Lecture #12

Announcement

- Midterm 1 on Tues. 3/2/04, 9:30-11
- A-M last initials in 10 Evans
- N-Z initials in Sibley auditorium
- Closed book, no electronic devices
- One sheet 8.5x11 inch of your notes
- Covers material through op-amps, I.e. hw #1-4

OUTLINE

- RC,RL review
- Propagation delay
- Energy consumption of simple RC circuit
- Circuit transient response examples
- Midterm questions?

TRANSIENTS IN SINGLE-INDUCTOR OR SINGLE-CAPACITOR CIRCUITS - THE EASY WAY

1) Find Resistance seen from terminals of L or C (short voltage sources, open current sources).

2) The circuit time constant is L/R or RC (for every node, every current, every voltage).

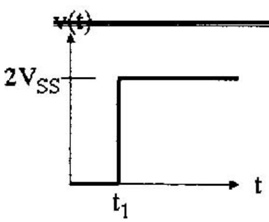
3) Use initial conditions and inductor/capacitor rules to find initial values of all transient variables. (Capacitor voltage and inductor current must be continuous.)

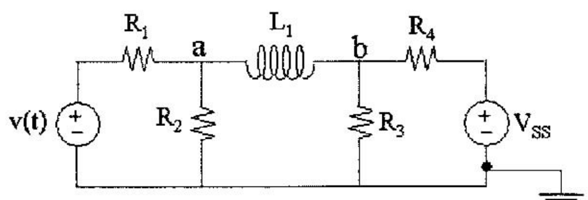
4) Find $t=\infty$ value of all variables by setting all time derivatives to zero.

5) Sketch the time-behavior of all transient variables, based on initial and final values and known time constant.

6) Write the equation for each transient variable by inspection.

EXAMPLE

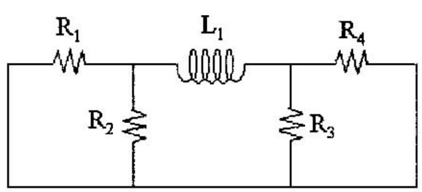




V_{SS}=4V R_1= R_2= R_3= R_4=1K L_1=10mH

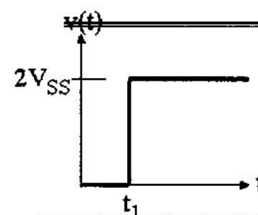
Solution: V_a, V_b= ?

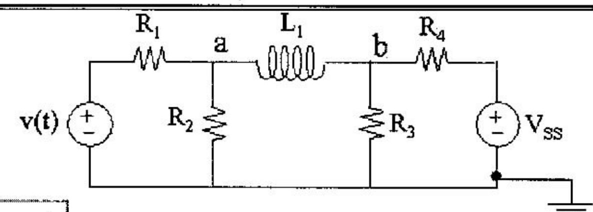
To find R_{eff}, short voltage sources and find R at terminals of L:



So τ =L/R=

EXAMPLE con't

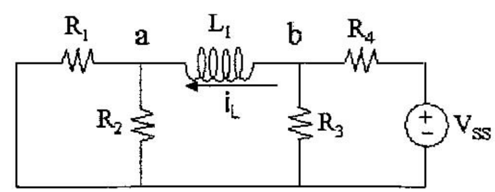




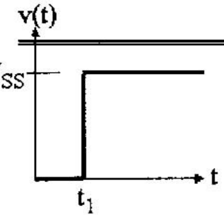
Find initial inductor current

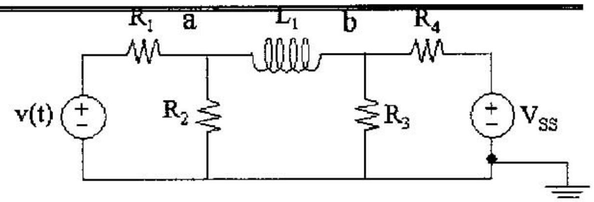
V_{SS}=4V R_1= R_2= R_3= R_4=1K L_1=10mH

For t < t_1, v(t)=0, d/dt =0 so inductor voltage is zero (like a wire):



