

Lecture #9

OUTLINE

- The capacitor
- The inductor

Reading

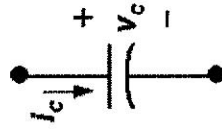
-Chapter 3

Symbol:  or 

Units: Farads (Coulombs/Volt)
(typical range of values: 1 pF to 1 μF)

Current-Voltage relationship:

$$i_c = \frac{dQ}{dt} = C \frac{dv_c}{dt} + v_c \frac{dC}{dt}$$



Note: Q (v_c) must be a continuous function of time

The Capacitor

Two conductors (a,b) separated by an insulator:
difference in potential = V_{ab}
 \Rightarrow equal & opposite charge Q on conductors

$$Q = CV_{ab}$$

(stored charge in terms of voltage)

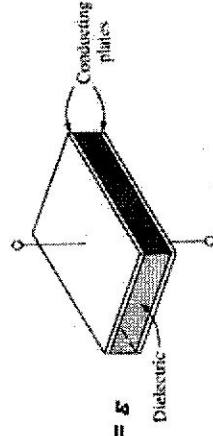
where C is the capacitance of the structure,
> positive (+) charge is on the conductor at higher potential

Parallel-plate capacitor:

- area of the plates = A
- separation between plates = d
- **dielectric permittivity** of insulator = ϵ

$$C = \frac{A\epsilon}{d}$$

\Rightarrow capacitance



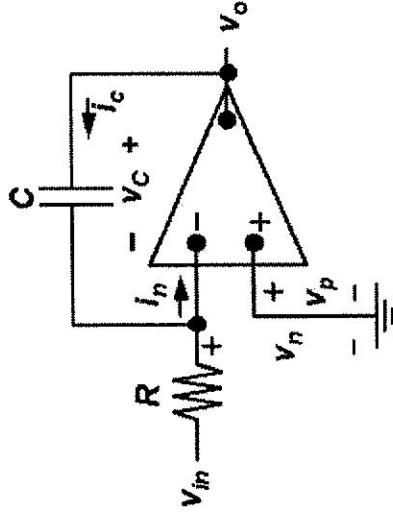
Voltage in Terms of Current

$$Q(t) = \int_0^t i_c(t) dt + Q(0)$$

$$v_c(t) = \frac{1}{C} \int_0^t i_c(t) dt + \frac{Q(0)}{C} = \frac{1}{C} \int_0^t i_c(t) dt + v_c(0)$$

Op-Amp Integrator

$$v_o(t) = -\frac{1}{RC} \int_0^t v_{in}(\tau) d\tau + v_c(0)$$



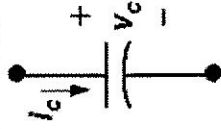
Stored Energy

You might think the energy stored on a capacitor is QV , which has the dimension of Joules. But during charging, the average voltage across the capacitor was only half the final value of V for a linear capacitor.

$$\text{Thus, energy is } \frac{1}{2} QV = \boxed{\frac{1}{2} CV^2}$$

Example: A 1 pF capacitance charged to 5 Volts has $\frac{1}{2}(5V)^2 (1pF) = 12.5 \text{ pJ}$

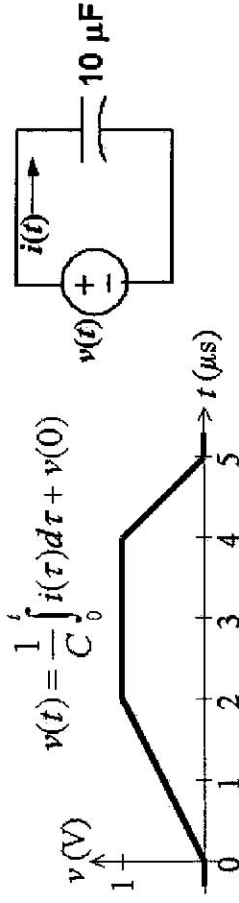
A more rigorous derivation



$$W = \int_{t=0}^{t=t_{\text{Final}}} v_c \cdot i_c dt = \int_{v=V_{\text{Initial}}}^{v=V_{\text{Final}}} v dv = \frac{1}{2} CV_{\text{Final}}^2 - \frac{1}{2} CV_{\text{Initial}}^2$$

