Lecture #10

OUTLINE

- Mutual inductance
- First-order circuits

Reading

Chapter 3, begin Chapter 4

Mutual Inductance

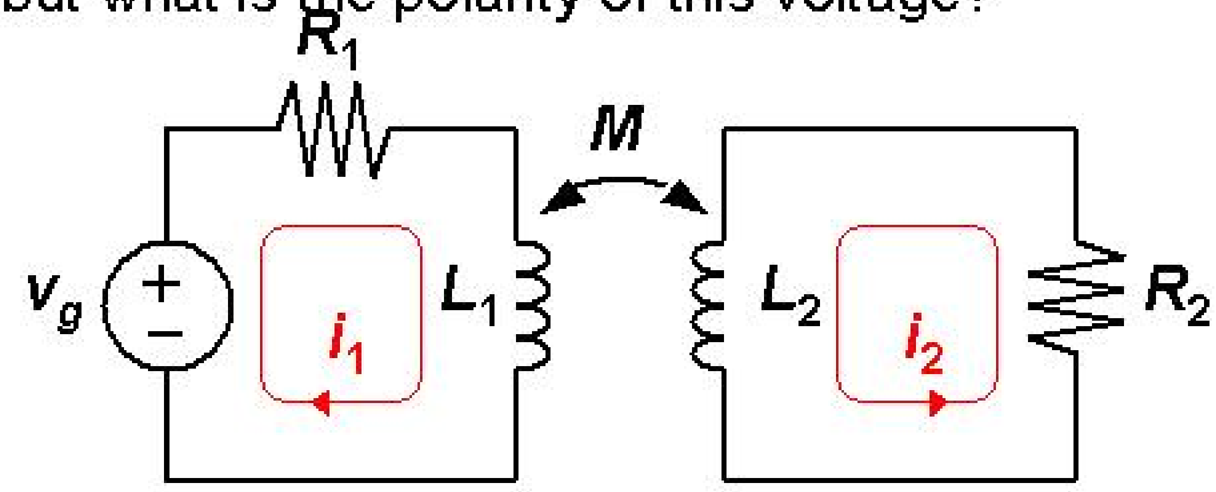
- Mutual inductance occurs when two windings are arranged so that they have a mutual flux linkage.
- The change in current in one winding causes a voltage drop to be induced in the other.

Example: Consider left-hand side of the diagram below

– self-induced voltage is $L_1(\mathrm{d}i_1/\mathrm{d}t)$

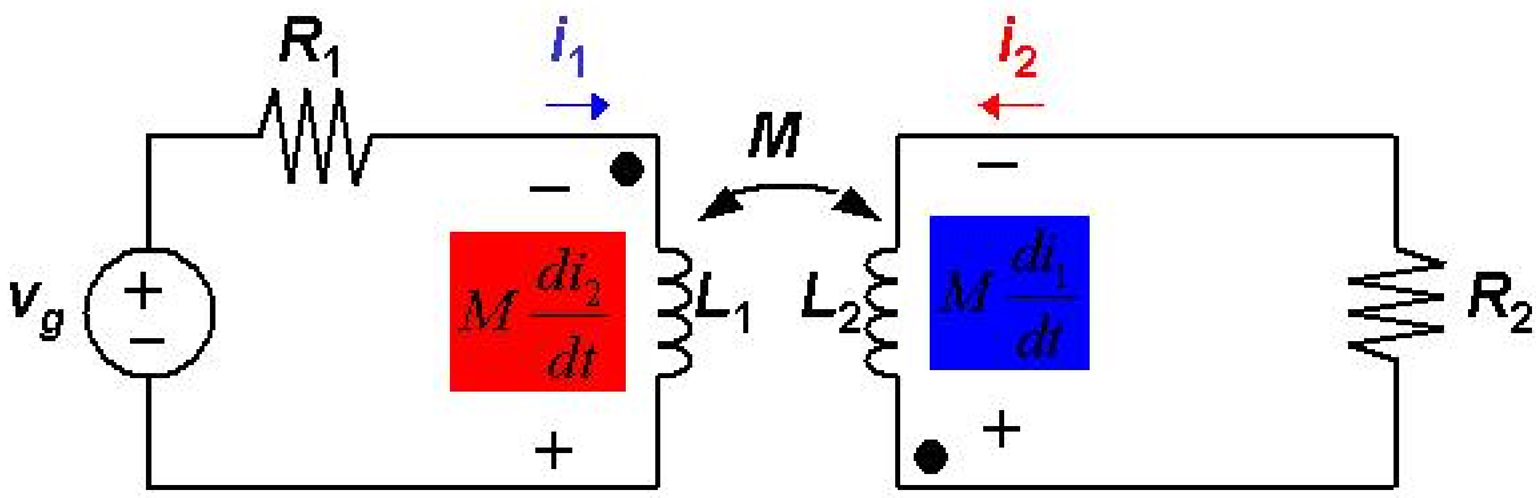
– mutually induced voltage is $M(\mathrm{d}i_2/\mathrm{d}t)$

