

Lecture #13

ANNOUNCEMENTS

- Reader with reference material from Howe&Sodini and from Rabaey et al available at Copy Central on Hearst Ave.

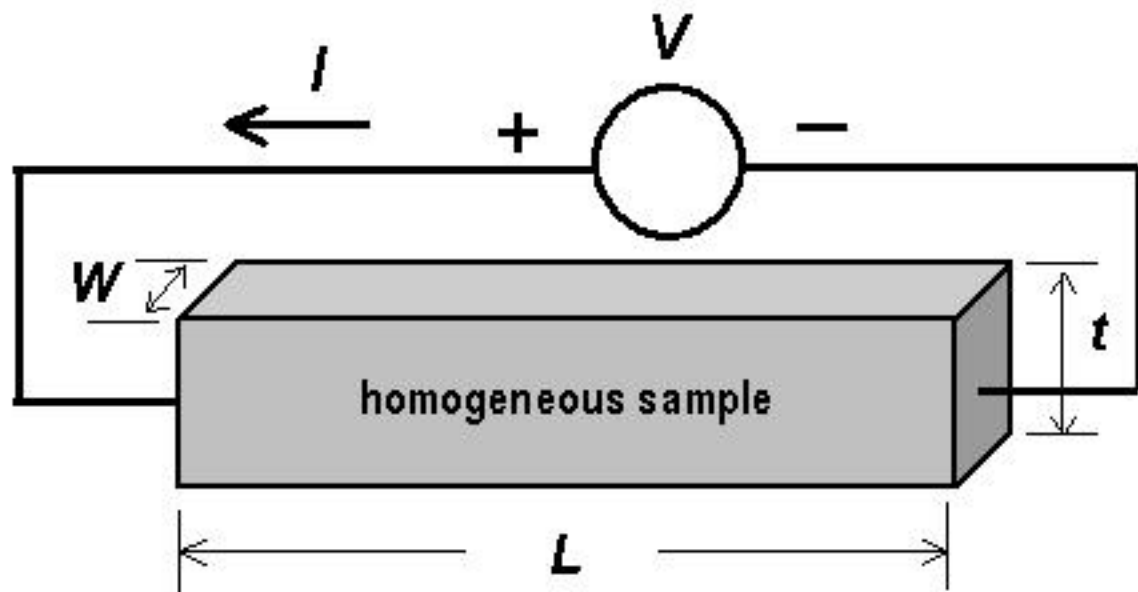
OUTLINE

- Semiconductor materials
- Properties of silicon
- Doping

Reading

Howe&Sodini: Ch. 2.1-2.4.1

Electrical Resistance



Resistance

$$R \equiv \frac{V}{I} = \rho \frac{L}{Wt}$$

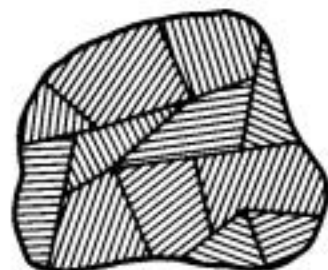
(Units: Ω)

where ρ is the **resistivity**

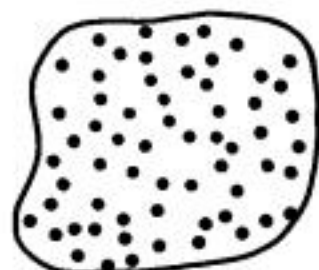
(Units: $\Omega\text{-cm}$)

What is a Semiconductor?

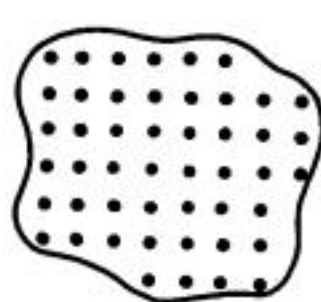
- Low resistivity => “conductor”
- High resistivity => “insulator”
- Intermediate resistivity => “semiconductor”
 - Generally, the semiconductor material used in integrated-circuit devices is crystalline
 - In recent years, however, non-crystalline semiconductors have become commercially very important



polycrystalline



amorphous



crystalline

Semiconductor Materials

Elemental:

Compound:

