

Lecture #17 (cont'd from #16)

OUTLINE

- MOSFET I_D vs. V_{GS} characteristic
- Circuit models for the MOSFET
 - resistive switch model
 - small-signal model

Reading

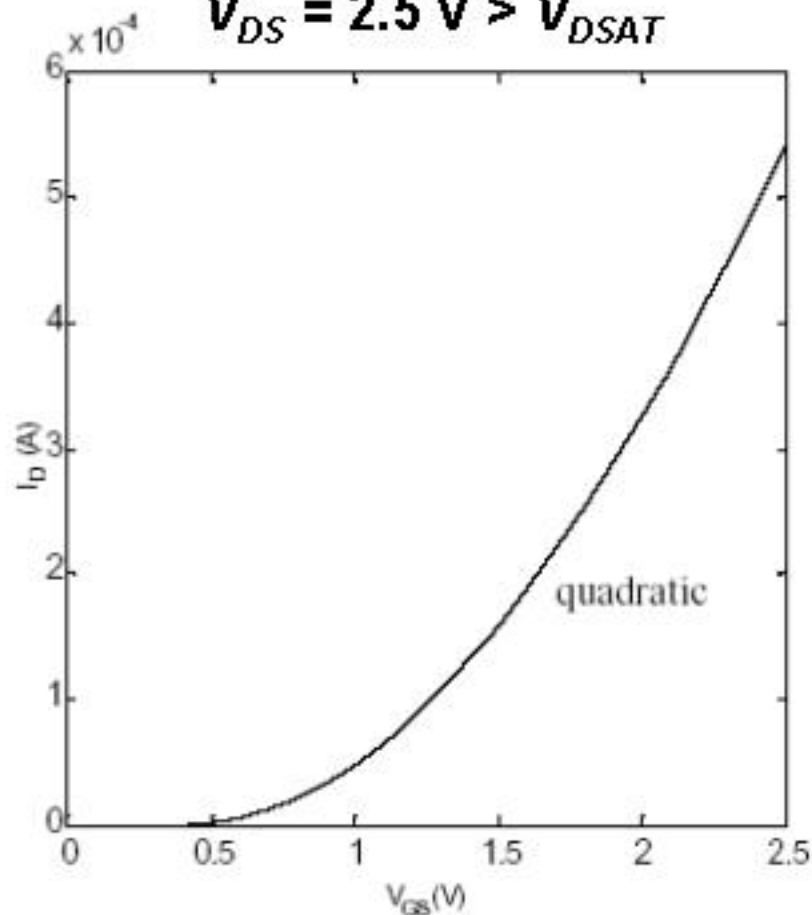
- **Rabaey et al.:** Chapter 3.3.2
- **Hambley:** Chapter 12 (through 12.5)