…but what is the polarity of this voltage?



The "Dot Convention"

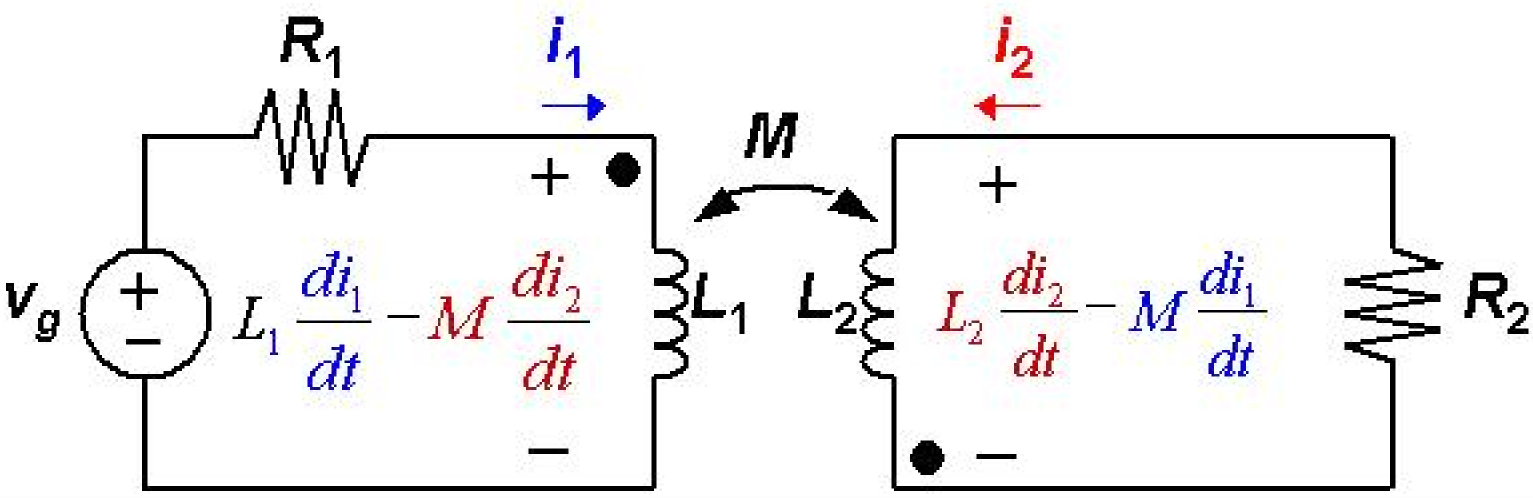
- If a current "enters" the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is positive at its dotted terminal.
- If a current "leaves" the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is negative at its dotted terminal.



Induced Voltage Drop

- The total induced voltage drop across a winding is equal to the sum of the self-induced voltage and the mutually induced voltage