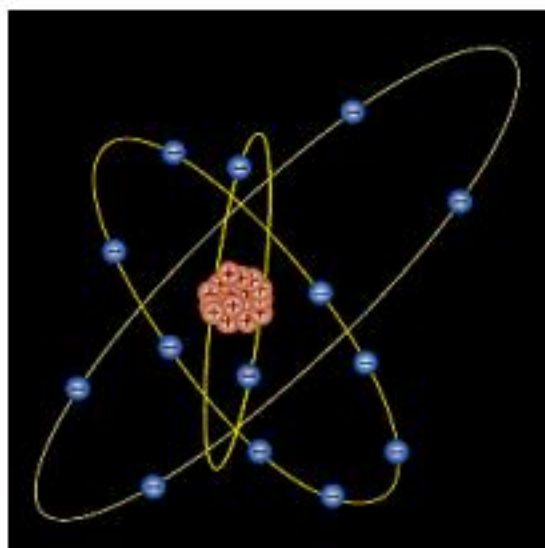
| | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|--|------------|----------|------------|-----------|------------|----------|------------|
| | | | | | | | 2 He |
| | | 5 B | 6 C | 7 N | 8 O | 9 F | 10 Ne |
| | | 13 Al | 14 Si | 15 P | 16 S | 17 Cl | 18 Ar |
| | 30 Zn | 31 Ga | 32 Ge | 33 As | 34 Se | 35 Br | 36 Kr |
| | 48 Cd | 49 In | 50 Sn | 51 Sb | 52 Te | 53 I | 54 Xe |
| | 80 Hg | 81 Tl | 82 Pb | 83 Bi | 84 Po | 85 At | 86 Rn |
| | 112 Uub | | 114 Uuq | | 116 Uuh | | 118 Uuo |
| | 66 Dy | 67 Ho | 68 Er | 69 Tm | 70 Yb | | |
| | 98 Cf | 99 Es | 100 Fm | 101 Md | 102 No | | |

The Silicon Atom

- 14 electrons occupying the 1st 3 energy levels:
 - 1s, 2s, 2p orbitals filled by 10 electrons
 - 3s, 3p orbitals filled by 4 electrons

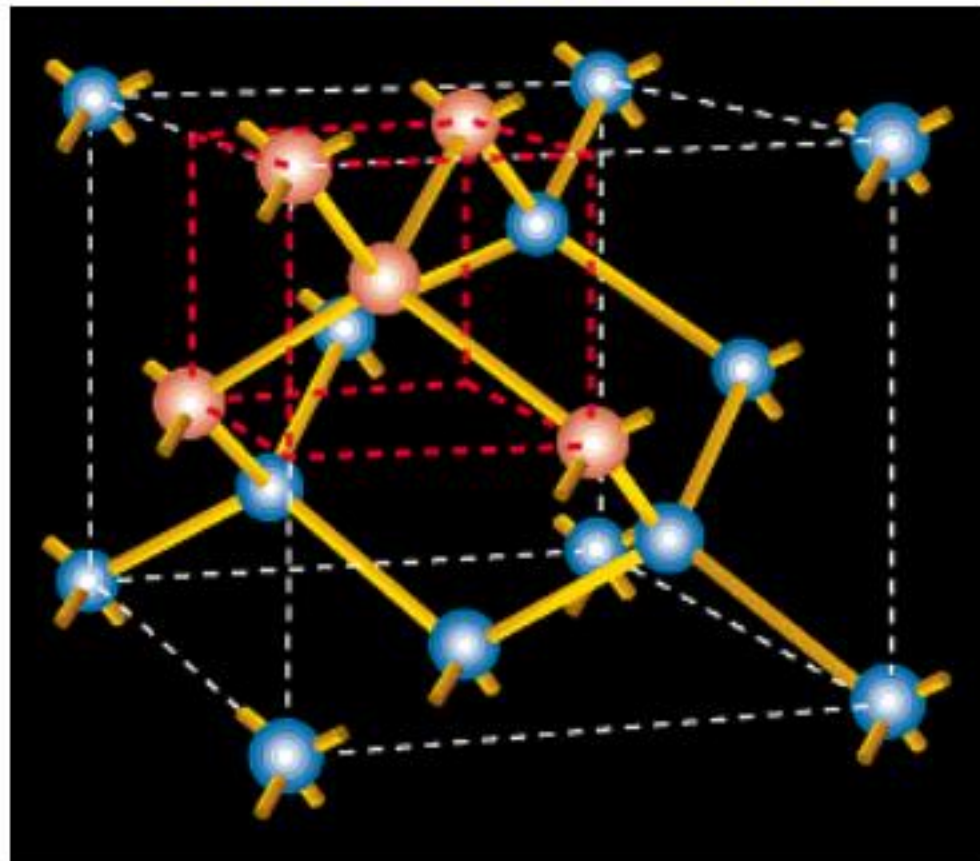
To minimize the overall energy, the 3s and 3p orbitals hybridize to form 4 tetrahedral 3sp orbitals