MOSFET I_D vs. V_{GS} Characteristic

- Typically, V_{DS} is fixed when I_D is plotted as a function of V_{GS}

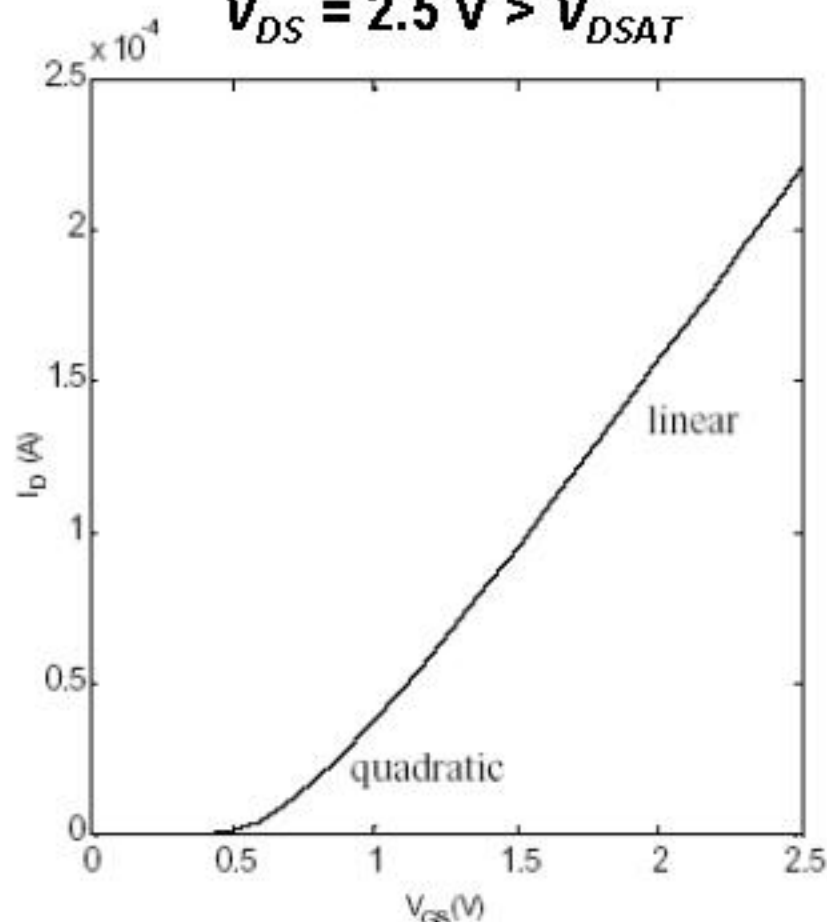
Long-channel MOSFET

$$V_{DS} = 2.5 \text{ V} > V_{DSAT}$$



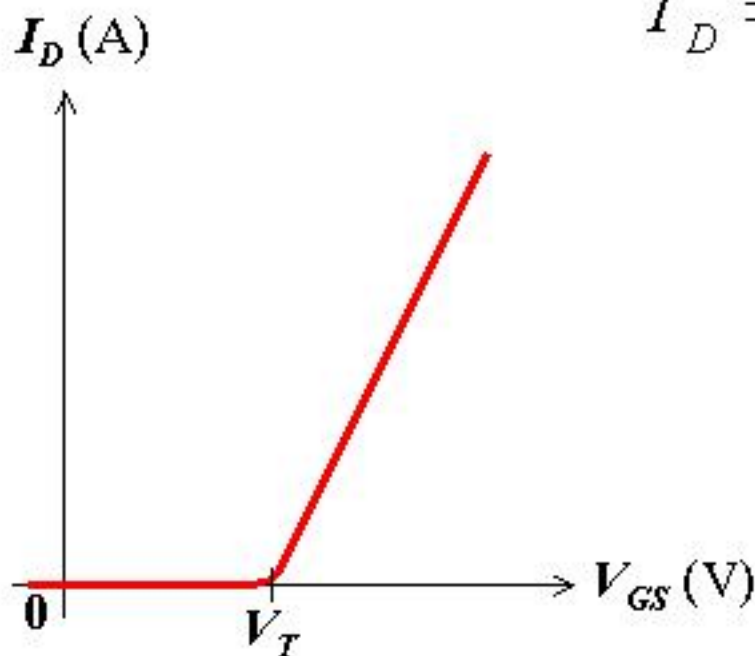
Short-channel MOSFET

$$V_{DS} = 2.5 \text{ V} > V_{DSAT}$$



MOSFET V_T Measurement

- V_T can be determined by plotting I_D vs. V_{GS} using a low value of V_{DS} :



$$I_D = k'_n \frac{W}{L} \left[V_{GS} - V_T - \frac{V_{DS}}{2} \right] V_{DS}$$

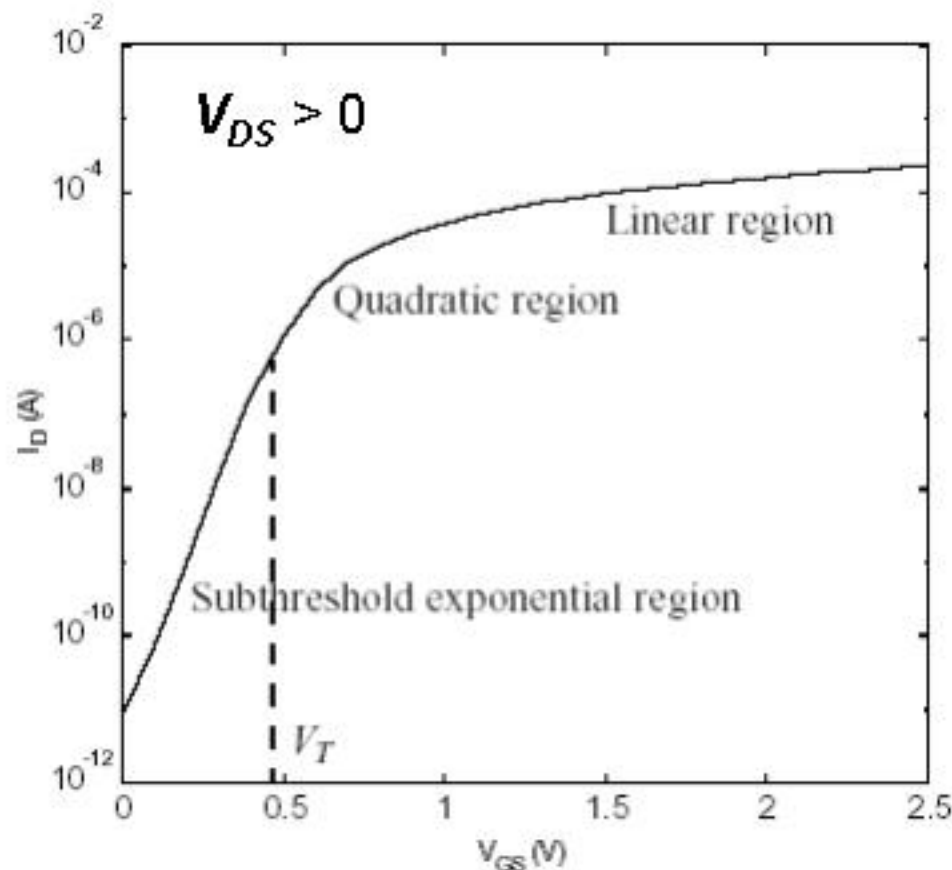
Subthreshold Conduction (Leakage Current)

- The transition from the ON state to the OFF state is gradual. This can be seen more clearly when I_D is plotted on a logarithmic scale:

- In the subthreshold ($V_{GS} < V_T$) region,

$$I_D \propto \exp\left(\frac{qV_{GS}}{nkT}\right)$$

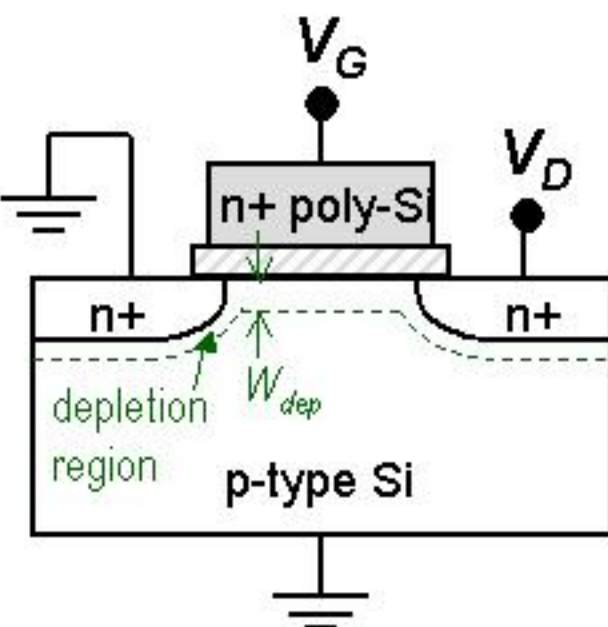
This is essentially the channel-source pn junction current.
(Some electrons diffuse from the source into the channel, if this pn junction is forward biased.)