EXAMPLE con't

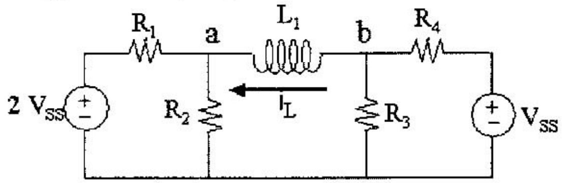




Find initial values

V_{SS}=4V $R_1= R_2= R_3= R_4$=1K L_1=10mH

For $t = t_1^+$, $v(t)=2V_{SS}$, i_L = V_{SS} /4 x 1/ $R_1 || R_2$ =2mA



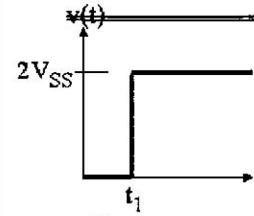
For $t = t_1^+$, KCL requires that $(2V_{SS} - V_a)/ R_1$ + 2mA= V_a / R_2 so

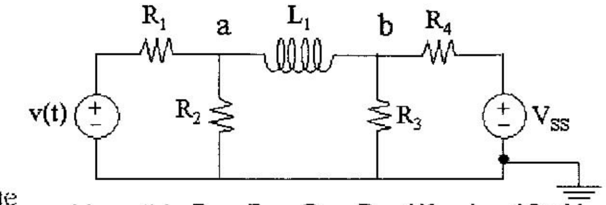
V_a = at $t = t_1^+$

Similarly, KCL requires that $(V_{SS} - V_b)/ R_4 = V_b / R_3$ + 2mA

so V_b = at $t = t_1^+$

EXAMPLE con't

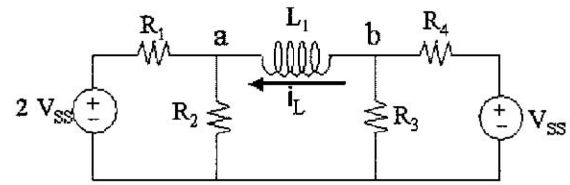




Find asymptotic value

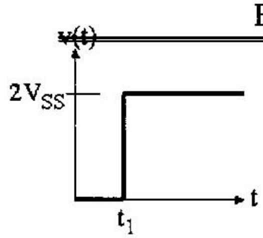
V_{SS}=4V $R_1= R_2= R_3= R_4$=1K L_1=10mH

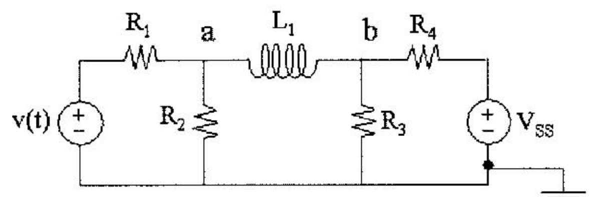
As $t \to \infty$, $v(t)=2V_{SS}$, di_L/dt= 0 so $V_a = V_b$. L_1 acts like simple wire again.



Thus For t >> t_1 $V_a = V_b$ =

EXAMPLE continued





V_{SS}=4V $R_1= R_2= R_3= R_4$=1K L_1=10mH

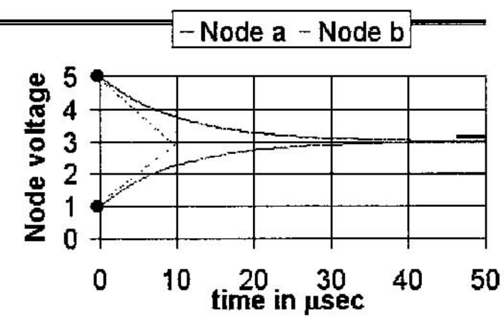
Table of initial and final values

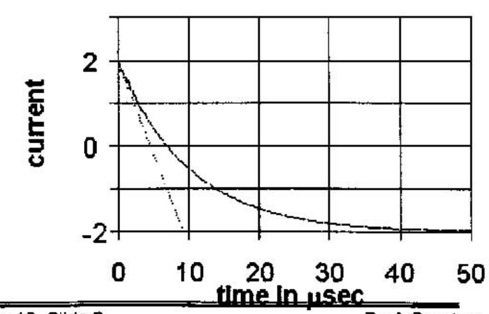
Variable	v_a	v_b	i_L
$t = t_1^+$	5V	1V	2mA
$t = \infty$	3V	3V	-2mA
τ	10µs	10µs	10µs

We plot these three variables on the following page:

EXAMPLE, continued

The voltage at node a and b are constructed by plotting initial and final value, and using the initial slope, shown as dotted line.





