches a limit of about 8.09 V.
Problem 10°

a) \[ \begin{array}{ccccccc}
1 & 4 & 2 & 5 & 3 \\
\end{array} \]

Fund. cut sets:

Cutting d separates 1 from 4, 2, 5, 3.
To fully separate: cut \( \text{Ed}, b, a \).

Cutting e separates 1, 4, 3 from 2, 5, 3.
To fully separate: cut \( \text{El}, a, b, h \).

Cutting f separates 1, 4, 2, 3 from 5, 3.
To fully separate: cut \( \text{Ef}, h, c, a \).

Cutting g separates 1, 4, 2, 3, 5 from 3, 3.
To fully separate: cut \( \text{Ga}, c, g \).

b) Your answers will vary depending on the reference node; replace the appropriate node voltage with zero.

Set \( \text{Ed}, b, a \): current through d unknown (voltage source) but it tells us \( V_1 - V_4 = 10 \).

Set \( \text{El}, a, b, h \): \( 2A + \frac{V_1 - V_2}{2} + \frac{V_4 - V_2}{2} + \frac{V_4 - V_5}{10} = 0 \)

\[ \begin{array}{cccc}
\text{a} & \text{b} & \text{e} & \text{h} \\
\end{array} \]
Set \( f, h, c, a \): 
\[
\frac{v_2 - v_5}{20} + \frac{v_4 - v_5}{10} + \frac{v_2 - v_3}{4} + 2A = 0
\]

Set \( a, c, g \): 
\[
2A + \frac{v_2 - v_3}{4} + \frac{v_5 - v_3}{5} = 0
\]