**Lecture #12**

**Announcement**
- Midterm 1 on Tues. 3/2/04, 9:30-11
- A-M last initials in 10 Evans
- N-Z initials in Sibley auditorium
- Closed book, no electronic devices
- One sheet 8.5x11 inch of your notes
- Covers material through op-amps, i.e. hw #1-4

**OUTLINE**
- RC,RL review
- Propagation delay
- Energy consumption of simple RC circuit
- Circuit transient response examples
- Midterm questions?

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**EXEMPLARY**

Solution: $V_a$, $V_b$ = ?

To find $R_{eff}$, short voltage sources and find $R$ at terminals of $L$:

So $\tau = L/R$ =

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**EXAMPLE con’t**

For $t < t_1$, $v(t)$=0, $d/dt = 0$ so inductor voltage is zero (like a wire):

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**TRANSIENTS IN SINGLE-INDUCTOR OR SINGLE-CAPACITOR CIRCUITS - THE EASY WAY**

1) Find Resistance seen from terminals of $L$ or $C$ (short voltage sources, open current sources).

2) The circuit time constant is $L/R$ or $RC$ (for every node, every current, every voltage).

3) Use initial conditions and inductor/capacitor rules to find initial values of all transient variables. (Capacitor voltage and inductor current must be continuous.)

4) Find $t=\infty$ value of all variables by setting all time derivatives to zero.

5) Sketch the time-behavior of all transient variables, based on initial and final values and known time constant.

6) Write the equation for each transient variable by inspection.
EXAMPLE con’t

Find initial values

For \( t = t_1^+ \), \( v(t) = 2V_{SS} \), \( i = V_{SS} / 4 \times 1 / R_1 || R_2 = 2mA \)

\[ 2V_{SS} \quad \frac{\Delta}{\Delta} \quad i_L \quad \frac{\Delta}{\Delta} \quad 2V_{SS} \]

For \( t = t_1^+ \), KCL requires that \( (2V_{SS} - V_a) \gamma R_4 = 2mA \)

\[ V_a = \frac{V_{SS} - 2V_a}{2mA} \]

Similarly, KCL requires that \( V_{SS} = V_a \gamma R_4 \)

so \( V_a = \frac{V_{SS} - 2V_a}{2mA} \)

at \( t = t_1^+ \)

Similarly, KCL requires that \( V_{SS} - V_a \gamma R_4 = V_a \gamma R_3 + 2mA \)

At \( t = t_1^+ \)

Thus, \( V_a = V_{SS} \)

EXAMPLE, continued

The voltage at node a and b are constructed by plotting initial and final value, and using the initial slope, shown as dotted line.

Table of initial and final values

<table>
<thead>
<tr>
<th>Variable</th>
<th>( v_i )</th>
<th>( v_f )</th>
<th>( i_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = t_1^+ )</td>
<td>5V</td>
<td>1V</td>
<td>2mA</td>
</tr>
<tr>
<td>( t = \infty )</td>
<td>3V</td>
<td>3V</td>
<td>-2mA</td>
</tr>
<tr>
<td>( t = \infty )</td>
<td>10( \mu )s</td>
<td>10( \mu )s</td>
<td>10( \mu )s</td>
</tr>
</tbody>
</table>

We plot these three variables on the following page:
Voltage Ranges for Digital Signals

- A digital signal varies with time, typically between ground (0 Volts) and the power supply voltage ($V_{\text{supply}}$).

- A digital voltage signal has two defined states “high” (corresponding to logical state 1) or “low” (corresponding to logical state 0)

- Each of the two states corresponds to a range of voltages, for example:

  - **logical 1 state**: voltage $> V_{\text{supply}}/2$
  - **logical 0 state**: voltage $< V_{\text{supply}}/2$

### Propagation Delay $t_p$

- The propagation delay $t_p$ of a logic gate defines how quickly the output voltage responds to a change in input voltage. It is measured between the 50% transition points of the input and output voltage waveforms.