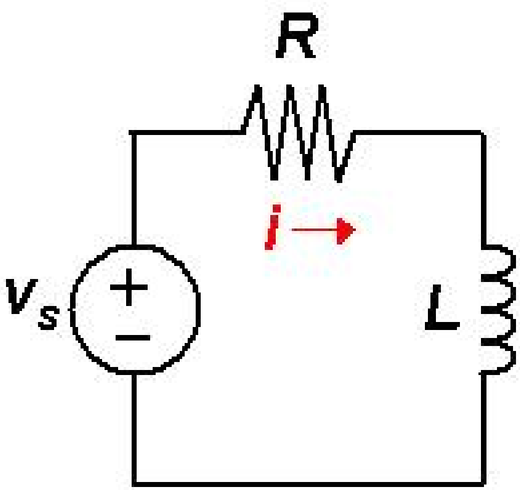
Example (cont'd): Apply KVL to loops

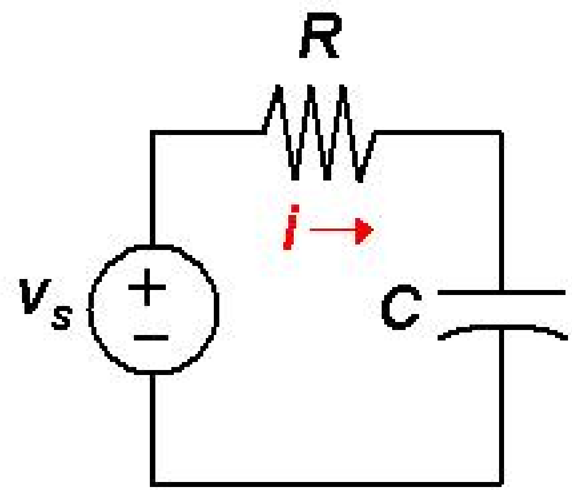


Relationship between *M* and L_1, L_2

First-Order Circuits

- A circuit which contains only sources, resistors and an inductor is called an ***RL circuit***.
- A circuit which contains only sources, resistors and a capacitor is called an ***RC circuit***.
- RL and RC circuits are called first-order circuits because their voltages and currents are described by first-order differential equations.



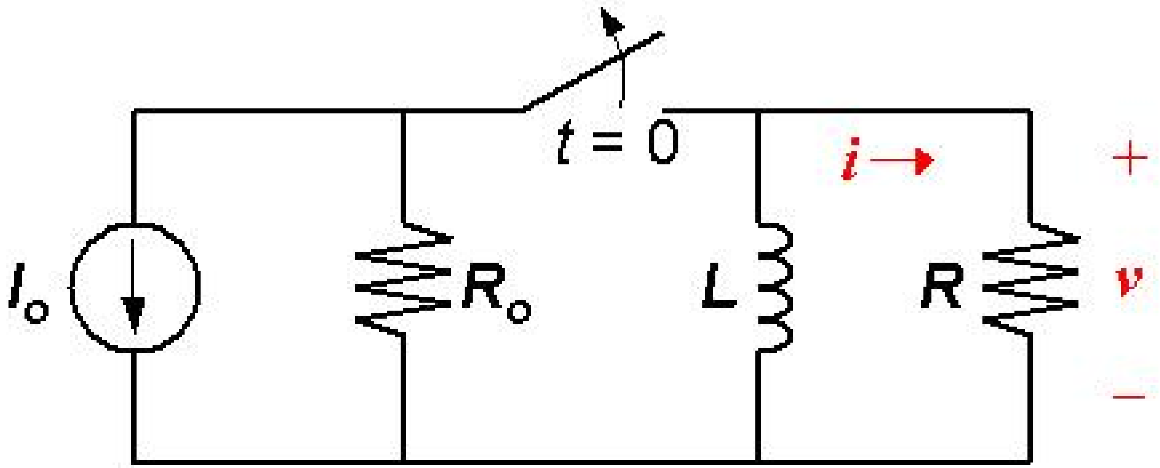


- The ***natural response*** of an RL or RC circuit is its behavior (*i.e.* current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources).

- The ***step response*** of an RL or RC circuit is its behavior when a voltage or current source **step** is applied to the circuit, or immediately after a switch state is changed.

Natural Response of an RL Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:



Notation:

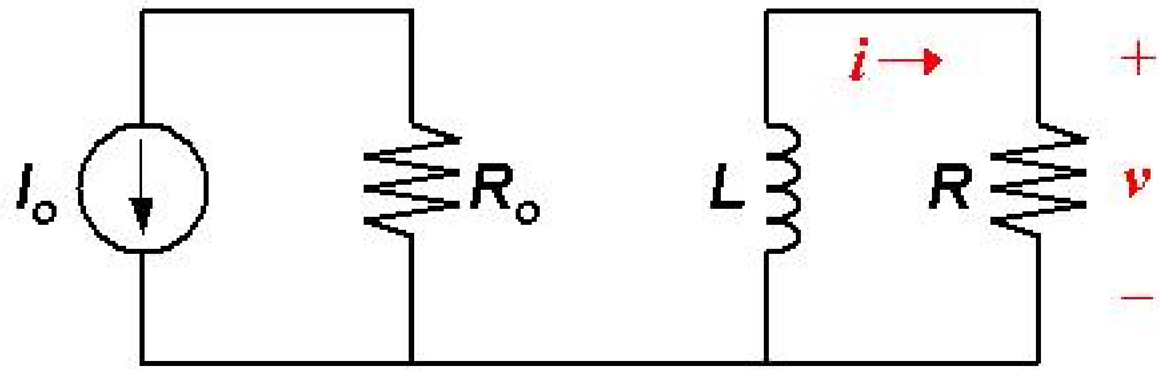
0^- is used to denote the time just prior to switching

