Problem 1:

\[ V_{GS} = 3\, \text{V} \implies \text{transistor is not cutoff} \quad (V_{GS} > V_{THN} = 1\, \text{V}) \]

Assume saturation (since it is easiest).

\[ I_D = I_{DSAT} = \frac{1}{2} \frac{W}{L} \mu A \text{Cox} (V_{GS} - V_{THN})^2 \]
\[ = \frac{1}{2} \left( 1 \text{mA/V}^2 \right) (3\, \text{V} - 1\, \text{V})^2 \]
\[ = 2\, \text{mA} \]

\[ V_{DS} = 12\, \text{V} - 2k\Omega \cdot I_D \quad (\text{KVL around outside loop}) \]
\[ = 12\, \text{V} - 2k\Omega \cdot 2\, \text{mA} = 8\, \text{V} \]

Check that these answers are valid for saturation:

Saturation requires \( V_{DS} \geq V_{GS} - V_{THN} \)

Indeed \( 8\, \text{V} \geq 3\, \text{V} - 1\, \text{V} \) so answers are correct.
Problem 2:

Since no current goes into gate, \( R_1 + R_2 \) have same current.

\[ V_{GS} = \frac{R_2}{R_1 + R_2} \cdot 12 \text{ V} \]

by voltage division.

Saturation current should be 8mA.

\[ 8 \text{ mA} = I_{DSAT} = \frac{12}{L} \mu A/cm^2 \cdot (V_{GS} - V_{TH})^2 \]

\[ = \frac{1}{2} \cdot 1 \text{ mA/cm}^2 \cdot (V_{GS} - 1 \text{ V})^2 \]

\[ (V_{GS} - 1 \text{ V})^2 = 16 \text{ V}^2 \implies V_{GS} = 5 \text{ V} \]

\[ 5 \text{ V} = V_{GS} = \frac{R_2}{R_1 + R_2} \cdot 12 \text{ V} \implies \frac{R_2}{R_1 + R_2} = \frac{5}{12} \]

Many solutions possible, for example, \( R_2 = 5 \text{ k}\Omega \), \( R_1 = 7 \text{ k}\Omega \).
Problem 3°.

When \( V_{IN} = V_{OUT} = 0 \),
\( V_{GS_P} = V_{DS_P} = -3 \text{ V} \)
\( V_{GS_N} = V_{DS_N} = 2 \text{ V} \)
Both transistors in saturation.

\[
I_D(N) = I_{DSAT}(N) \quad \quad I_D(P) = I_{DSAT}(P)
\]

By KCL, \( I_D(P) + I_D(N) = 0 \) \implies I_D(N) = -I_D(P)

\[
I_{DSAT}(N) = \frac{1}{2} \frac{W_N}{L_N} \mu_n C_0 (V_{GS_N} - V_{TH_N})^2
= \frac{1}{2} \cdot \frac{1}{2.5 \mu m} \cdot W_N \cdot 50000 \text{ mm}^2 \cdot \frac{5 \text{ fF}}{\text{mm}^2} (2 \text{ V} - 1 \text{ V})^2
= 50 W_N
\]

\[
I_{DSAT}(P) = -\frac{1}{2} \frac{W_P}{L_P} \mu_p C_0 (V_{GS_P} - V_{TH_P})^2
= -\frac{1}{2} \cdot \frac{1}{2.5 \mu m} \cdot W_P \cdot 25000 \text{ mm}^2 \cdot \frac{5 \text{ fF}}{\text{mm}^2} (3 \text{ V} - 1 \text{ V})^2
= -100 W_P
\]

\[I_D(N) = -I_D(P) \implies I_{DSAT_N} = -I_{DSAT_P} \implies 50 W_N = 100 W_P\]

\[
\frac{W_N}{W_P} = 2
\]
Possible answers: \( W_N = 2 \mu m \), \( W_P = 1 \mu m \)
Problem 4.