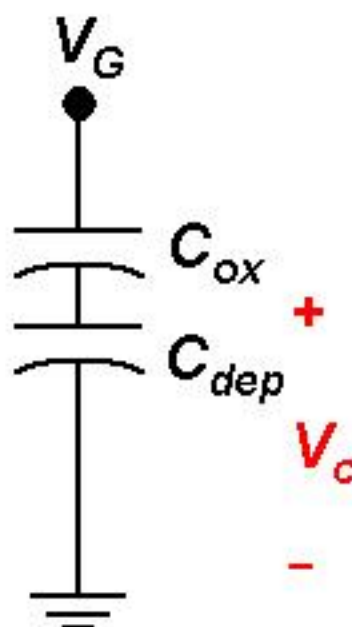
Qualitative Explanation for Subthreshold Leakage

- The channel V_c (at the Si surface) is capacitively coupled to the gate voltage V_G :

DEVICE



CIRCUIT MODEL



Using the capacitive voltage divider formula:

$$\Delta V_c = \frac{C_{ox}}{C_{ox} + C_{dep}} \Delta V_G$$

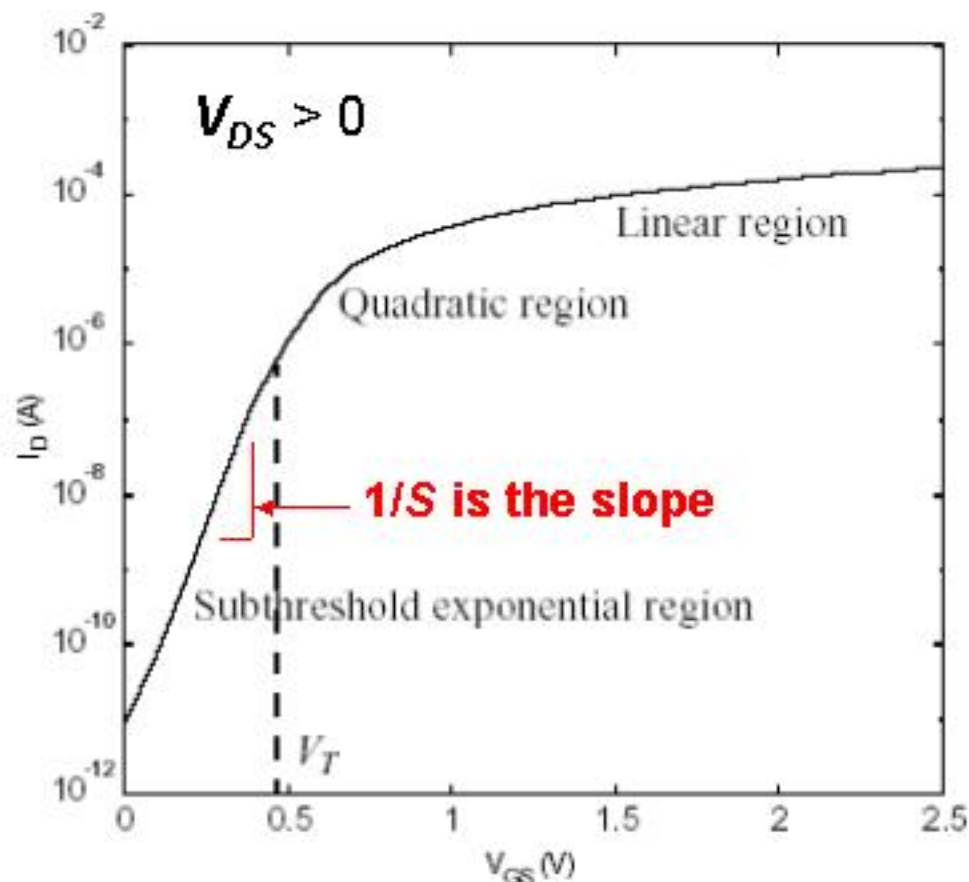
The forward bias on the channel-source pn junction increases with V_G scaled by the factor $C_{ox} / (C_{ox} + C_{dep})$

$$C_{dep} = \frac{\epsilon_{Si}}{W_{dep}} \propto \sqrt{\frac{1}{N_A}}$$

$$\Rightarrow n = \frac{C_{ox} + C_{dep}}{C_{ox}} = 1 + \frac{C_{dep}}{C_{ox}}$$

Slope Factor (or Subthreshold Swing) S

- S is defined to be the inverse slope of the log (I_D) vs. V_{GS} characteristic in the subthreshold region:



$$S \equiv n \left(\frac{kT}{q} \right) \ln(10)$$

Units: Volts per decade

Note that $S \geq 60$ mV/dec at room temperature:

$$\left(\frac{kT}{q} \right) \ln(10) = 60 \text{ mV}$$

V_T Design Trade-Off

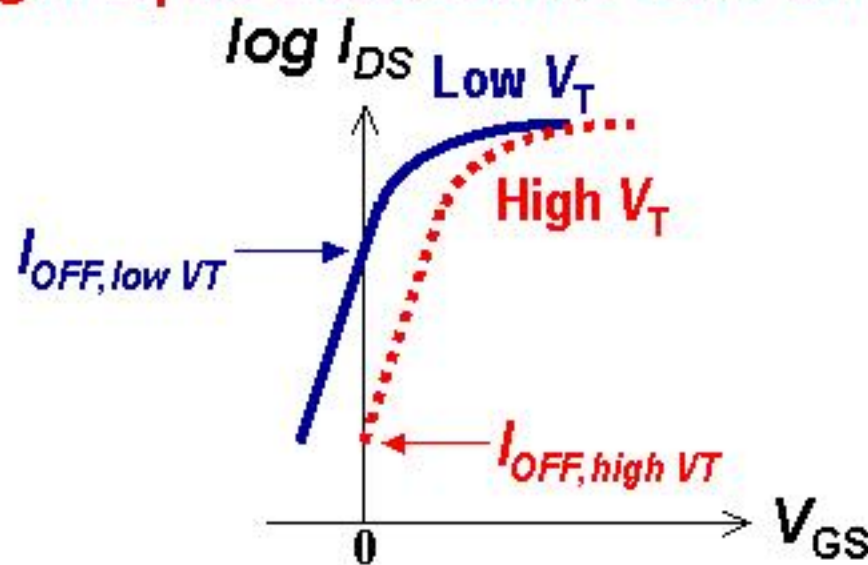
(Important consideration for digital-circuit applications)

- Low V_T is desirable for high ON current

$$I_{DSAT} \propto (V_{DD} - V_T)^\eta \quad 1 < \eta < 2$$

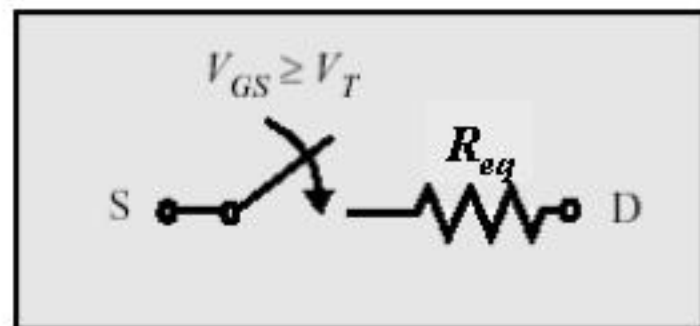
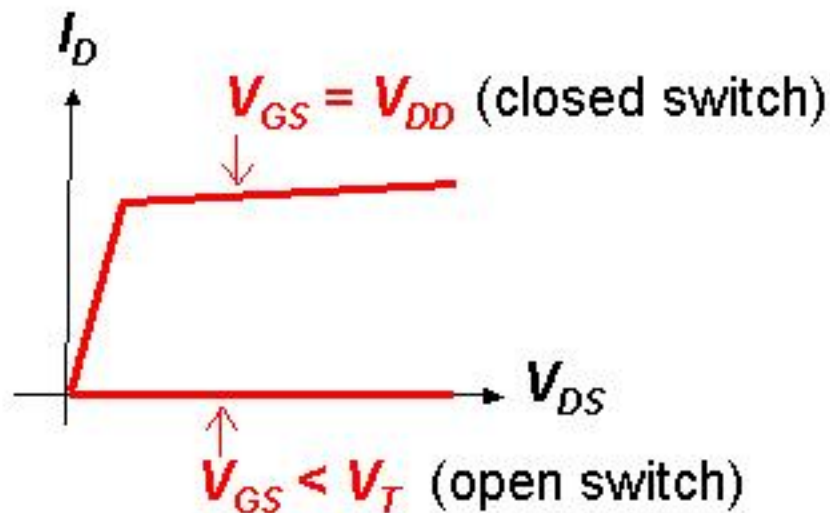
where V_{DD} is the power-supply voltage

... but high V_T is needed for low OFF current



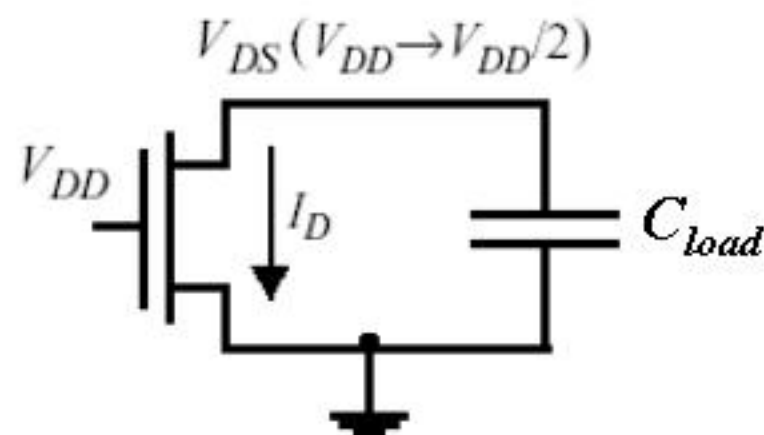
The MOSFET as a Resistive Switch

- For digital circuit applications, the MOSFET is either OFF ($V_{GS} < V_T$) or ON ($V_{GS} = V_{DD}$). Thus, we only need to consider two I_D vs. V_{DS} curves:
 - the curve for $V_{GS} < V_T$
 - the curve for $V_{GS} = V_{DD}$