Each has one electron and is capable of forming a bond with a neighboring atom



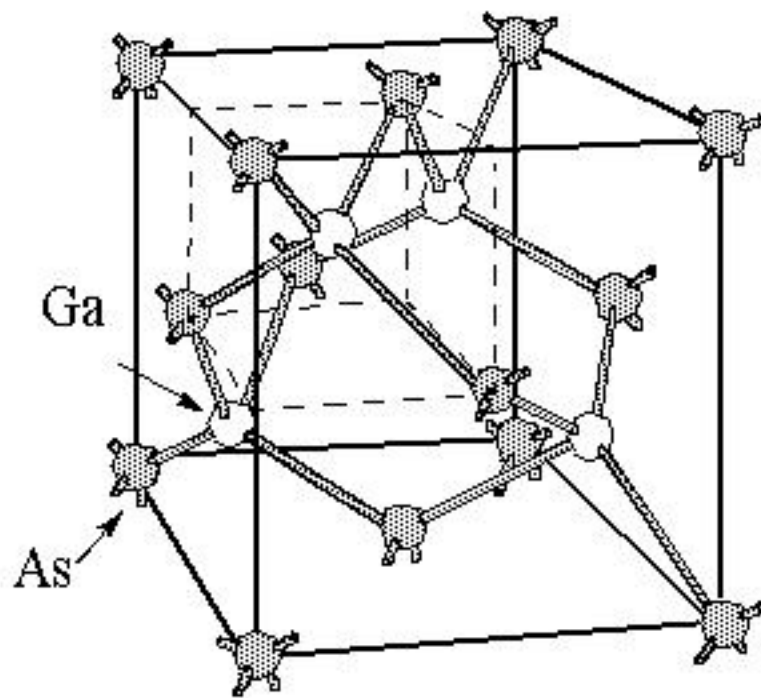
The Si Crystal

“diamond cubic” lattice



- Each Si atom has 4 nearest neighbors
- lattice constant = 5.431Å

Compound Semiconductors



- “zinc blende” structure
- III-V compound semiconductors: GaAs, GaP, GaN, *etc.*
 - ✓ important for optoelectronics and high-speed ICs

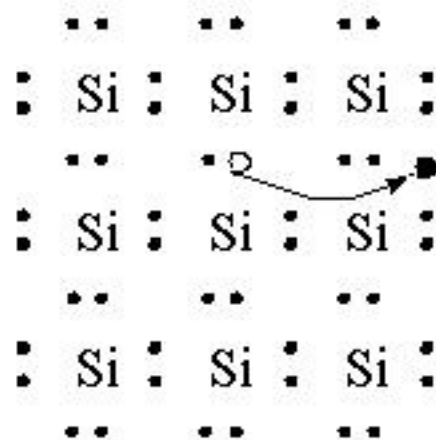
Electronic Properties of Si

- **Silicon is a semiconductor material.**
Pure Si has relatively high resistivity at room temperature.
- **There are 2 types of mobile charge-carriers in Si:**
Conduction electrons are negatively charged.
Holes are positively charged.
- **The concentration of conduction electrons & holes in a semiconductor can be affected in several ways:**
 1. by adding special impurity atoms (*dopants*)
 2. by applying an electric field
 3. by changing the temperature
 4. by irradiation

Conduction Electrons and Holes

When an electron breaks loose and becomes a **conduction electron**, a **hole** is also created.

2-D representation



Note: A hole (along with its associated positive charge) is mobile!

Definition of Parameters

n = number of mobile electrons per cm^3

p = number of holes per cm^3

n_i = intrinsic carrier concentration ($\#/\text{cm}^3$)

In a pure semiconductor,

$$n = p = n_i$$

Generation

- We have seen that conduction (mobile) electrons and holes can be created in pure (intrinsic) silicon by **thermal generation**.
 - Thermal generation rate increases exponentially with temperature T
- Another type of generation process which can occur is **optical generation**
 - The energy absorbed from a photon frees an electron from covalent bond
 - In Si, the minimum energy required is **1.1eV**, which corresponds to $\sim 1 \mu\text{m}$ wavelength (infrared region)
- Note that conduction electrons and holes are continuously generated, if $T > 0$

Recombination

- When a conduction electron and hole meet, each one is eliminated. The energy lost by the conduction electron (when it “falls” back into the covalent bond) can be released in 2 ways:
 1. to the semiconductor lattice (vibrations)
“thermal recombination” → semiconductor is heated
 2. to photon emission
“optical recombination” → light is emitted
 - Optical recombination is negligible in Si. It is significant in compound semiconductor materials, and is the basis for light-emitting diodes and laser diodes.