Vin = 3.5 V

VGS(N) = 3.5 V

VGS(P) = -1.5 V

Looks like "Region D"

(NMOS fully on; PMOS barely on)

\[ I_{DP} = I_{DSATP} = -\frac{1}{2} \frac{W}{L} \mu_p C_0x (V_{GSP} - V_{THP})^2 \]

\[ = -V_2 \cdot 1 \text{mA/V}^2 (-1.5V - (-1V))^2 \]

\[ I_{DP} = -1.25 \text{mA} \]

\[ I_{DN} = \frac{W}{L} \mu_N C_0x (V_{GSN} - V_{THN} - \frac{V_{DSN}}{2}) V_{DSN} \]

\[ = 1 \text{mA/V}^2 (3.5V - 1V - \frac{V_{DSN}}{2}) V_{DSN} \]

By KCL, \[ I_{DN} = -I_{DP} \]

\[ -I_{DP} = 125 \mu A = I_{DN} = 1 \text{mA/V}^2 (2.5V - \frac{V_{DSN}}{2}) V_{DSN} \]

Solutions: \[ V_{DSN} = 0, 0.05 V, 4.95 V \]

4.95 V impossible for triode; \( 4.95V \leq V_{GSN} - V_{THN} = 2.5V \)

\[ V_{DSN} = 0.05 V \]

\[ V_{DSP} = V_{DSN} - V_{DD} = -4.95 V \]
Problem 5°

NMOS cut off! \(V_{GSN} = 0\, V < V_{THN}\)

PMOS "fully on" \(V_{GSP} = -5\, V\)

PMOS triode happens to be the right guess, but it's hard to guess this.

First I will show that PMOS triode is correct. Then I will show why PMOS saturation is wrong.

PMOS triode. (Also guess diode forward biased)

\[
I_{DP} = -\frac{W}{L} \mu p \, C_{ox} \left( V_{GSP} - V_{THP} - \frac{V_{DSP}}{2} \right) V_{DSP}
\]

\[
= -1mA/\mu m (-5\, V - -1\, V - \frac{V_{DSP}}{2}) V_{DSP}
\]

\[
I_{DP} + I_{DN} + I_{OUT} = 0 \quad \text{by KCL}
\]

\[
I_{DP} = -I_{OUT} \quad I_{OUT} = \frac{V_{DSN} - 0.7}{1k}
\]
\[ V_{DSN} = V_{DD} + V_{DSP} = 5\, V + V_{DSP} \]

Plug equations 1, 2, and 3 into \( I_{DP} \) triode eqn -

\[ \frac{5V + V_{DSP} - 0.7V}{1\, k\Omega} = -1mA/\sqrt{2} \left( -4V - \frac{V_{DSP}}{2} \right) V_{DSP} \]

Solutions: \( V_{DSP} = 3 - 9.05V, -0.95V \)

\( V_{DSP} = -9.05V < V_{GSP} - V_{THP} = -4V \)

so this is impossible for triode mode.

\( V_{DSP} = -0.95V \) is possible for triode mode.

\[ I_{DP} = -I_{OUT} = - \frac{5V + V_{DSP} - 0.7V}{1\, k\Omega} = -3.35\, mA \]

Power absorbed by PMOS transistor:

\[ P_{PLOS} = I_{DP} \cdot V_{DSN} = (-3.35\, mA)(0.95V) = 3.18\, mW \]

Power absorbed by NMOS transistor:

\[ P_{NMOS} = I_{DN} \cdot V_{PSN} = 0 \, W \quad (I_{DN} = 0\, A \text{ since NMOS cutoff}) \]

Power absorbed by resistor:

\[ P_R = I^2R = (3.35\, mA)^2 \cdot 1k\Omega = 11.02\, mW \]

Power absorbed by diode:

\[ P_D = VI = 0.7\, V \cdot 3.35\, mA = 2.34\, mW \]
Now to show PMOS saturation is wrong:

If PMOS saturation,

\[ I_{DP} = -\frac{1}{2} \frac{W}{L} \mu_p C_OX (V_{GPS} - V_{THP})^2 \]

\[ = -\frac{1}{2} \text{mA} \sqrt{2}(5V - -1V)^2 = 8\text{mA} \]

Thus current must flow in the branch with the resistor + diode (since NMOS cutoff \( I_{DN} = 0\text{A} \)). So diode is forward biased. (Voltage 0.7V.)

Resistor voltage is \( 8\text{mA} \cdot 1\text{k} = 8\text{V} \).

\[ V_{DSP} = 8\text{V} + 0.7\text{V} - 5\text{V} = 3.7\text{V} \]

which is impossible for saturation;

\[ V_{DSP} \text{ must be less than } V_{GPS} - V_{THP} = -4\text{V} \]