Equivalent Resistance R_{eq}

- In a digital circuit, an n-channel MOSFET in the ON state is typically used to discharge a capacitor connected to its drain terminal:
 - gate voltage $V_G = V_{DD}$
 - source voltage $V_S = 0$ V
 - drain voltage V_D initially at V_{DD} , discharging toward 0 V



$$I_{DSATn} = \frac{k'_n}{2} \frac{W}{L} (V_{DD} - V_{Th})^2 \longrightarrow$$

The value of R_{eq} should be set to the value which gives the correct propagation delay (time required for output to fall to $\frac{1}{2}V_{DD}$):

$$R_{eq} \cong \frac{3}{4} \frac{V_{DD}}{I_{DSATn}} \left(1 - \frac{5}{6} \lambda_n V_{DD} \right)$$

Typical MOSFET Parameter Values

- For a given MOSFET fabrication process technology, the following parameters are known:
 - V_T (~ 0.5 V)
 - C_{ox} and k' (< 0.001 A/V²)
 - V_{DSAT} (≤ 1 V)
 - λ (≤ 0.1 V⁻¹)

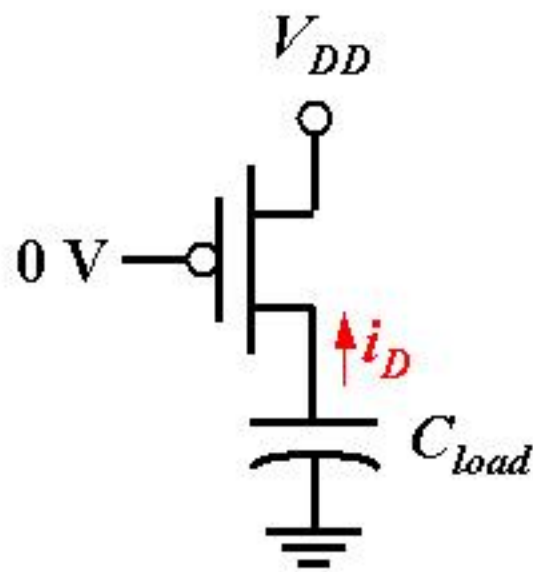
Example R_{eq} values for 0.25 μ m technology ($W = L$):

V_{DD} (V)	1	1.5	2	2.5
NMOS (k Ω)	35	19	15	13
PMOS (k Ω)	115	55	38	31

How can R_{eq} be decreased?

P-Channel MOSFET Example

- In a digital circuit, a p-channel MOSFET in the ON state is typically used to charge a capacitor connected to its drain terminal:
 - gate voltage $V_G = 0 \text{ V}$
 - source voltage $V_S = V_{DD}$ (power-supply voltage)
 - drain voltage V_D initially at 0 V , charging toward V_{DD}



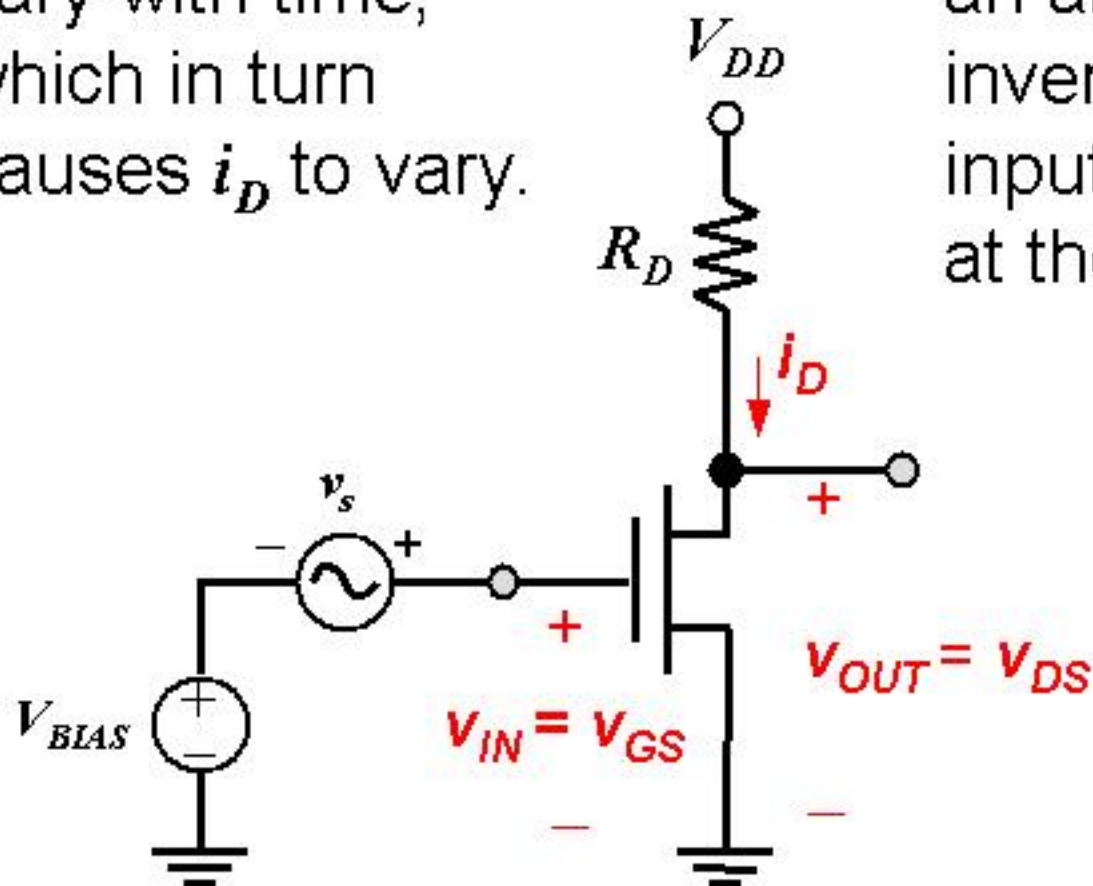
$$R_{eq} \cong \frac{3}{4} \frac{V_{DD}}{|I_{DSATp}|} \left(1 - \frac{5}{6} \lambda_p V_{DD} \right)$$

$$I_{DSAT} = -\frac{k'_p}{2} \frac{W}{L} \left(V_{DD} - |V_{Tp}| \right)^2$$

Common-Source (CS) Amplifier

- The input voltage v_s causes v_{GS} to vary with time, which in turn causes i_D to vary.