Generation and Recombination Rates

- The generation rate is dependent on temperature T , but it is independent of n and p : $G = G_{thermal}(T) + G_{optical}$

- The recombination rate is proportional to both n and p :

$$R \propto np$$

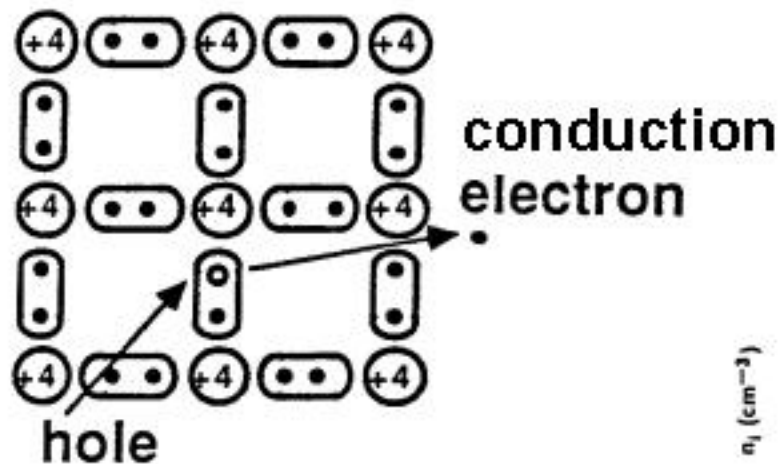
- In steady state, a balance exists between the generation and recombination rates.

$$G = R \quad \Rightarrow \quad np = f(T)$$

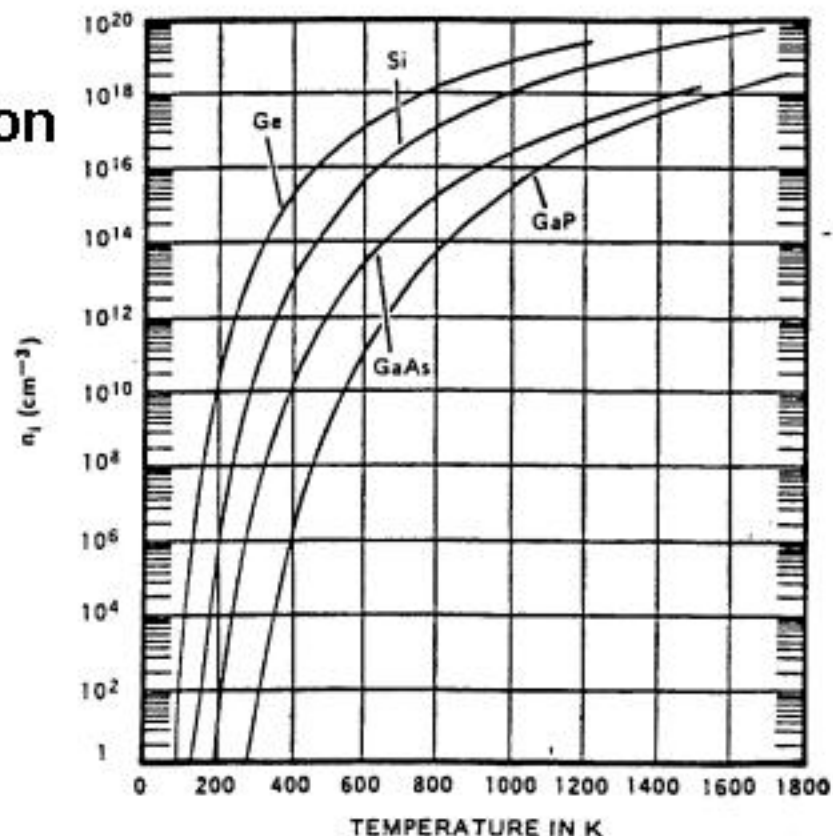
- A special case of the steady-state condition is **thermal equilibrium**: no optical or electrical sources

$$np = n_i^2(T)$$

Pure Si



Covalent (shared e^-) bonds exist between Si atoms in a crystal. Since the e^- are loosely bound, some will be free at any T , creating hole electron pairs.