  **Example**: Output voltage changing from “low” to “high”

  \[
  V_{\text{out}}(t) = V_{\text{supply}}\left(1 - e^{-t/(0.69R_pC)}\right)
  \]

### Formula for Propagation Delay $t_p$

- A logic gate can display different response times for rising or falling input waveforms, so two definitions of propagation delay are necessary.

  \[
  t_p = \frac{t_{p,\text{LH}} + t_{p,\text{HL}}}{2}
  \]

  **Example**: Output voltage changing from “high” to “low”

  \[
  V_{\text{out}}(t) = V_{\text{high}} e^{-t/(0.69R_pC)}
  \]

### Energy Consumption of Simple RC Circuit

- In charging a capacitor, the energy which is delivered to the capacitor is

  \[
  \frac{1}{2} CV_{\text{supply}}^2
  \]

- The energy delivered by the source is

  **How much energy is delivered to the resistor $R_p$?**

  ![Energy Consumption Diagram](image)
In discharging a capacitor, the energy which is delivered to the resistor $R_N$ is $\frac{1}{2} CV^2_{\text{supply}}$.

Thus, in one complete cycle (charging and discharging), the total energy delivered by the voltage source is $C V_{\text{supply}}^2$.

**DRAM (Dynamic Memory Device) Example**

- The operation of a DRAM cell (which stores one bit of information) can be modeled as an RC circuit:

  ![RC Circuit Diagram]

  - Suppose the bit line is pre-charged to 1 V before the cell is read, and that the cell is programmed to 2 V. What is the final value of the bit-line voltage, after the switch is closed?

**DRAM Example (cont'd)**

- The charges stored on $C_{\text{cell}}$ and $C_{\text{bit-line}}$ prior to reading are $Q_{\text{cell, initial}} = C_{\text{cell}} V_{\text{cell, initial}} = (10^{-13} \text{ F})(2 \text{ V}) = 2 \times 10^{-13} \text{ C}$ and $Q_{\text{bit-line, initial}} = C_{\text{bit-line}} V_{\text{bit-line, initial}} = (10^{-12} \text{ F})(1 \text{ V}) = 1 \times 10^{-12} \text{ C}$.

  $Q_{\text{total, initial}} = Q_{\text{cell, initial}} + Q_{\text{bit-line, initial}} = 1.2 \times 10^{-12} \text{ C}$

- The final voltages on each capacitor are equal.

  $Q_{\text{total, final}} = C_{\text{cell}} V_{\text{final}} + C_{\text{bit-line}} V_{\text{final}}$

- Total charge is conserved:

  $Q_{\text{total, final}} = (C_{\text{cell}} + C_{\text{bit-line}}) V_{\text{final}} = Q_{\text{total, initial}}$

  $V_{\text{final}} = \frac{Q_{\text{total, initial}}}{C_{\text{cell}} + C_{\text{bit-line}}} = \frac{1.2 \times 10^{-12} \text{ C}}{1.1 \times 10^{-12} \text{ F}} \approx 1.09 \text{ Volts}$

**DRAM Example (cont'd)**

- Sketch the bit-line voltage waveform:

  ![Bit-Line Voltage Waveform]

- Is energy conserved? Explain.
Plan for coming weeks

- We will be studying semiconductor devices and technology for the next several weeks
  - How does a transistor work?
    (need to learn about semiconductors and diode devices first)
  - How are transistors used as amplifiers?
    • modeled as dependent current source
  - How are transistors used to implement digital logic gates?
    • modeled as resistive switch
    (circuit performance is limited by RC delay)

RC Circuit Transient Analysis Example

The switch is closed for $t < 0$, and then opened at $t = 0$.
Find the voltage $v_c(t)$ for $t \geq 0$.

1. Determine the initial voltage $v_c(0)$

2. Determine the final voltage $v_c(\infty)$

3. Calculate the time constant $\tau$

$v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)] e^{-t/\tau}$