- The changing voltage drop across R_D causes an amplified (and inverted) version of the input signal to appear at the drain terminal.

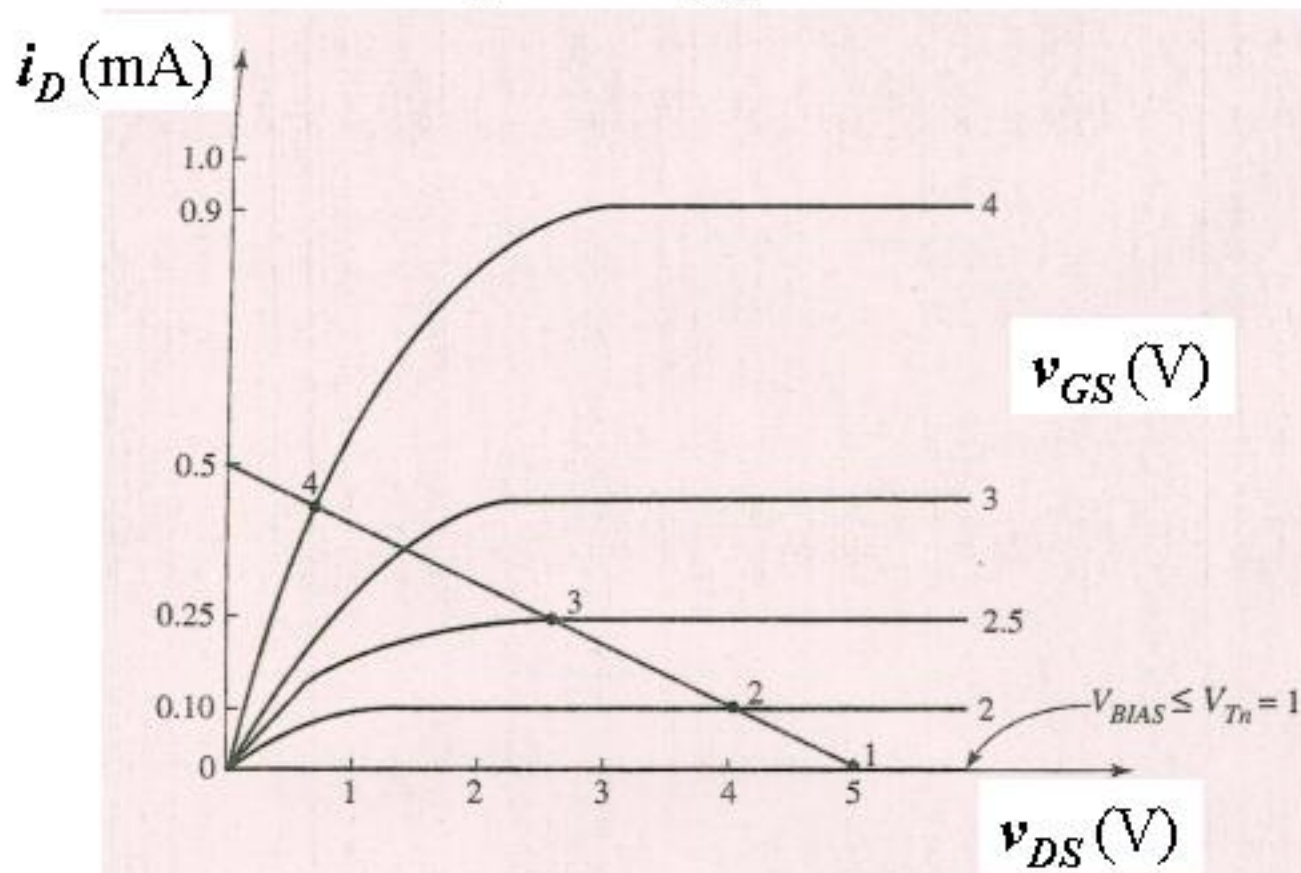


Notation

- Subscript convention:
 - $V_{DS} \equiv V_D - V_S$, $V_{GS} \equiv V_G - V_S$, *etc.*
- Double-subscripts denote DC sources:
 - V_{DD} , V_{CC} , I_{SS} , *etc.*
- To distinguish between DC and incremental components of an electrical quantity, the following convention is used:
 - DC quantity: upper-case letter with upper-case subscript
 - I_D , V_{DS} , *etc.*
 - Incremental quantity: lower-case letter with lower-case subscript
 - i_d , v_{ds} , *etc.*
 - Total (DC + incremental) quantity:
lower-case letter with upper-case subscript
 - i_D , v_{DS} , *etc.*