Voltage Ranges for Digital Signals

- A digital signal varies with time, typically between between ground (0 Volts) and the power supply voltage (V_{supply}).
- A digital voltage signal has two defined states "high" (corresponding to logical state **1**) or "low" (corresponding to logical state **0**)
- Each of the two states corresponds to a range of voltages, for example:

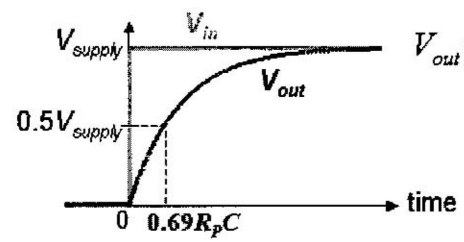
logical 1 state: voltage > $V_{supply}/2$

logical 0 state: voltage < $V_{supply}/2$

Propagation Delay t_p

- The propagation delay t_p of a logic gate defines how quickly the output voltage responds to a change in input voltage. It is measured between the 50% transition points of the input and output voltage waveforms.

Example: Output voltage changing from "low" to "high"



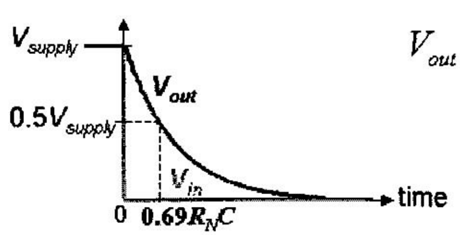
$$V_{out}(t) = V_{supply}\left(1 - e^{-t/R_pC}\right)$$

Formula for Propagation Delay t_p

- A logic gate can display different response times for rising or falling input waveforms, so two definitions of propagation delay are necessary.

$$t_p = \frac{t_{pLH} + t_{pHL}}{2}$$

Example: Output voltage changing from "high" to "low"

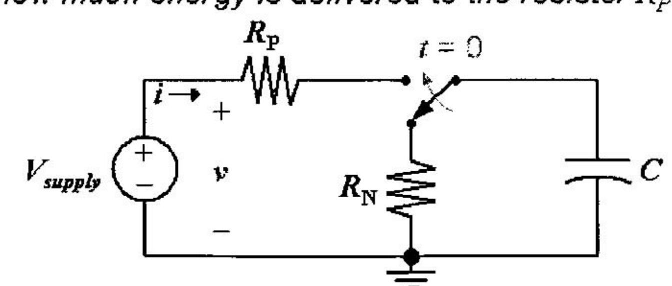


$$V_{out}(t) = V_{high}e^{-t/R_NC}$$

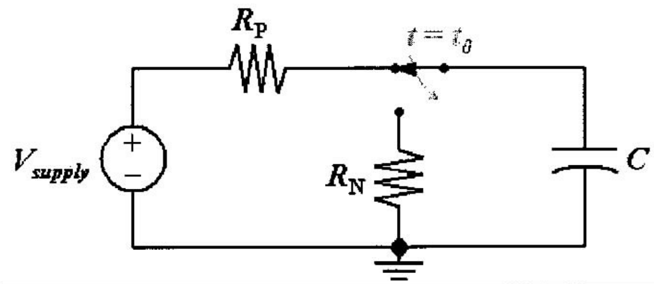
Energy Consumption of Simple RC Circuit

- In charging a capacitor, the energy which is delivered to the capacitor is $\frac{1}{2}CV_{supply}^2$
- The energy delivered by the source is

How much energy is delivered to the resistor R_p?