0^+ is used to denote the time immediately after switching

- The current flowing in the inductor at $t = 0^-$ is I_o

Solving for the Current ($t \geq 0$)

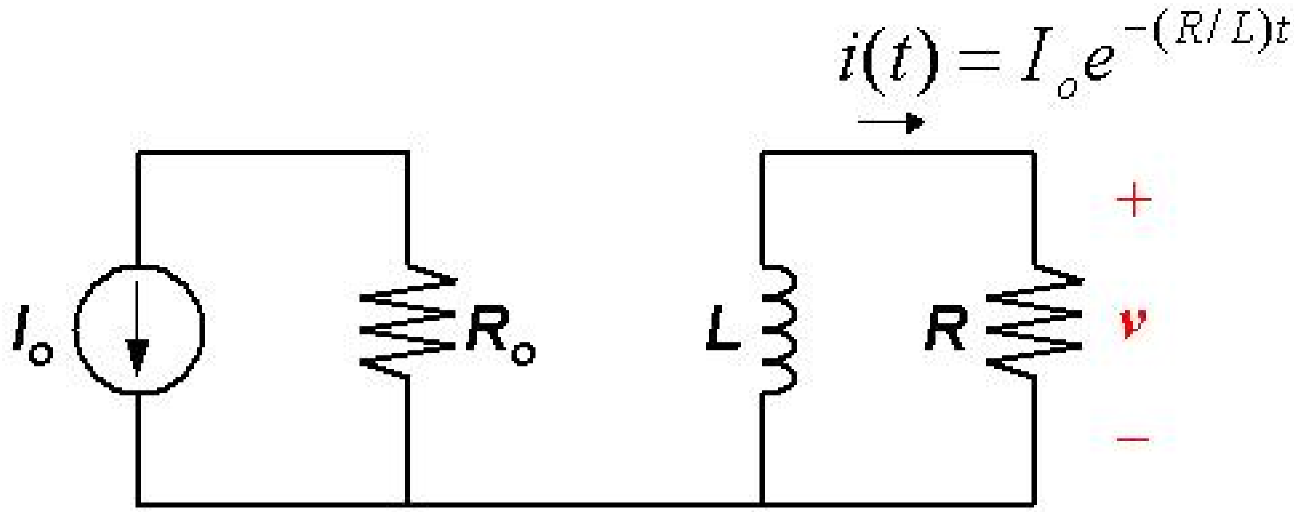
- For t > 0, the circuit reduces to



- Applying KVL to the LR circuit:

- Solution: $i(t) = i(0)e^{-(R/L)t}$

Solving for the Voltage ($t > 0$)



- Note that the voltage changes abruptly:

$$v(0^-) = 0$$

$$\text{for } t > 0,\; v(t) = iR = I_o R e^{-(R/L)t}$$

$$\Rightarrow\; v(0^+) = I_o R$$

Time Constant τ

- In the example, we found that

$$i(t) = I_o e^{-(R/L)t} \quad \text{and} \quad v(t) = I_o R e^{-(R/L)t}$$

- Define the ***time constant*** $\tau = \frac{L}{R}$
  - At $t = \tau$, the current has reduced to $1/e$ (~0.37) of its initial value.
  - At $t = 5\tau$, the current has reduced to less than 1% of its initial value.