Load-Line Analysis of CS Amplifier

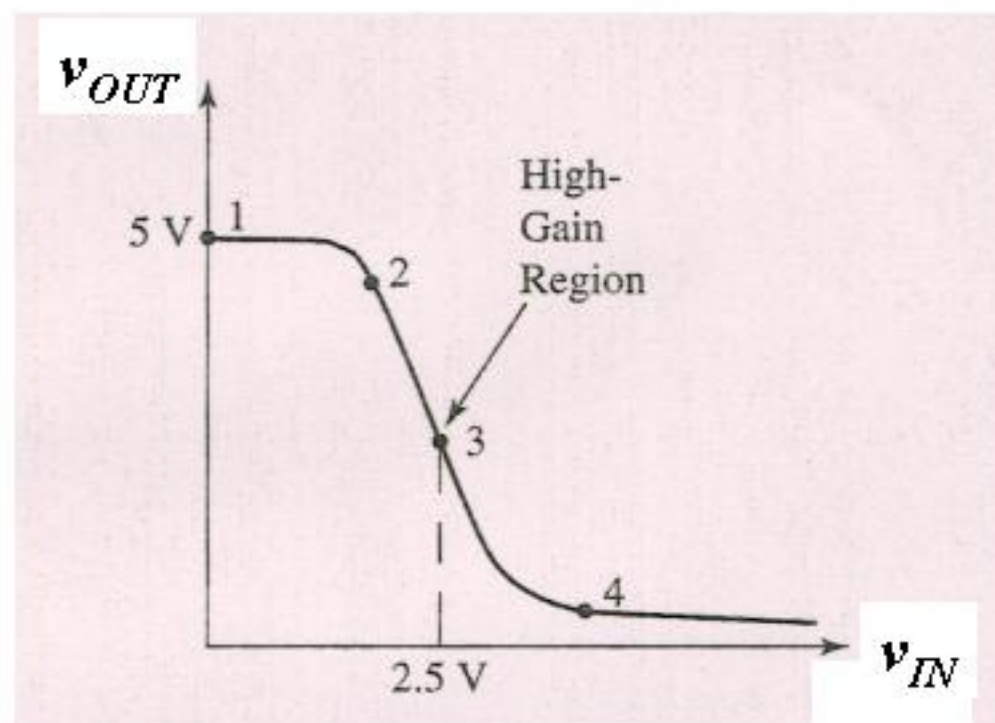
- The operating point of the circuit can be determined by finding the intersection of the appropriate MOSFET i_D vs. v_{DS} characteristic and the load line:



load-line equation:

$$V_{DD} = R_D i_D + v_{DS}$$

Voltage Transfer Function



Goal:

Operate the amplifier in the high-gain region, so that small changes in v_{IN} result in large changes in v_{OUT}

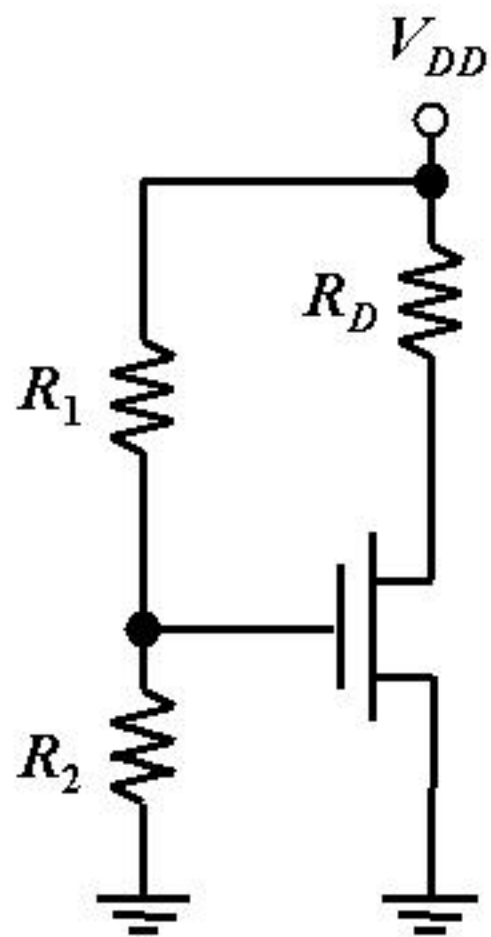
- (1): transistor biased in cutoff region
- (2): $v_{IN} > V_T$; transistor biased in saturation region
- (3): transistor biased in saturation region
- (4): transistor biased in "resistive" or "triode" region

Quiescent Operating Point

- The operating point of the amplifier for zero input signal ($v_s = 0$) is often referred to as the **quiescent operating point**. (Another word: **bias**.)
 - The bias point should be chosen so that the output voltage is approximately centered between V_{DD} and 0 V.
 - v_s varies the input voltage around the input bias point.

Note: The relationship between v_{OUT} and v_{IN} is not linear; this can result in a distorted output voltage signal. If the input signal amplitude is very small, however, we can have amplification with negligible distortion.