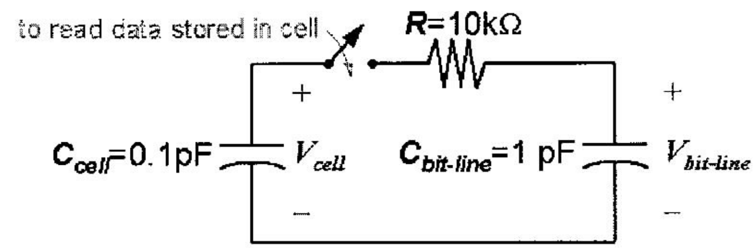


- In discharging a capacitor, the energy which is delivered to the resistor R_N is $\frac{1}{2}CV_{supply}^2$
- Thus, in one complete cycle (charging and discharging), the total energy delivered by the voltage source is CV_{supply}^2



DRAM (Dynamic Memory Device) Example

- The operation of a DRAM cell (which stores one bit of information) can be modeled as an RC circuit:



- Suppose the bit line is pre-charged to 1 V before the cell is read, and that the cell is programmed to 2 V. What is the final value of the bit-line voltage, after the switch is closed?

DRAM Example (cont'd)

- The charges stored on C_{cell} and $C_{bit\text{-}line}$ prior to reading are $Q_{cell,initial} = C_{cell}V_{cell,initial} = (10^{-13}\,\mathrm{F})(2\,\mathrm{V}) = 2\times10^{-13}C$ and $Q_{bit-line,initial} = C_{bit-line}V_{bit-line,initial} = (10^{-12}\,\mathrm{F})(1\,\mathrm{V}) = 1\times10^{-12}C$

$$Q_{total,initial} = Q_{cell,initial} + Q_{bit-line,initial} = 1.2\times10^{-12}C$$

- The final voltages on each capacitor are equal.

$$\Rightarrow \; Q_{total,final} = C_{cell}V_{final} + C_{bit-line}V_{final}$$

- Total charge is conserved:

$$Q_{total,final} = (C_{cell} + C_{bit-line})V_{final} = Q_{total,initial}$$

$$V_{final} = \frac{Q_{total,initial}}{C_{cell} + C_{bit-line}} = \frac{1.2\times10^{-12}\,\mathrm{C}}{1.1\times10^{-12}\,\mathrm{F}} \cong 1.09\ \mathrm{Volts}$$

DRAM Example (cont'd)

- Sketch the bit-line voltage waveform

- Is energy conserved? Explain.

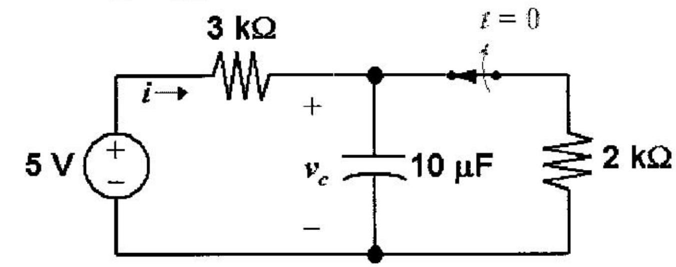
Plan for coming weeks

- **We will be studying semiconductor devices and technology for the next several weeks**
  - How does a transistor work?
    (need to learn about semiconductors and diode devices first)
  - How are transistors used as amplifiers?
    - modeled as dependent current source
  - How are transistors used to implement digital logic gates?
    - modeled as resistive switch

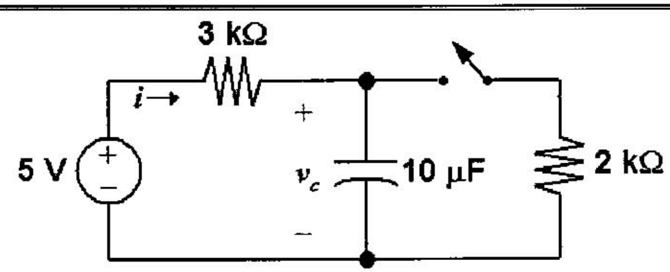
    (circuit performance is limited by RC delay)

RC Circuit Transient Analysis Example

The switch is closed for $t < 0$, and then opened at $t = 0$.
Find the voltage $v_c(t)$ for $t \geq 0$.



1. Determine the initial voltage $v_c(0)$



2. Determine the final voltage $v_c(\infty)$

3. Calculate the time constant τ

$$v_c(t) = v_c(\infty) + \left[v_c(0) - v_c(\infty)\right] e^{-t/\tau}$$