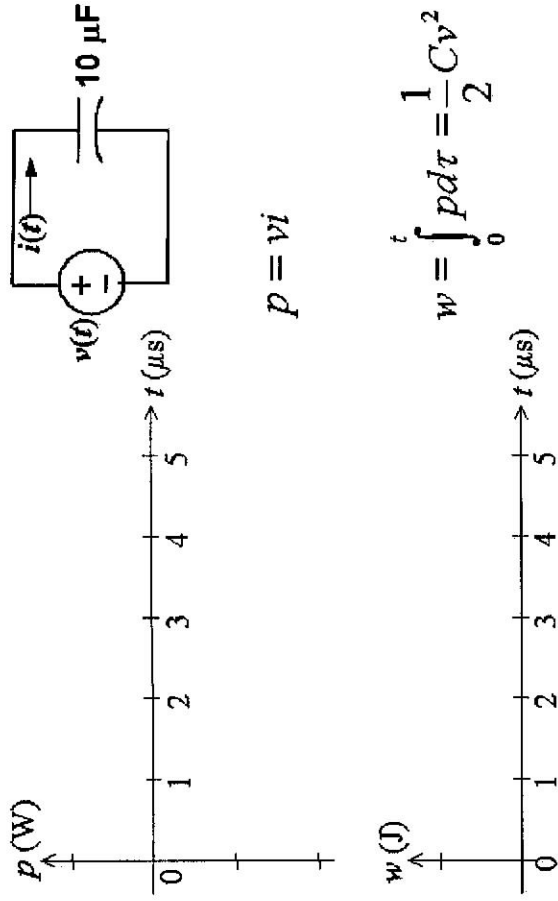
$$W = \int_{v=V_{\text{Initial}}}^{v=V_{\text{Final}}} CV dv = \frac{1}{2} CV_{\text{Final}}^2 - \frac{1}{2} CV_{\text{Initial}}^2$$

Example: Current, Power & Energy for a Capacitor

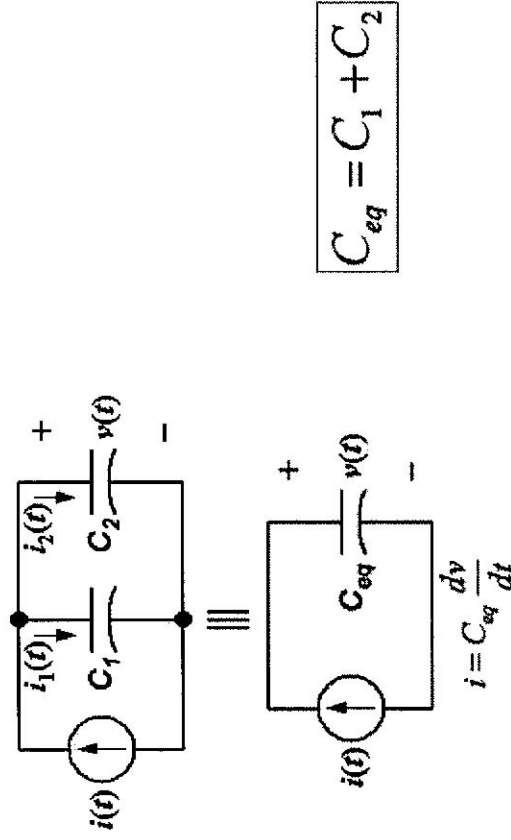


v_c must be a continuous function of time; however, i_c can be discontinuous.