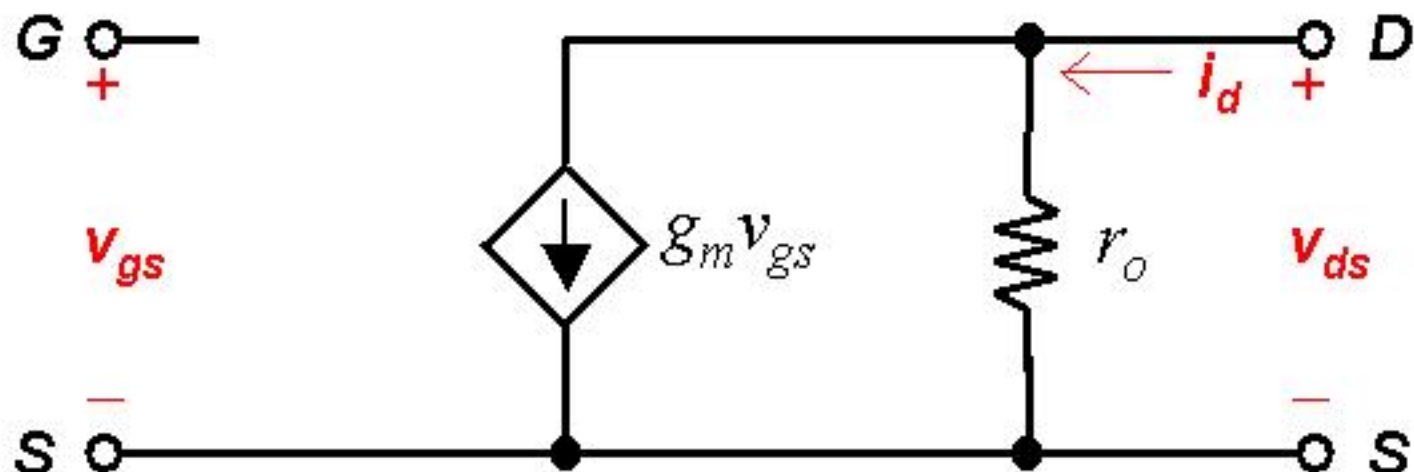
Bias Circuit Example



Rules for Small-Signal Analysis

- A DC supply voltage source acts as a short circuit
 - Even if AC current flows through the DC voltage source, the AC voltage across it is zero.
- A DC supply current source acts as an open circuit
 - Even if AC voltage is applied across the current source, the AC current through it is zero.

NMOSFET Small-Signal Model

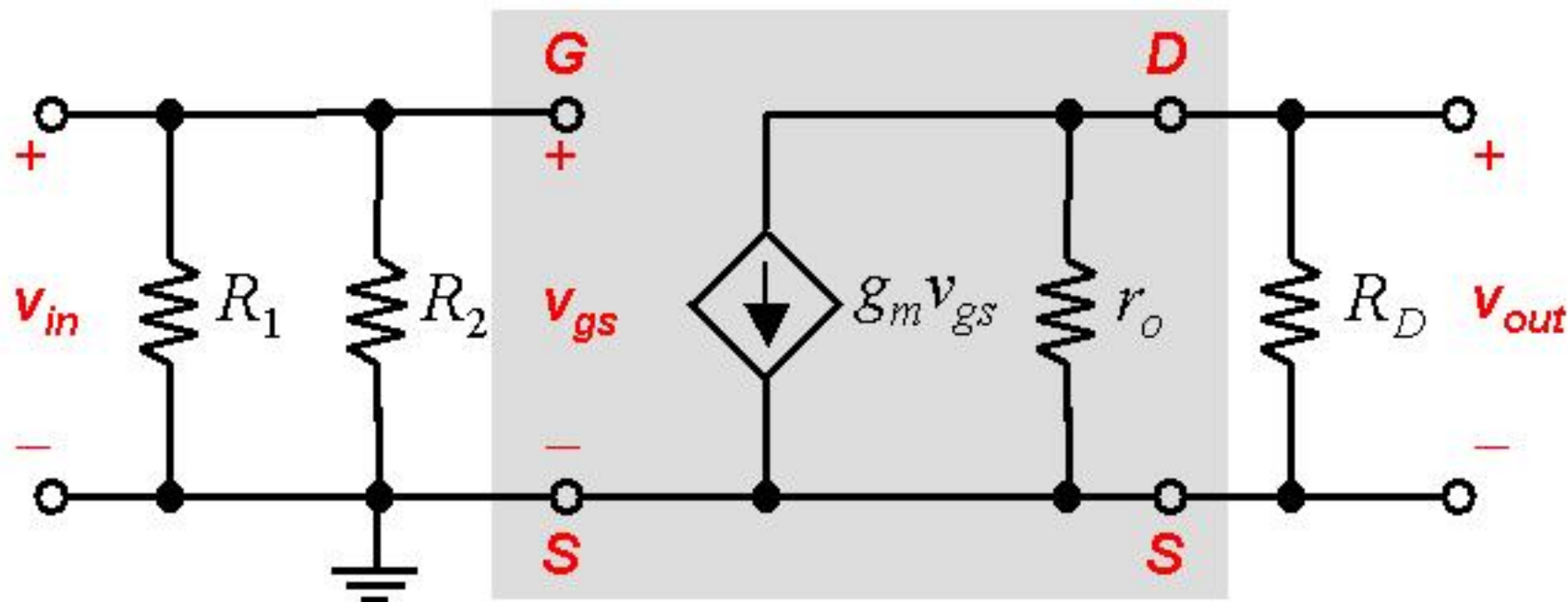


$$i_d = \frac{\partial i_D}{\partial v_{GS}} v_{gs} + \frac{\partial i_D}{\partial v_{DS}} v_{ds} = g_m v_{gs} + g_o v_{ds}$$

$$g_m \equiv \frac{\partial i_D}{\partial v_{GS}} \cong \frac{W}{L} k' (V_{GS} - V_T) \quad \text{transconductance}$$

$$g_o \equiv \frac{\partial i_D}{\partial v_{DS}} \cong \lambda I_D \quad \text{output conductance}$$

Small-Signal Equivalent Circuit



$$v_{out} = -g_m v_{gs} (r_o \parallel R_D)$$

voltage gain $A_v = \frac{v_{out}}{v_{in}} = -g_m (r_o \parallel R_D)$