Si:

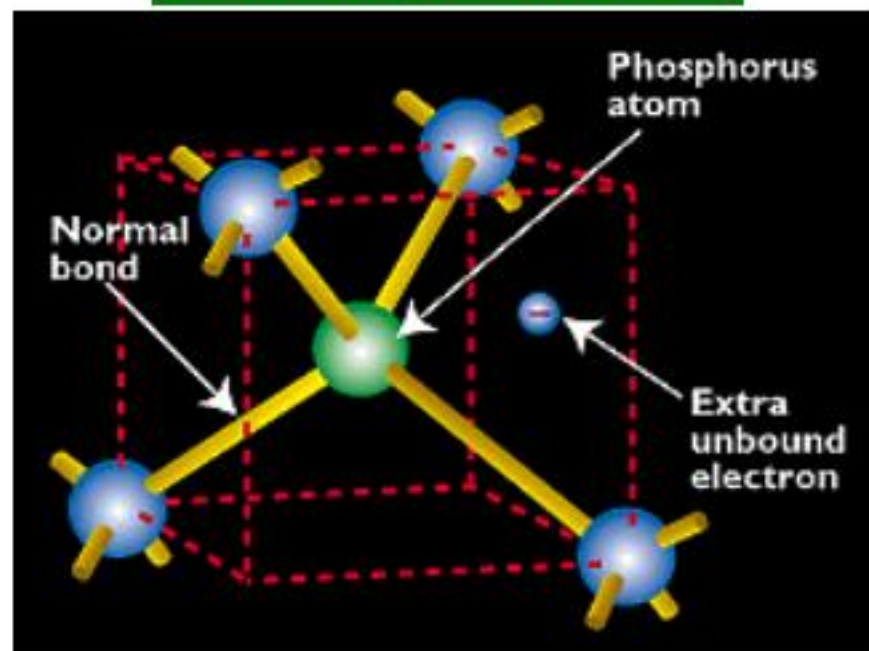
$$n_i = 3.9 \times 10^{16} T^{3/2} e^{-\frac{0.605\text{eV}}{kT}} / \text{cm}^3$$

$$n_i \cong 10^{10} \text{ cm}^{-3} \text{ at room temperature}$$

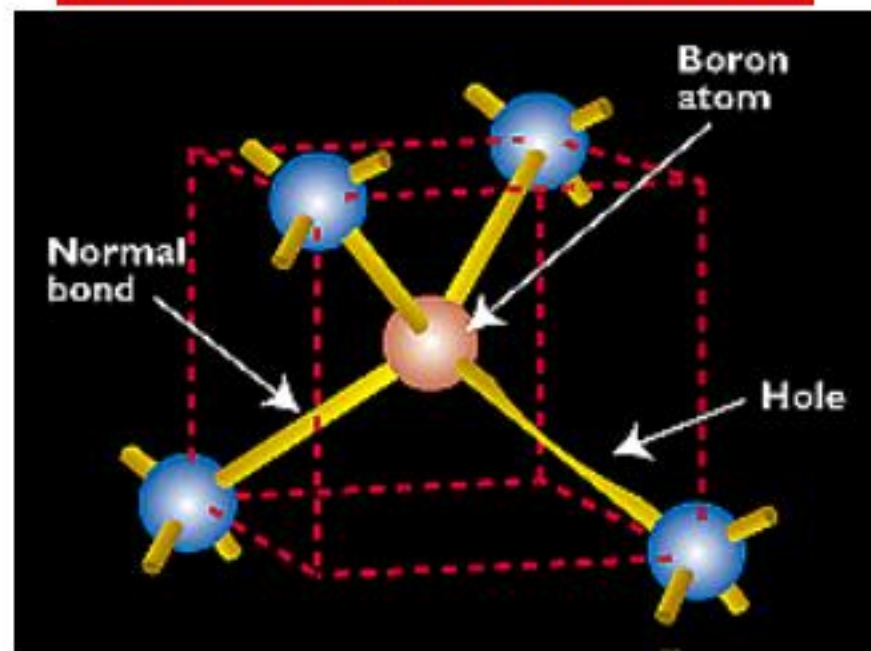
Doping

By substituting a Si atom with a special impurity atom (**Column V** or **Column III** element), a conduction electron or hole is created.

