

## Lecture #20

### ANNOUNCEMENT

- Midterm 2 thurs. april 15, 9:40-11am.
- A-M initials in 10 Evans
- N-Z initials in Sibley auditorium
- Closed book, except for two 8.5 x 11 inch cheat sheets
- Covers HW's 5-9; L's, C's, 1<sup>st</sup>-order ckts, semiconductor devices, diode ckts, mosfet model, common source amplifier

### OUTLINE

- Synthesis of logic circuits
- Minimization of logic circuits

Reading: Hambley Ch. 7 through 7.5

### Boolean Algebra Relations

$$A \cdot A = A$$

$$A + A = A$$

$$A \cdot \bar{A} = 0$$

$$A + \bar{A} = 1$$

$$A \cdot 1 = A$$

$$A + 1 = 1$$

$$A \cdot 0 = 0$$

$$A + 0 = A$$

$$A \cdot B = B \cdot A$$

$$A + B = B + A$$

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

$$A + (B + C) = (A + B) + C$$

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

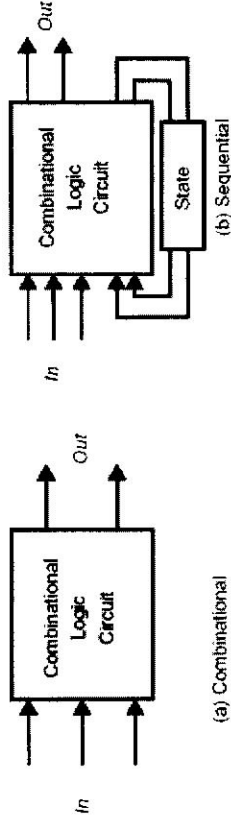
$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

$$\overline{\bar{A} \cdot \bar{B}} = A + B$$

} De Morgan's laws

## Combinational Logic Circuits

- Logic gates combine several logic-variable inputs to produce a logic-variable output.
- **Combinational logic circuits** are "memoryless" because their output value at a given instant depends only on the input values at that instant.



(a) Combinational

(b) Sequential

- Next time, we will study **sequential logic circuits** that possess memory because their present output value depends on previous as well as present input values.

## Boolean Expression Example

$$F = A \cdot \bar{B} \cdot C + A \cdot B \cdot C + (C + D) \cdot (\bar{D} + E)$$

$$F = C \cdot (A + \bar{D} + E) + D \cdot E$$

## Logical Sufficiency of NAND Gates

- If the inputs to a NAND gate are tied together, an inverter results
- From De Morgan's laws, the OR operation can be realized by inverting the input variables and combining the results in a NAND gate.
- Since the basic logic functions (AND, OR, and NOT) can be realized by using only NAND gates, NAND gates are sufficient to realize any combinational logic function.

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## Logical Sufficiency of NOR Gates

- Show how to realize the AND, OR, and NOT functions using only NOR gates
- Since the basic logic functions (AND, OR, and NOT) can be realized by using only NOR gates, NOR gates are sufficient to realize any combinational logic function.

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## Synthesis of Logic Circuits

Suppose we are given a truth table for a logic function. Is there a method to implement the logic function using basic logic gates?

**Answer:** There are lots of ways, but one simple way is the "sum of products" implementation method:

- 1) Write the sum of products expression based on the truth table for the logic function
  - 2) Implement this expression using standard logic gates.
- We may not get the most efficient implementation this way, but we can simplify the circuit afterwards...

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## Logic Synthesis Example: Adder

Input			Output	
A	B	C	$S_1$	$S_0$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$S_1$  using sum-of-products:

- 1) Find where  $S_1$  is 1
- 2) Write down each product of inputs which create a 1  
 $\bar{A} B C$     $A \bar{B} C$   
 $A B \bar{C}$     $A B C$
- 3) Sum all of the products  
 $\bar{A} B C + A \bar{B} C + A B \bar{C} + A B C$
- 4) Draw the logic circuit

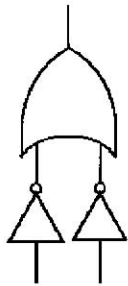
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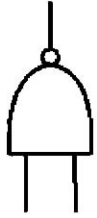
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## NAND Gate Implementation

- De Morgan's law tells us that



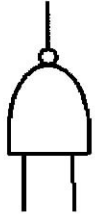
is the same as



- By definition,



is the same as

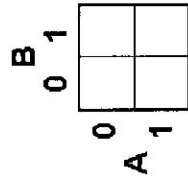


→ All sum-of-products expressions can be implemented with only NAND gates.

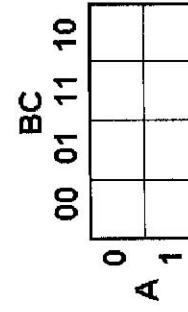
## Karnaugh Maps

- Graphical approach to minimizing the number of terms in a logic expression:
  - Map the truth table into a Karnaugh map (see below)
  - For each 1, circle the biggest block that includes that 1
  - Write the product that corresponds to that block.
  - Sum all of the products

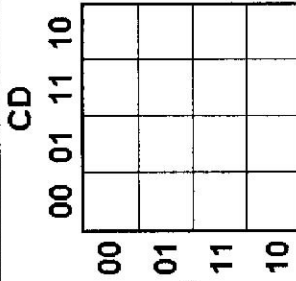
2-variable Karnaugh Map



3-variable Karnaugh Map



4-variable Karnaugh Map



## Creating a Better Circuit

### What makes a digital circuit better?

- Fewer number of gates
- Fewer inputs on each gate
  - multi-input gates are slower
- Let's see how we can simplify the sum-of-products expression for  $S_1$ , to make a better circuit...
  - Use the Boolean algebra relations

## Karnaugh Map Example

Input		Output		
A	B	C	$S_1$	$S_0$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Simplification of expression for  $S_1$ :

	BC			
A	00	01	11	10
0	0	0	0	1
1	0	0	1	1

BC AC AC AB

$$S_1 = AB + BC + AC$$

## Further Comments on Karnaugh Maps

- The algebraic manipulations needed to simplify a given expression are not always obvious. Karnaugh maps make it easier to minimize the number of terms in a logic expression.
- Terminology:
  - "2-cube": 2 squares that have a common edge ( $\rightarrow$  product of 3 variables)
  - "4-cube": 4 squares with common edges ( $\rightarrow$  product of 2 variables)
- In locating cubes on a Karnaugh map, the map should be considered to fold around from top to bottom, and from left to right.
  - Squares on the right-hand side are considered to be adjacent to those on the left-hand side.
  - Squares on the top of the map are considered to be adjacent to those on the bottom.
- Example:
  - The four squares in the map corners form a 4-cube

