Lecture #13

ANNOUNCEMENTS

- Reader with reference material from Howe&Sodini and from Rabaey et al available at Copy Central on Hearst Ave.

OUTLINE

- Semiconductor materials
- Properties of silicon
- Doping

Reading

Howe&Sodini: Ch. 2.1-2.4.1

What is a Semiconductor?

- Low resistivity => “conductor”
- High resistivity => “insulator”
- Intermediate resistivity => “semiconductor”
  - Generally, the semiconductor material used in integrated-circuit devices is crystalline
  - In recent years, however, non-crystalline semiconductors have become commercially very important

![Image of polycrystalline, amorphous, and crystalline materials]

Electrical Resistance

\[ R \equiv \frac{V}{I} = \frac{L}{Wt} \]  
(Units: \( \Omega \))

\[ \rho \] is the resistivity (Units: \( \Omega \cdot \text{cm} \))

Semiconductor Materials

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Compound:
The Silicon Atom

- 14 electrons occupying the 1st 3 energy levels:
  - 1s, 2s, 2p orbitals filled by 10 electrons
  - 3s, 3p orbitals filled by 4 electrons
  
  To minimize the overall energy, the 3s and 3p orbitals hybridize to form 4 tetrahedral 3sp orbitals
  
  Each has one electron and is capable of forming a bond with a neighboring atom

The Si Crystal

- "diamond cubic" lattice
  
  - Each Si atom has 4 nearest neighbors
  - lattice constant = 5.431 Å

Compound Semiconductors

- "zinc blende" structure
- III-V compound semiconductors: GaAs, GaP, GaN, etc.
- Important for optoelectronics and high-speed ICs

Electronic Properties of Si

- Silicon is a semiconductor material. Pure Si has relatively high resistivity at room temperature.

- There are 2 types of mobile charge-carriers in Si: Conduction electrons are negatively charged. Holes are positively charged.

- The concentration of conduction electrons & holes in a semiconductor can be affected in several ways:
  1. by adding special impurity atoms (dopants)
  2. by applying an electric field
  3. by changing the temperature
  4. by irradiation
Conduction Electrons and Holes

2-D representation

When an electron breaks loose and becomes a conduction electron, a hole is also created.

Note: A hole (along with its associated positive charge) is mobile!

Definition of Parameters

\[ n = \text{number of mobile electrons per cm}^3 \]
\[ p = \text{number of holes per cm}^3 \]
\[ n_i = \text{intrinsic carrier concentration (#/cm}^3) \]

In a pure semiconductor,

\[ n = p = n_i \]

Generation

- We have seen that conduction (mobile) electrons and holes can be created in pure (intrinsic) silicon by thermal generation.
  - Thermal generation rate increases exponentially with temperature \( T \)
- Another type of generation process which can occur is optical generation
  - The energy absorbed from a photon frees an electron from covalent bond
    - In Si, the minimum energy required is 1.1 eV, which corresponds to \(~1 \text{ } \mu\text{m}\) wavelength (infrared region)
- Note that conduction electrons and holes are continuously generated, if \( T > 0 \)

Recombination

- When a conduction electron and hole meet, each one is eliminated. The energy lost by the conduction electron (when it "falls" back into the covalent bond) can be released in 2 ways:
  1. to the semiconductor lattice (vibrations) "thermal recombination" \( \rightarrow \) semiconductor is heated
  2. to photon emission "optical recombination" \( \rightarrow \) light is emitted
    - Optical recombination is negligible in Si. It is significant in compound semiconductor materials, and is the basis for light-emitting diodes and laser diodes.
Generation and Recombination Rates

- The generation rate is dependent on temperature $T$, but it is independent of $n$ and $p$: $G = G_{\text{thermal}}(T) + G_{\text{optical}}$
- The recombination rate is proportional to both $n$ and $p$: $R \propto np$
- In steady state, a balance exists between the generation and recombination rates. $G = R \Rightarrow np = f(T)$
- A special case of the steady-state condition is thermal equilibrium: no optical or electrical sources
  \[ np = n_i^2(T) \]

Doping

By substituting a Si atom with a special impurity atom (Column V or Column III element), a conduction electron or hole is created.