Donors: P, As, Sb



Acceptors: B, Al, Ga, In



Dopant concentrations typically range from 10^{14} cm^{-3} to 10^{20} cm^{-3}

Charge-Carrier Concentrations

N_D : ionized donor concentration (cm^{-3})

N_A : ionized acceptor concentration (cm^{-3})

Charge neutrality condition: $N_D + p = N_A + n$

At thermal equilibrium, $np = n_i^2$ (“Law of Mass Action”)

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

Note: Carrier concentrations depend on *net dopant concentration* ($N_D - N_A$) !

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

N-type and P-type Material

If $N_D \gg N_A$ (so that $N_D - N_A \gg n_i$):

$$n \cong N_D - N_A \quad \text{and} \quad p \cong \frac{n_i^2}{N_D - N_A}$$

$n \gg p \rightarrow$ material is “n-type”

If $N_A \gg N_D$ (so that $N_A - N_D \gg n_i$):

$$p \cong N_A - N_D \quad \text{and} \quad n \cong \frac{n_i^2}{N_A - N_D}$$

$p \gg n \rightarrow$ material is “p-type”

Terminology

intrinsic semiconductor: “undoped” semiconductor
electrical properties are native to the material

extrinsic semiconductor: doped semiconductor
electrical properties are controlled by the added impurity atoms

donor: impurity atom that increases the electron concentration
group V elements (P, As)

acceptor: impurity atom that increases the hole concentration
group III elements (B, In)

n-type material: semiconductor containing more electrons than holes

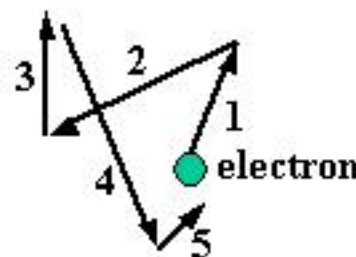
p-type material: semiconductor containing more holes than electrons

majority carrier: the most abundant carrier in a semiconductor sample

minority carrier: the least abundant carrier in a semiconductor sample

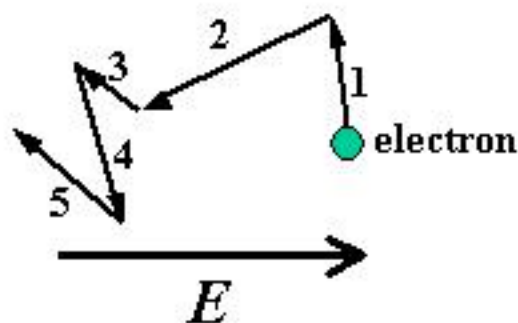
Carrier Scattering

- **Mobile electrons and atoms in the Si lattice are always in random thermal motion.**
 - Average velocity of thermal motion for electrons in Si:
 $\sim 10^7$ cm/s @ 300K
 - Electrons make frequent collisions with the vibrating atoms
 - “lattice scattering” or “phonon scattering”
 - Other scattering mechanisms:
 - deflection by ionized impurity atoms
 - deflection due to Coulombic force between carriers
- **The average current in any direction is zero, if no electric field is applied.**



Carrier Drift

- When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:



- **Electrons *drift* in the direction opposite to the E -field**
→ **Current flows**
- ❖ Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as classical particles moving at a constant average *drift velocity*.

