Lecture #13

ANNOUNCEMENTS

- Reader with reference material from Howe&Sodini and from Rabaey et al available at Copy Central on Hearst Ave.

OUTLINE

- Semiconductor materials
- Properties of silicon
- Doping

Reading

Howe&Sodini: Ch. 2.1-2.4.1
Electrical Resistance

\[ R = \frac{V}{I} = \frac{L}{Wt} \rho \]

(Uits: \( \Omega \))

where \( \rho \) is the resistivity

(Uits: \( \Omega \cdot \text{cm} \))
What is a Semiconductor?

- Low resistivity => "conductor"
- High resistivity => "insulator"
- Intermediate resistivity => "semiconductor"
  - Generally, the semiconductor material used in integrated-circuit devices is crystalline
    - In recent years, however, non-crystalline semiconductors have become commercially very important
## Semiconductor Materials

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The Silicon Atom

- 14 electrons occupying the 1st 3 energy levels:
  - 1s, 2s, 2p orbitals filled by 10 electrons
  - 3s, 3p orbitals filled by 4 electrons

To minimize the overall energy, the 3s and 3p orbitals hybridize to form 4 tetrahedral 3sp orbitals

Each has one electron and is capable of forming a bond with a neighboring atom
The Si Crystal

“diamond cubic” lattice

- Each Si atom has 4 nearest neighbors
- lattice constant = 5.431 Å
Compound Semiconductors

- "zinc blende" structure
- III-V compound semiconductors: GaAs, GaP, GaN, etc.
  - important for optoelectronics and high-speed ICs
Electronic Properties of Si

- Silicon is a semiconductor material. Pure Si has relatively high resistivity at room temperature.

- There are 2 types of mobile charge-carriers in Si:
  - *Conduction electrons* are negatively charged.
  - *Holes* are positively charged.

- The concentration of conduction electrons & holes in a semiconductor can be affected in several ways:
  1. by adding special impurity atoms (*dopants*)
  2. by applying an electric field
  3. by changing the temperature
  4. by irradiation
Conduction Electrons and Holes

When an electron breaks loose and becomes a conduction electron, a hole is also created.

Note: A hole (along with its associated positive charge) is mobile!
Definition of Parameters

\[ n = \text{number of mobile electrons per cm}^3 \]
\[ p = \text{number of holes per cm}^3 \]
\[ n_i = \text{intrinsic carrier concentration (#/cm}^3) \]

In a pure semiconductor,
\[ n = p = n_i \]
Generation

- We have seen that conduction (mobile) electrons and holes can be created in pure (intrinsic) silicon by **thermal generation**.
  - Thermal generation rate increases exponentially with temperature $T$.

- Another type of generation process which can occur is **optical generation**.
  - The energy absorbed from a photon frees an electron from covalent bond.
    - In Si, the minimum energy required is $1.1\text{eV}$, which corresponds to $\sim 1 \mu\text{m}$ wavelength (infrared region).

- Note that conduction electrons and holes are continuously generated, if $T > 0$. 
Recombination

- When a conduction electron and hole meet, each one is eliminated. The energy lost by the conduction electron (when it “falls” back into the covalent bond) can be released in 2 ways:
  1. to the semiconductor lattice (vibrations)
     "thermal recombination” → semiconductor is heated
  2. to photon emission
     "optical recombination” → light is emitted
- Optical recombination is negligible in Si. It is significant in compound semiconductor materials, and is the basis for light-emitting diodes and laser diodes.
Generation and Recombination Rates

• The generation rate is dependent on temperature $T$, but it is independent of $n$ and $p$: $G = G_{\text{thermal}}(T) + G_{\text{optical}}$

• The recombination rate is proportional to both $n$ and $p$: $R \propto np$

• In steady state, a balance exists between the generation and recombination rates.

\[ G = R \quad \Rightarrow \quad np = f(T) \]

• A special case of the steady-state condition is thermal equilibrium: no optical or electrical sources

\[ np = n_i^2(T) \]
Covalent (shared e\(^-\)) bonds exists between Si atoms in a crystal. Since the e\(^-\) are loosely bound, some will be free at any \(T\), creating hole electron pairs.

\[
\frac{S_i}{n_i} = 3.9 \times 10^{16} T^{3/2} e^{-\frac{0.605\text{eV}}{kT}} \text{ /cm}^3
\]

\(n_i \approx 10^{10} \text{ cm}^{-3}\) at room temperature
Doping

By substituting a Si atom with a special impurity atom (Column V or Column III element), a conduction electron or hole is created.