Donors: P, As, Sb

Acceptors: B, Al, Ga, In

Charge-Carrier Concentrations

$N_D$: ionized donor concentration (cm$^{-3}$)
$N_A$: ionized acceptor concentration (cm$^{-3}$)

Charge neutrality condition: $N_D + p = N_A + n$

At thermal equilibrium, $np = n_i^2$ ("Law of Mass Action")

\[ n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} \]

\[ p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2} \]

Note: Carrier concentrations depend on net dopant concentration $(N_D - N_A)$!
N-type and P-type Material

If $N_D \gg N_A$ (so that $N_D - N_A \gg n_i$):

$$n \approx N_D - N_A \quad \text{and} \quad p \approx \frac{n_i^2}{N_D - N_A}$$

$n \gg p \rightarrow \text{material is "n-type"}$

If $N_A \gg N_D$ (so that $N_A - N_D \gg n_i$):

$$p \approx N_A - N_D \quad \text{and} \quad n \approx \frac{n_i^2}{N_A - N_D}$$

$p \gg n \rightarrow \text{material is "p-type"}$

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Terminology

- **intrinsic semiconductor**: "undoped" semiconductor
  electrical properties are native to the material

- **extrinsic semiconductor**: doped semiconductor
  electrical properties are controlled by the added impurity atoms

- **donor**: impurity atom that increases the electron concentration
  group V elements (P, As)

- **acceptor**: impurity atom that increases the hole concentration
  group III elements (B, In)

- **n-type material**: semiconductor containing more electrons than holes
- **p-type material**: semiconductor containing more holes than electrons

- **majority carrier**: the most abundant carrier in a semiconductor sample
- **minority carrier**: the least abundant carrier in a semiconductor sample

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Carrier Scattering

- **Mobile electrons and atoms in the Si lattice are always in random thermal motion.**
  - Average velocity of thermal motion for electrons in Si:
    $-10^7 \text{ cm/s} @ 300K$
  - Electrons make frequent collisions with the vibrating atoms
    - "lattice scattering" or "phonon scattering"
  - Other scattering mechanisms:
    - deflection by ionized impurity atoms
    - deflection due to Coulombic force between carriers

- The average current in any direction is zero, if no electric field is applied.

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Carrier Drift

- When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:

![Electron drift diagram](image)

- Electrons drift in the direction opposite to the $E$-field
  - Current flows

- Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as classical particles moving at a constant average drift velocity.
Drift Velocity and Carrier Mobility

Mobile charge-carrier drift velocity is proportional to applied E-field:

\[ |v| = \mu E \]

\(\mu\) is the **mobility**

(Units: cm²/V·s)

![Graph showing mobility vs. total impurity concentration](image)

Note: Carrier mobility depends on total dopant concentration \((N_d + N_i)\)

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Current Density

The current density \(J\) is the current per unit area \((J = I / A; A\) is the cross-sectional area of the conductor\)

If we have \(N\) positive charges per unit volume moving with average speed \(v\) in the +x direction, then the current density in the +x direction is just \(J = qNv\)

\[ J = qNv \]

Example:

- 2 x 10¹⁶ holes/cm³ moving to the right at 2 x 10⁴ cm/sec
- \(J = 1.6 \times 10^{-19} \times 2 \times 10^{16} \times 2 \times 10^4 = 64\) A/cm²

Suppose this occurs in a conductor 2 µm wide and 1 µm thick:

\[ I = J \times A = 64 \times (2 \times 10^4 \times 1 \times 10^{-4}) = 1.28\ \mu\text{A} \]

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Electrical Conductivity \(\sigma\)

When an electric field is applied, current flows due to drift of mobile electrons and holes:

- **Electron current density:** \(J_n = (-q)n\nu_n = qn\mu_n E\)
- **Hole current density:** \(J_p = (+q)p\nu_p = qp\mu_p E\)
- **Total current density:** \(J = J_n + J_p = (qn\mu_n + qp\mu_p)E\)

**Conductivity**

\(\sigma \equiv qn\mu_n + qp\mu_p\)

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Electrical Resistivity \(\rho\)

\(\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}\)

- \(\rho \equiv \frac{1}{qn\mu_n}\) for n-type mat'l
- \(\rho \equiv \frac{1}{qp\mu_p}\) for p-type mat'l

(Units: ohm-cm)
**Example**

Consider a Si sample doped with $10^{16}$/cm$^3$ Boron. What is its resistivity?

**Answer:**

\[ N_A = 10^{16}$/cm$^3$, N_D = 0 \quad (N_A \gg N_D \rightarrow \text{p-type}) \]

\[ \rightarrow p \approx 10^{16}$/cm$^3$ and \quad n \approx 10^4$/cm$^3 \]

\[ \rho = \frac{1}{q\eta\mu_n + q\rho\mu_p} \approx \frac{1}{q\rho\mu_p} \]

\[ = \left[1.6 \times 10^{-19} \times (10^{16})(450)\right]^{-1} = 1.4\ \Omega\cdot\text{cm} \]

From $\mu$ vs. $(N_A + N_D)$ plot

**Example (cont'd)**

Consider the same Si sample, doped *additionally* with $10^{17}$/cm$^3$ Arsenic. What is its resistivity?

**Answer:**

\[ N_A = 10^{16}$/cm$^3$, N_D = 10^{17}$/cm$^3 \quad (N_D \gg N_A \rightarrow \text{n-type}) \]

\[ \rightarrow n \approx 9 \times 10^{16}$/cm$^3$ and \quad p \approx 1.1 \times 10^3$/cm$^3 \]

\[ \rho = \frac{1}{q\eta\mu_n + q\rho\mu_p} \approx \frac{1}{q\eta\mu_n} \]

\[ = \left[1.6 \times 10^{-19} \times (9 \times 10^{16})(700)\right]^{-1} = 0.10\ \Omega\cdot\text{cm} \]

The sample is converted to n-type material by adding more donors than acceptors, and is said to be "compensated".

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**Sheet Resistance $R_s$**

\[ R = \rho \frac{L}{Wt} = R_s \frac{L}{W} \quad \Rightarrow \quad R_s = \frac{\rho}{t} \quad \text{(Unit: ohms/square)} \]

$R_s$ is the resistance when $W = L$

- The $R_s$ value for a given layer in an IC technology is used
  - for design and layout of resistors
  - for estimating values of parasitic resistance in a circuit

  \[ R = R_s \quad R = R_s/2 \quad R = 2R_s \quad R = 3R_s \quad R \approx 2.6R_s \]

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**Integrated-Circuit Resistors**

The resistivity $\rho$ and thickness $t$ are fixed for each layer in a given manufacturing process.

A circuit designer specifies the length $L$ and width $W$, to achieve a desired resistance $R$

\[ R = R_s \left( \frac{L}{W} \right) \]

**Example:** Suppose we want to design a 5 k\(\Omega\) resistor using a layer of material with $R_s = 200 \\Omega/\square$

**Resistor layout (top view)**

**Space-efficient layout**
Summary

- **Crystalline Si:**
  - 4 valence electrons per atom
  - diamond lattice: each atom has 4 nearest neighbors
  - $5 \times 10^{22}$ atoms/cm$^3$

- **In a pure Si crystal, conduction electrons and holes are formed in pairs.**
  - Holes can be considered as positively charged mobile particles which exist inside a semiconductor.
  - Both holes and electrons can conduct current.

- **Dopants in Si:**
  - Reside on lattice sites (substituting for Si)
  - Group V elements contribute conduction electrons, and are called *donors*
  - Group III elements contribute holes, and are called *acceptors*