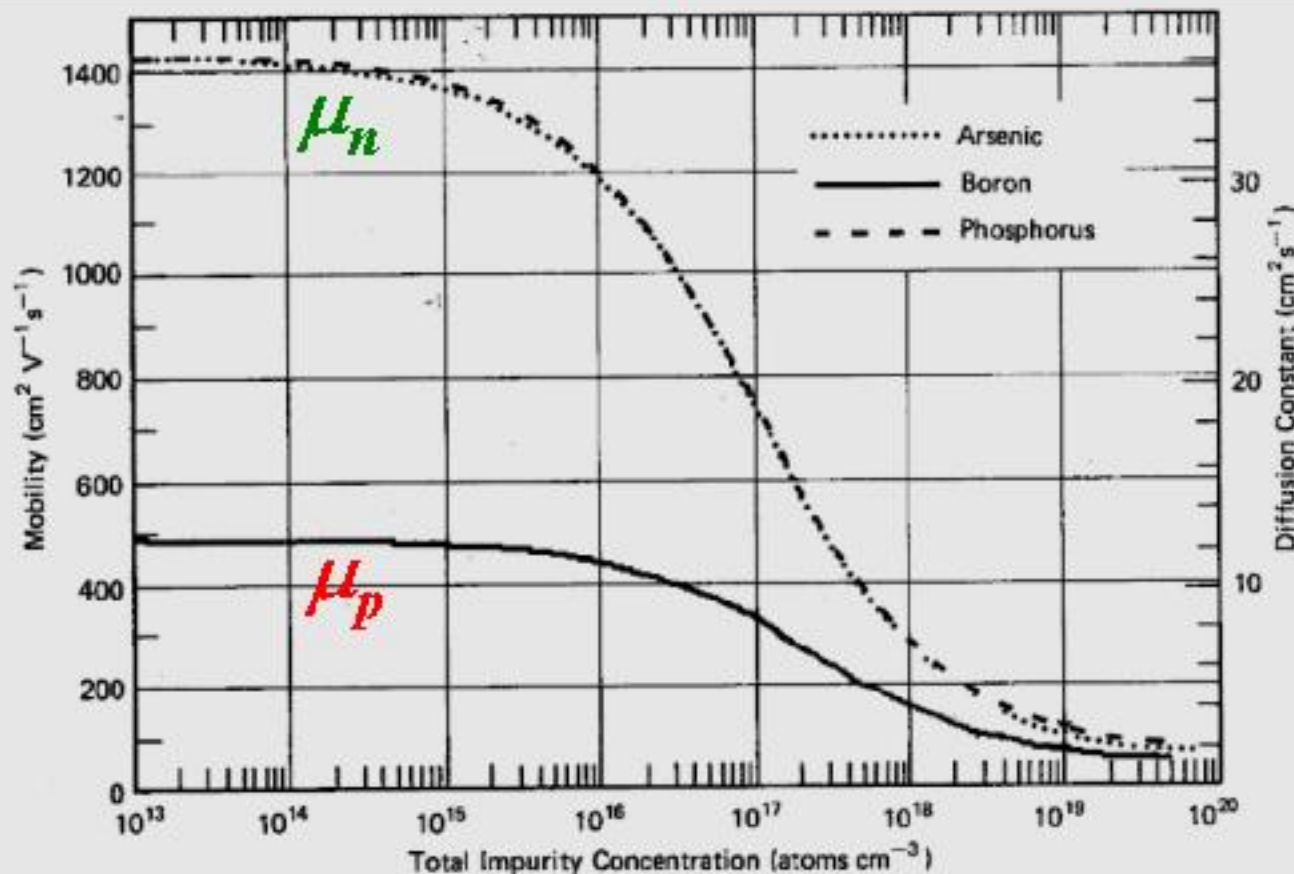
Drift Velocity and Carrier Mobility

Mobile charge-carrier drift velocity is proportional to applied E -field:

$$|v| = \mu E$$

μ is the **mobility**

(Units: $\text{cm}^2/\text{V}\cdot\text{s}$)

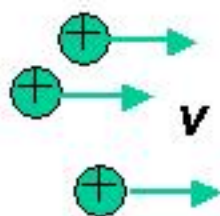


Note: Carrier mobility depends on *total* dopant concentration ($N_D + N_A$) !

Current Density

The current density \mathbf{J} is the current per unit area
($\mathbf{J} = I / \mathbf{A}$; \mathbf{A} is the cross-sectional area of the conductor)

If we have N positive charges per unit volume moving with average speed v in the $+x$ direction, then the current density in the $+x$ direction is just $\mathbf{J} = qNv$

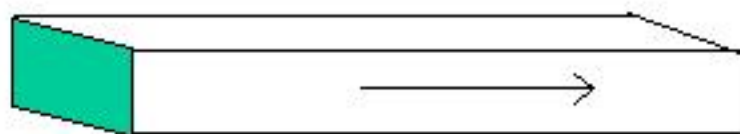


Example:

2×10^{16} holes/cm³ moving to the right at 2×10^4 cm/sec

$$\mathbf{J} = 1.6 \times 10^{-19} \times 2 \times 10^{16} \times 2 \times 10^4 = 64 \text{ A/cm}^2$$

Suppose this occurs in a conductor $2 \mu\text{m}$ wide and $1 \mu\text{m}$ thick:



$$\begin{aligned} I &= J \times A = 64 \times (2 \times 10^{-4} \times 1 \times 10^{-4}) \\ &= 1.28 \mu\text{A} \end{aligned}$$