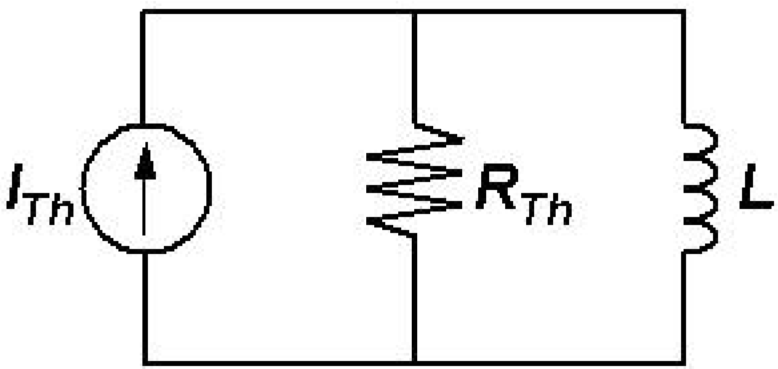
Transient *vs.* Steady-State Response

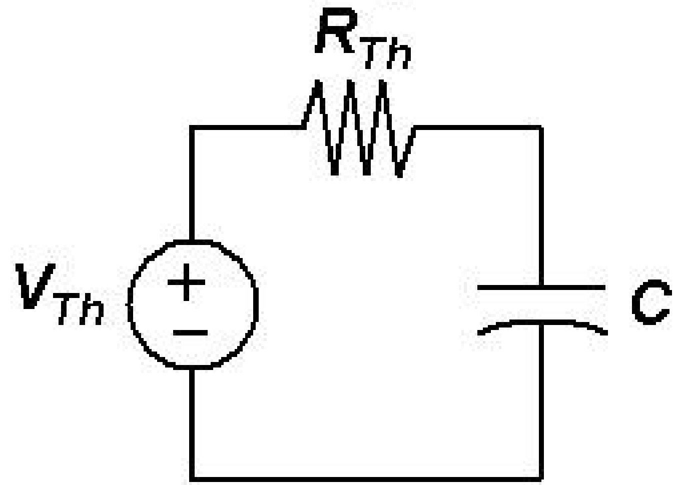
- The momentary behavior of a circuit (in response to a change in stimulation) is referred to as its ***transient response***.

- The behavior of a circuit a long time (many time constants) after the change in voltage or current is called the ***steady-state response***.

Review (Conceptual)

- Any* first-order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.

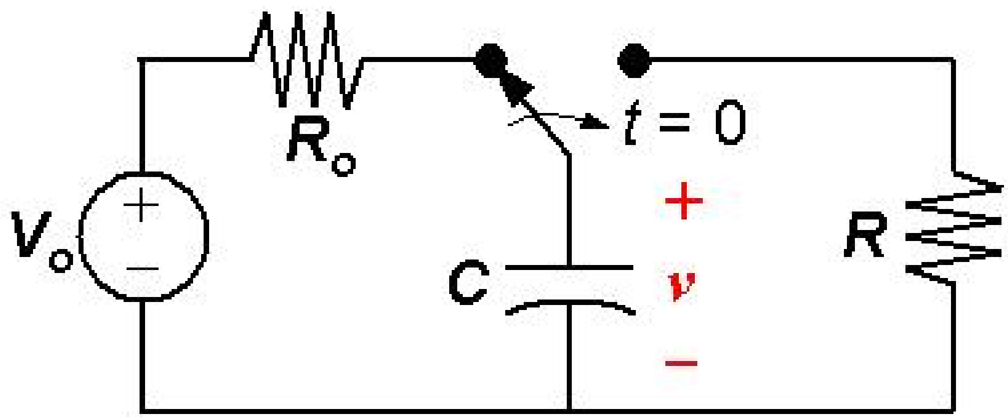




  – In steady state, an inductor behaves like a short circuit
  – In steady state, a capacitor behaves like an open circuit

Natural Response of an RC Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:



Notation:

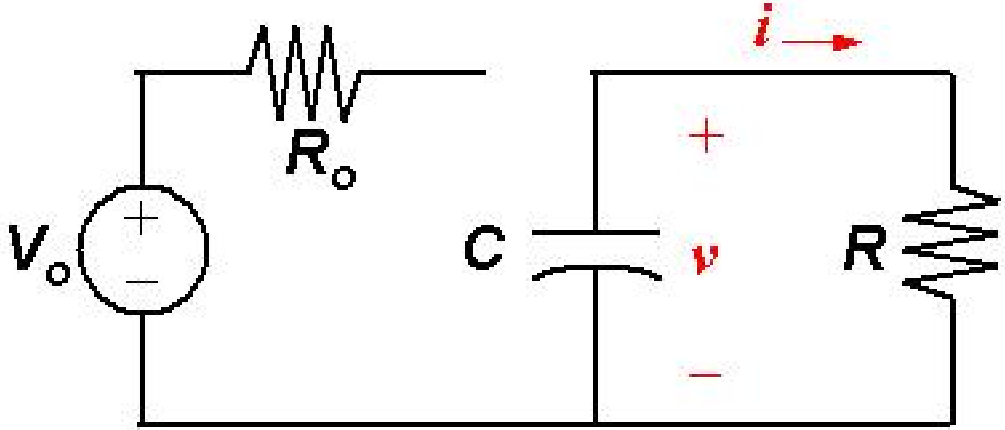
0^- is used to denote the time just prior to switching