Note: In "steady state" (dc operation), time derivatives are zero
 $\rightarrow C$ is an open circuit

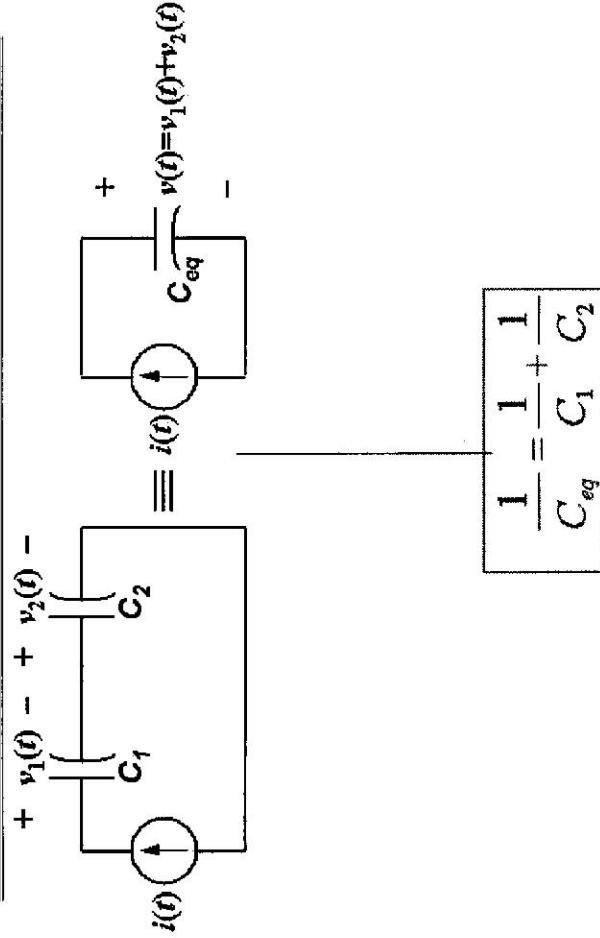


Capacitors in Parallel



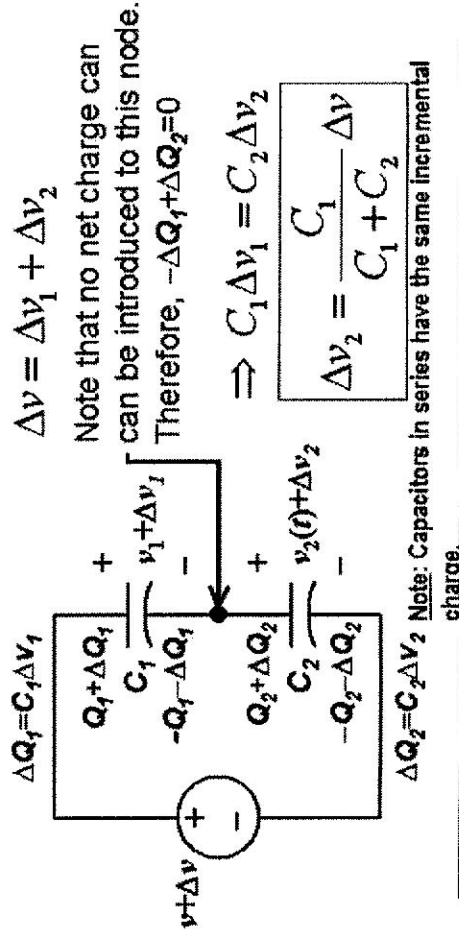
Equivalent capacitance of capacitors in parallel is the sum

Capacitors in Series



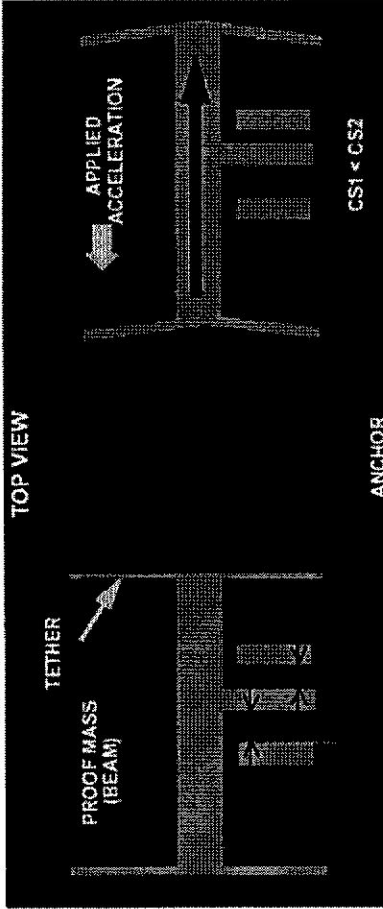
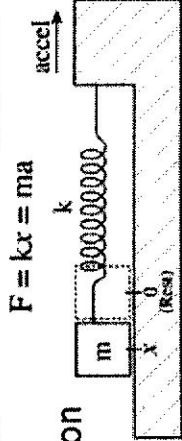
Capacitive Voltage Divider