Electrical Conductivity σ

When an electric field is applied, current flows due to drift of mobile electrons and holes:

electron current density: $J_n = (-q)nv_n = qn\mu_n E$

hole current density: $J_p = (+q)pv_p = qp\mu_p E$

total current density: $J = J_n + J_p = (qn\mu_n + qp\mu_p)E$

$$J = \sigma E$$

conductivity

$$\sigma \equiv qn\mu_n + qp\mu_p$$

Electrical Resistivity ρ

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

$$\rho \cong \frac{1}{qn\mu_n} \quad \text{for n-type mat'l}$$

$$\rho \cong \frac{1}{qp\mu_p} \quad \text{for p-type mat'l}$$

(Units: ohm-cm)

Example


Consider a Si sample doped with $10^{16}/\text{cm}^3$ Boron.
What is its resistivity?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 0 \quad (N_A \gg N_D \rightarrow \text{p-type})$$

$$\rightarrow p \approx 10^{16}/\text{cm}^3 \quad \text{and} \quad n \approx 10^4/\text{cm}^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qp\mu_p}$$
$$= \left[(1.6 \times 10^{-19})(10^{16})(450) \right]^{-1} = 1.4 \Omega\text{-cm}$$

From μ vs. $(N_A + N_D)$ plot 

Example (cont'd)

Consider the same Si sample, doped *additionally* with $10^{17}/\text{cm}^3$ Arsenic. What is its resistivity?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 10^{17}/\text{cm}^3 \quad (N_D \gg N_A \rightarrow \text{n-type})$$

$$\rightarrow n \approx 9 \times 10^{16}/\text{cm}^3 \quad \text{and} \quad p \approx 1.1 \times 10^3/\text{cm}^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \approx \frac{1}{qn\mu_n}$$

$$= \left[(1.6 \times 10^{-19})(9 \times 10^{16})(700) \right]^{-1} = 0.10 \, \Omega\text{-cm}$$

The sample is converted to n-type material by adding more donors than acceptors, and is said to be “compensated”.

Sheet Resistance R_s

$$R = \rho \frac{L}{Wt} = R_s \frac{L}{W} \Rightarrow \boxed{R_s \equiv \frac{\rho}{t}} \quad (\text{Unit: ohms/square})$$

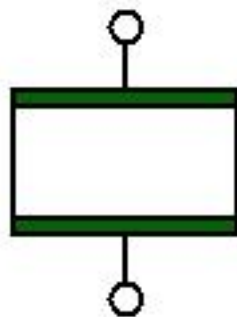
R_s is the resistance when $W = L$

- The R_s value for a given layer in an IC technology is used
 - for design and layout of resistors
 - for estimating values of parasitic resistance in a circuit

$R = R_s$



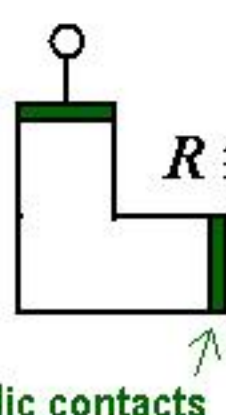
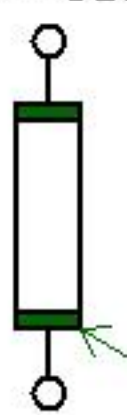
$R = R_s/2$



$R = 2R_s$



$R = 3R_s$

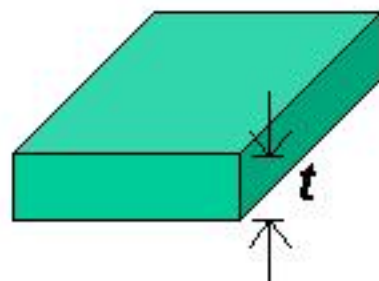


$R \cong 2.6R_s$

Integrated-Circuit Resistors

The resistivity ρ and thickness t are fixed for each layer in a given manufacturing process

A circuit designer specifies the length L and width W , to achieve a desired resistance R



$$R = R_s \left(\frac{L}{W} \right)$$

fixed designable

Example: Suppose we want to design a $5 \text{ k}\Omega$ resistor using a layer of material with $R_s = 200 \text{ }\Omega/\square$

Resistor layout (top view)



Space-efficient layout



Summary

- **Crystalline Si:**
 - 4 valence electrons per atom
 - diamond lattice: each atom has 4 nearest neighbors
 - 5×10^{22} atoms/cm³
- **In a pure Si crystal, conduction electrons and holes are formed in pairs.**
 - Holes can be considered as positively charged mobile particles which exist inside a semiconductor.
 - **Both holes and electrons can conduct current.**
- **Dopants in Si:**
 - Reside on lattice sites (substituting for Si)
 - Group V elements contribute conduction electrons, and are called *donors*
 - Group III elements contribute holes, and are called *acceptors*