Today we will

- Put a twist on our normally linear operational amplifier circuits to make them perform nonlinear computations
- Make a linear circuit model for the nonlinear NMOS transistor (Preview of EE 105)

Next time we will

- Show how we can design a pipelined computer datapath at the transistor level
- Use a relay to design an analog circuit that counts

Trying to expose you to various complicated circuits/topics to use the tools you've developed and prepare you for final exam...

NONLINEAR OPERATIONAL AMPLIFIERS

When I put a nonlinear device in an operational amplifier circuit, I can compute a nonlinear function.

Consider the following circuit using the realistic diode model:
NONLINEAR OPERATIONAL AMPLIFIERS

What if I switch the positions of the resistor and the diode (and make sure $V_{IN} \geq 0$ V)?

Changing the position of the elements inverted the function performed!
This circuit acts like a constant current source, as long as the transistor remains in saturation mode. But this hides the fact that \( I_{\text{DSAT}} \) depends on \( V_{\text{GS}} \); it is really a *voltage-dependent current source*!

If \( V_{\text{GS}} \) is not constant, the model fails. What if \( V_{\text{GS}} \) changes? What if there is noise in the circuit?

If \( V_{\text{GS}} \) changes a little bit, so does \( I_D \).
THE SMALL-SIGNAL MODEL

Let's include the effect of noise in $V_{GS}$. Suppose we have tried to set $V_{GS}$ to some value $V_{GS,DC}$ with a fixed voltage source, but some noise $\Delta V_{GS}$ gets added in. $V_{GS} = V_{GS,DC} + \Delta V_{GS}$

$$I_{DSAT,DC} = f(V_{GS,DC})$$

We get the predicted $I_{DSAT,DC}$ plus a change due to noise, $\Delta I_{DSAT}$.

No current flows into or out of the gate because of the opening.

To be even more accurate, we could add in the effect of $\lambda$.

When $\lambda$ is nonzero, $I_D$ increases linearly with $V_{DS}$ in saturation.

We can model this with a resistor from drain to source:

$$\Delta I_{DSAT,DC} = g(\Delta V_{GS})$$

The resistor will make more current flow from drain to source as $V_{DS}$ increases.
THE SMALL-SIGNAL MODEL

How do we find the values for the model?

\[ I_{\text{DSAT,DC}} = \frac{1}{2} \frac{W}{L} \mu N \cdot C_{\text{OX}} \cdot (V_{\text{GS,DC}} - V_{\text{TH}})^2 \]

This is a constant depending on \( V_{\text{GS,DC}} \).

\[ \Delta I_{\text{DSAT}} \approx \frac{\partial I_{\text{DSAT}}}{\partial V_{\text{GS}}} (V_{\text{GS,DC}}) \Delta V_{\text{GS}} \]

This is a first-order Taylor series approximation which works out to

\[ \Delta I_{\text{DSAT}} = \frac{W}{L} \mu N \cdot C_{\text{OX}} \cdot (V_{\text{GS,DC}} - V_{\text{TH}}) \Delta V_{\text{GS}} \]

We often refer to \( \frac{W}{L} \mu N \cdot C_{\text{OX}} \cdot (V_{\text{GS,DC}} - V_{\text{TH}}) \) as \( g_m \), so

\[ \Delta I_{\text{DSAT}} = g_m \Delta V_{\text{GS}}. \]

Including the effect of \( \lambda \) via \( r_o \), the added current contributed by the resistor is

\[ I_{r_o} = \frac{1}{2} \frac{W}{L} \mu N \cdot C_{\text{OX}} \cdot (V_{\text{GS}} - V_{\text{TH}})^2 \lambda V_{DS} \]

To make things much easier, since the \( \lambda \) effect is small anyway, we neglect the effect of \( \Delta V_{GS} \) in the resistance, so the current is

\[ I_{r_o} \approx \frac{1}{2} \frac{W}{L} \mu N \cdot C_{\text{OX}} \cdot (V_{\text{GS,DC}} - V_{\text{TH}})^2 \lambda V_{DS} = I_{\text{DSAT,DC}} \lambda V_{DS} \]

This leads to

\[ r_0 = \frac{V_{DS}}{I_{r_o}} = (\lambda I_{\text{DSAT,DC}})^{-1} \]
EXAMPLE

Revisit the example of Lecture 20, but now the 3 V source can have noise up to ± 0.1 V.

Find the range of variation for \( I_D \).

\[ V_{TH(N)} = 1 \text{ V}, \]
\[ W/L \mu_n C_{OX} = 500 \mu \text{ A/V}^2, \]
\[ \lambda = 0 \text{ V}^{-1}. \]

We figured out that saturation is the correct mode in Lecture 20.

The “noiseless” value of \( I_D \) is

\[ I_D = \frac{V_G - V_S}{R_{DS}} \]

The variation in \( I_D \) due to noise:

\[ I_D \text{ could vary between:} \]

Will saturation mode be maintained?