Q: Suppose the voltage applied across a series combination of capacitors is changed by Δv . How will this affect the voltage across each individual capacitor?



Application Example: MEMS Accelerometer

- Capacitive position sensor used to measure acceleration (by measuring force on a proof mass)



FIXED OUTER PLATES

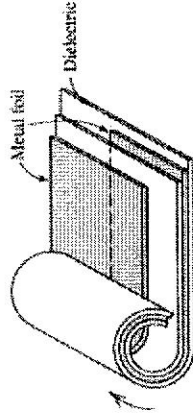
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Practical Capacitors

- A capacitor can be constructed by interleaving the plates with two dielectric layers and rolling them up, to achieve a compact size.
- To achieve a small volume, a very thin dielectric with a high dielectric constant is desirable. However, dielectric materials break down and become conductors when the electric field (units: V/cm) is too high.
 - Real capacitors have maximum voltage ratings
 - An engineering trade-off exists between compact size and high voltage rating



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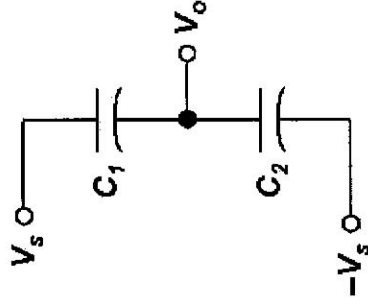
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Sensing the Differential Capacitance

- Begin with capacitances electrically discharged
- Fixed electrodes are then charged to $+V_s$ and $-V_s$
- Movable electrode (proof mass) is then charged to V_o

Circuit model



$$V_o = -V_s + \frac{C_1}{C_1 + C_2} (2V_s) = \frac{C_1 - C_2}{C_1 + C_2} V_s$$

$$\frac{V_o}{V_s} = \frac{\frac{\epsilon A}{g_1} - \frac{\epsilon A}{g_2}}{\frac{\epsilon A}{g_1} + \frac{\epsilon A}{g_2}} = \frac{g_2 - g_1}{g_2 + g_1} = \frac{g_2 - g_1}{const}$$

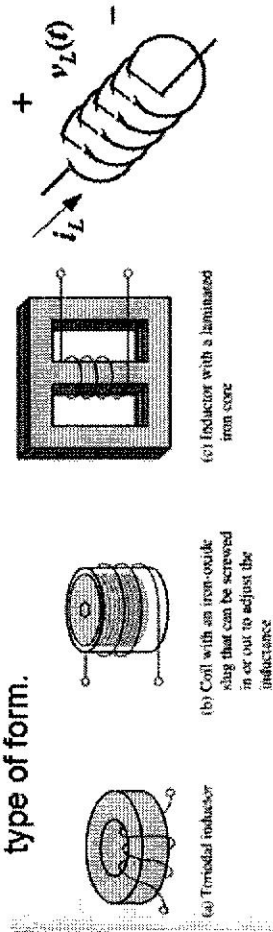
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The Inductor

- An inductor is constructed by coiling a wire around some type of form.