**Donors:** P, As, Sb

**Acceptors:** B, Al, Ga, In

Dopant concentrations typically range from $10^{14}$ cm$^{-3}$ to $10^{20}$ cm$^{-3}$
Charge-Carrier Concentrations

\( N_D \): ionized donor concentration \( \text{(cm}^{-3}\text{)} \)
\( N_A \): ionized acceptor concentration \( \text{(cm}^{-3}\text{)} \)

Charge neutrality condition: \( N_D + p = N_A + n \)

At thermal equilibrium, \( n p = n_i^2 \) \( \text{("Law of Mass Action")} \)

\[
\begin{align*}
n &= \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} \\
p &= \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}
\end{align*}
\]

Note: Carrier concentrations depend on net dopant concentration \( (N_D - N_A) \)!
N-type and P-type Material

If $N_D \gg N_A$ (so that $N_D - N_A \gg n_i$):

\[
 n \approx N_D - N_A \quad \text{and} \quad p \approx \frac{n_i^2}{N_D - N_A}
\]

$n \gg p \Rightarrow \text{material is "n-type"} $

If $N_A \gg N_D$ (so that $N_A - N_D \gg n_i$):

\[
 p \approx N_A - N_D \quad \text{and} \quad n \approx \frac{n_i^2}{N_A - N_D}
\]

$p \gg n \Rightarrow \text{material is "p-type"} $
Terminology

**intrinsic** semiconductor: “undoped” semiconductor
   electrical properties are native to the material

**extrinsic** semiconductor: doped semiconductor
   electrical properties are controlled by the added impurity atoms

**donor**: impurity atom that increases the electron concentration
   group V elements (P, As)

**acceptor**: impurity atom that increases the hole concentration
   group III elements (B, In)

**n-type** material: semiconductor containing more electrons than holes
**p-type** material: semiconductor containing more holes than electrons

**majority carrier**: the most abundant carrier in a semiconductor sample
**minority carrier**: the least abundant carrier in a semiconductor sample
Carrier Scattering

- Mobile electrons and atoms in the Si lattice are always in random thermal motion.
  - Average velocity of thermal motion for electrons in Si:
    \[ \sim 10^7 \text{ cm/s @ 300K} \]
  - Electrons make frequent collisions with the vibrating atoms
    • "lattice scattering" or "phonon scattering"
  - Other scattering mechanisms:
    • deflection by ionized impurity atoms
    • deflection due to Coulombic force between carriers

- The average current in any direction is zero, if no electric field is applied.
Carrier Drift

- When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:

![Diagram showing electron drift]

- Electrons drift in the direction opposite to the $E$-field → Current flows

- Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as classical particles moving at a constant average drift velocity.
Drift Velocity and Carrier Mobility

Mobile charge-carrier drift velocity is proportional to applied $E$-field:

$$|v| = \mu E$$

$\mu$ is the *mobility* (Units: cm$^2$/V·s)

Note: Carrier mobility depends on total dopant concentration ($N_D + N_A$)!
Current Density

The current density $J$ is the current per unit area ($J = I / A$; $A$ is the cross-sectional area of the conductor)

If we have $N$ positive charges per unit volume moving with average speed $v$ in the $+x$ direction, then the current density in the $+x$ direction is just $J = qNv$

Example:

$2 \times 10^{16}$ holes/cm$^3$ moving to the right at $2 \times 10^4$ cm/sec

$J = 1.6 \times 10^{-19} \times 2 \times 10^{16} \times 2 \times 10^4 = 64$ A/cm$^2$

Suppose this occurs in a conductor 2 $\mu$m wide and 1 $\mu$m thick:

$I = J \times A = 64 \times (2 \times 10^{-4} \times 1 \times 10^{-4})$

$= 1.28$ $\mu$A
Electrical Conductivity $\sigma$

When an electric field is applied, current flows due to drift of mobile electrons and holes:

**Electron current density:**
$$J_n = (-q)nv_n = qn\mu_n E$$

**Hole current density:**
$$J_p = (+q)pv_p = qp\mu_p E$$

**Total current density:**
$$J = J_n + J_p = (qn\mu_n + qp\mu_p)E$$

**Conductivity**
$$\sigma \equiv qn\mu_n + qp\mu_p$$
Electrical Resistivity $\rho$

\[ \rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p} \]

$\rho \approx \frac{1}{qn\mu_n}$ for n-type mat'l

$\rho \approx \frac{1}{qp\mu_p}$ for p-type mat'l

(Units: ohm-cm)
Consider a Si sample doped with $10^{16}$/cm$^3$ Boron. What is its resistivity?

**Answer:**

$N_A = 10^{16}$/cm$^3$, $N_D = 0$ \hspace{1cm} ($N_A >> N_D \rightarrow p$-type)

$\rightarrow p \approx 10^{16}$/cm$^3$ and $n \approx 10^4$/cm$^3$

$$\rho = \frac{1}{q \mu_n n + q \mu_p p} \approx \frac{1}{q \mu_p p}$$

$$= \left[ (1.6 \times 10^{-19}) (10^{16}) (450) \right]^{-1} = 1.4 \ \Omega \cdot \text{cm}$$

From $\mu$ vs. $(N_A + N_D)$ plot
Consider the same Si sample, doped additionally with $10^{17}$/cm$^3$ Arsenic. What is its resistivity?

**Answer:**

\[ N_A = 10^{16}$/cm$^3$, \quad N_D = 10^{17}$/cm$^3$ \quad (N_D \gg N_A \rightarrow \text{n-type}) \]

\[ \rightarrow n \approx 9 \times 10^{16}$/cm$^3$ \quad \text{and} \quad p \approx 1.1 \times 10^3$/cm$^3$ \]

\[ \rho = \frac{1}{q_n \mu_n + q_p \mu_p} \approx \frac{1}{q_n \mu_n} \]

\[ = \left[ (1.6 \times 10^{-19}) (9 \times 10^{16}) (700) \right]^{-1} = 0.10 \ \Omega \cdot \text{cm} \]

The sample is converted to n-type material by adding more donors than acceptors, and is said to be “compensated.”
Sheet Resistance $R_s$

$$R = \rho \frac{L}{Wt} = R_s \frac{L}{W} \implies R_s = \frac{\rho}{t}$$  (Unit: ohms/square)

$R_s$ is the resistance when $W = L$

- The $R_s$ value for a given layer in an IC technology is used
  - for design and layout of resistors
  - for estimating values of parasitic resistance in a circuit

$R = R_s$  
$R = R_s/2$ 
$R = 2R_s$  
$R = 3R_s$  
$R \approx 2.6R_s$
Integrated-Circuit Resistors

The resistivity $\rho$ and thickness $t$ are fixed for each layer in a given manufacturing process.

A circuit designer specifies the length $L$ and width $W$, to achieve a desired resistance $R$.

$$R = R_s \left( \frac{L}{W} \right)$$

**Example:** Suppose we want to design a 5 kΩ resistor using a layer of material with $R_s = 200 \ \Omega/\square$.

**Resistor layout (top view)**
Summary

- **Crystalline Si:**
  - 4 valence electrons per atom
  - Diamond lattice: each atom has 4 nearest neighbors
  - $5 \times 10^{22}$ atoms/cm$^3$

- In a pure Si crystal, conduction electrons and holes are formed in pairs.
  - Holes can be considered as positively charged mobile particles which exist inside a semiconductor.
  - Both holes and electrons can conduct current.

- **Dopants in Si:**
  - Reside on lattice sites (substituting for Si)
  - Group V elements contribute conduction electrons, and are called *donors*
  - Group III elements contribute holes, and are called *acceptors*