0^+ is used to denote the time immediately after switching

- The voltage on the capacitor at $t = 0^-$ is V_o

Solving for the Voltage ($t \geq 0$)

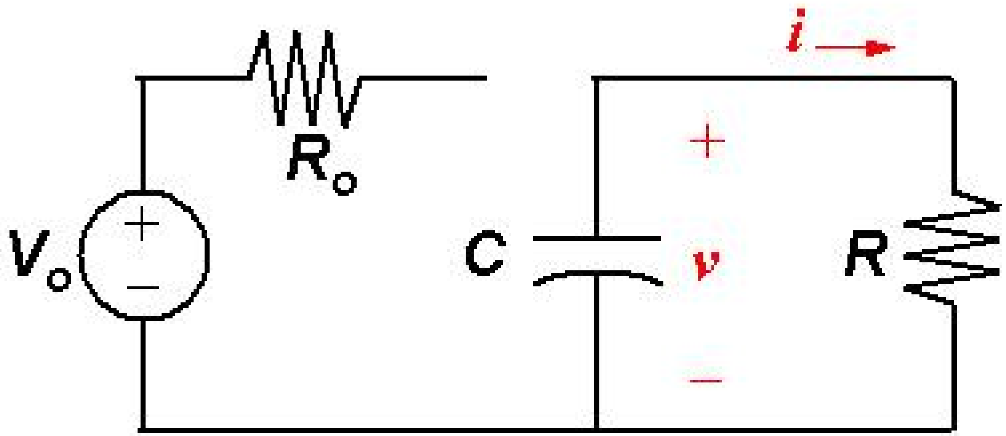
- For t > 0, the circuit reduces to



- Applying KCL to the RC circuit:

- Solution: $v(t) = v(0)e^{-t/RC}$

Solving for the Current (t > 0)



$$v(t) = V_o e^{-t/RC}$$

- Note that the current changes abruptly:

$$i(0^-) = 0$$

$$\text{for } \ t > 0,\ i(t) = \frac{v}{R} = \frac{V_o}{R} e^{-t/RC}$$

$$\Rightarrow \ i(0^+) = \frac{V_o}{R}$$

Time Constant τ

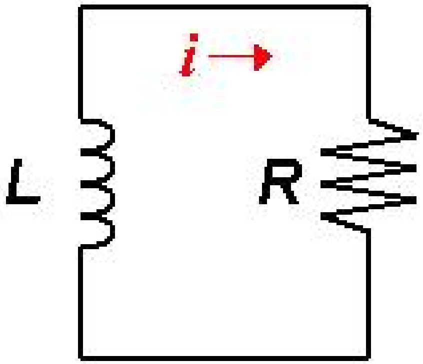
- In the example, we found that

$$v(t) = V_o e^{-t/RC} \quad \text{and} \quad i(t) = \frac{V_o}{R} e^{-t/RC}$$

- Define the ***time constant*** $\tau = RC$
  - At $t = \tau$, the voltage has reduced to $1/e$ (~0.37) of its initial value.
  - At $t = 5\tau$, the voltage has reduced to less than 1% of its initial value.

Natural Response Summary

RL Circuit



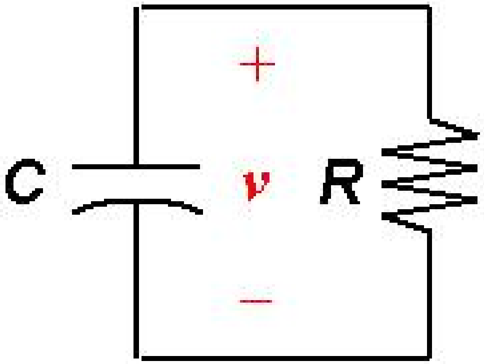
- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \dfrac{L}{R}$

RC Circuit



- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$