- Current flowing through the coil creates a magnetic field and a magnetic flux that links the coil: $L i_L$
- When the current changes, the magnetic flux changes \rightarrow a voltage across the coil is induced:

$$v_L(t) = L \frac{di_L}{dt}$$

Note: In "steady state" (dc operation), time derivatives are zero $\rightarrow L$ is a short circuit

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Symbol: 

Units: Henrys (Volts • second / Ampere)

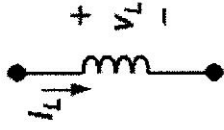
(typical range of values: μH to 10 H)

Current in terms of voltage:

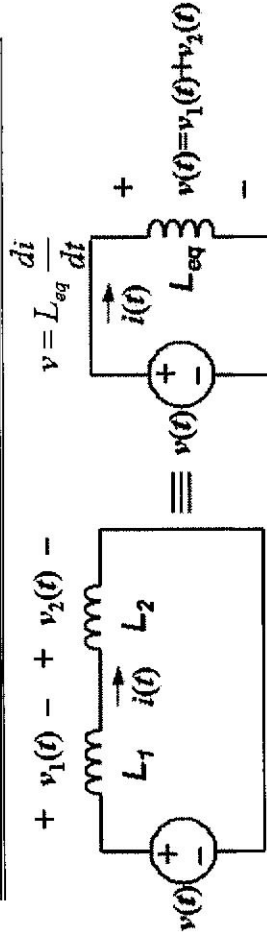
$$di_L = \frac{1}{L} v_L(t) dt$$

$$i_L(t) = \frac{1}{L} \int_{t_0}^t v_L(\tau) d\tau + i(t_0)$$

Note: i_L must be a continuous function of time



Inductors in Series



$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} = (L_1 + L_2) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2$$

Equivalent inductance of inductors in series is the sum

Stored Energy

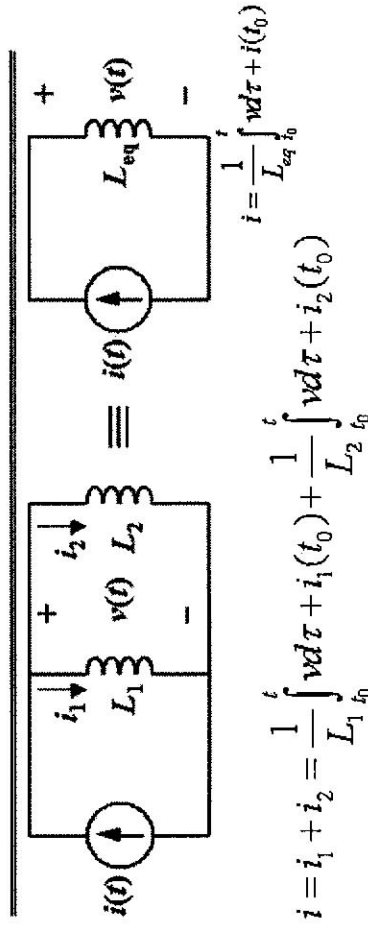
Consider an inductor having an initial current $i(t_0) = i_0$

$$p(t) = v(t)i(t) =$$

$$w(t) = \int_{t_0}^t p(\tau) d\tau =$$

$$w(t) = \frac{1}{2} Li^2 - \frac{1}{2} Li_0^2$$

Inductors in Parallel



$$i = i_1 + i_2 = \frac{1}{L_1} \int_{t_0}^t v d\tau + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^t v d\tau + i_2(t_0)$$

$$i = \left[\frac{1}{L_1} + \frac{1}{L_2} \right] \int_{t_0}^t v d\tau + [i_1(t_0) + i_2(t_0)]$$

$$\Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{with} \quad i(t_0) = i_1(t_0) + i_2(t_0)$$

Summary

Capacitor

$$i = C \frac{dv}{dt}$$

$$W = \frac{1}{2} C v^2$$

v cannot change instantaneously

i can change instantaneously

Do not short-circuit a charged capacitor (-> infinite current!)

n cap.'s in series: $\frac{1}{C_{eq}} = \sum_{i=1}^n \frac{1}{C_i}$

n cap.'s in parallel: $C_{eq} = \sum_{i=1}^n C_i$

Inductor

$$v = L \frac{di}{dt}$$

$$W = \frac{1}{2} L i^2$$

i cannot change instantaneously

v can change instantaneously

Do not open-circuit an inductor with current (-> infinite voltage!)

n ind.'s in series: $L_{eq} = \sum_{i=1}^n L_i$

n ind.'s in parallel: $\frac{1}{L_{eq}} = \sum_{i=1}^n \frac{1}{L_i}$