Introduction to circuit analysis

OUTLINE

- Electrical quantities and sign conventions (review)
- Ideal basic circuit elements:
  - Voltage and current sources
  - Electrical resistance (Ohm's law)
- Power calculations
- Kirchhoff's Laws

Reading

Chapter 1

Circuit Analysis

- **Circuit analysis is used to predict the behavior of the electric circuit, and plays a key role in the design process.**
  - Design process has analysis as fundamental 1st step
  - Comparison between desired behavior (specifications) and predicted behavior (from circuit analysis) leads to refinements in design
- **In order to analyze an electric circuit, we need to know the behavior of each circuit element (in terms of its voltage and current) AND the constraints imposed by interconnecting the various elements.**

Electric Current

Definition: rate of positive charge flow

Symbol: i

Units: Coulombs per second ≡ Amperes (A)

$$i = dq/dt$$

where q = charge (in Coulombs), t = time (in seconds)

Note: Current has polarity.

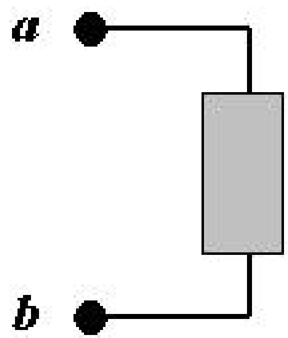
Electric Potential (Voltage)

- **Definition:** energy per unit charge
- **Symbol:** v
- **Units:** Joules/Coulomb ≡ Volts (V)

$$v = dw/dq$$

where w = energy (in Joules), q = charge (in Coulombs)

Note: Potential is always referenced to some point.



Subscript convention:

v_{ab} means the potential at a minus the potential at b.

$$v_{ab} \equiv v_a - v_b$$

Electric Power

- **Definition:** transfer of energy per unit time
- **Symbol:** p
- **Units:** Joules per second ≡ Watts (W)

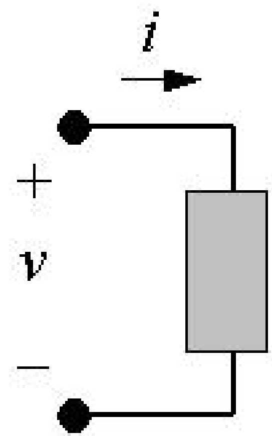
$$p = dw/dt = (dw/dq)(dq/dt) = vi$$

- **Concept:**

  As a positive charge q moves through a drop in voltage v, it loses energy

  - energy change = qv
  - rate is proportional to # charges/sec

The Ideal Basic Circuit Element



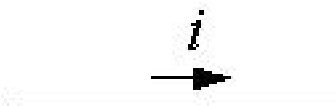
- Polarity reference for voltage can be indicated by plus and minus signs
- Reference direction for the current is indicated by an arrow

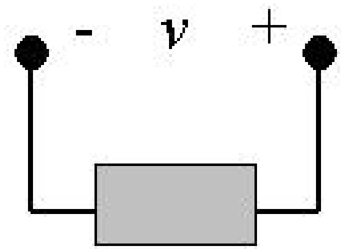
Attributes:

- Two terminals (points of connection)
- Mathematically described in terms of current and/or voltage
- Cannot be subdivided into other elements

A Note about Reference Directions

A problem like "Find the current" or "Find the voltage" is always accompanied by a definition of the direction:



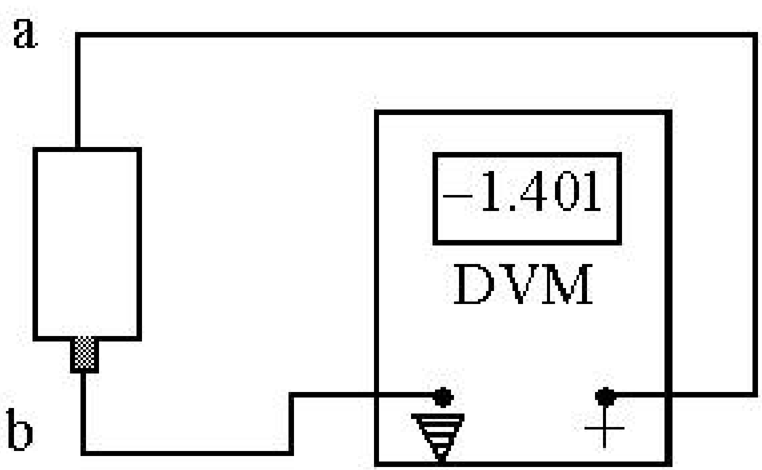


In this case, if the current turns out to be 1 mA flowing to the left, we would say i = -1 mA.

In order to perform circuit analysis to determine the voltages and currents in an electric circuit, you need to specify reference directions. There is no need to guess the reference direction so that the answers come out positive, however.

Sign Convention Example

Suppose you have an unlabelled battery and you measure its voltage with a digital voltmeter (DVM). It will tell you the **magnitude and sign** of the voltage.



With this circuit, you are measuring v_{ab}.

The DVM indicates –1.401, so v_a is lower than v_b by 1.401 V.

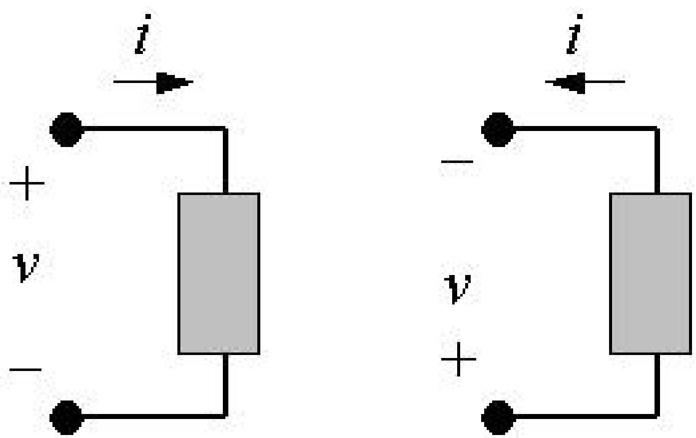
Which is the positive battery terminal?

Note that we have used the "ground" symbol (⏚) for the reference node on the DVM. Often it is labeled "C" for "common."

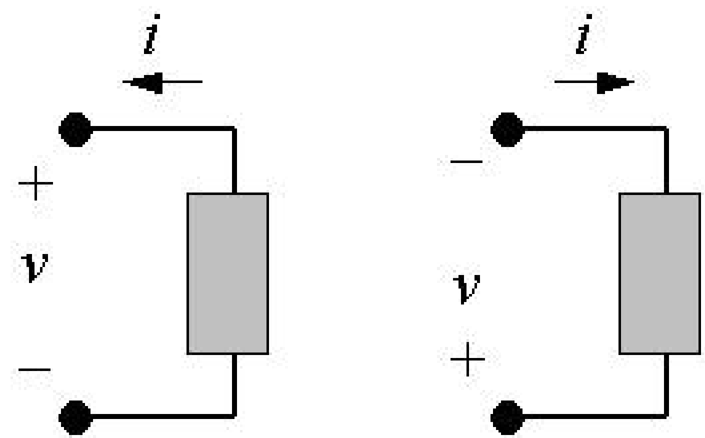
Sign Convention for Power

Passive sign convention

$$p = vi$$



$$p = -vi$$

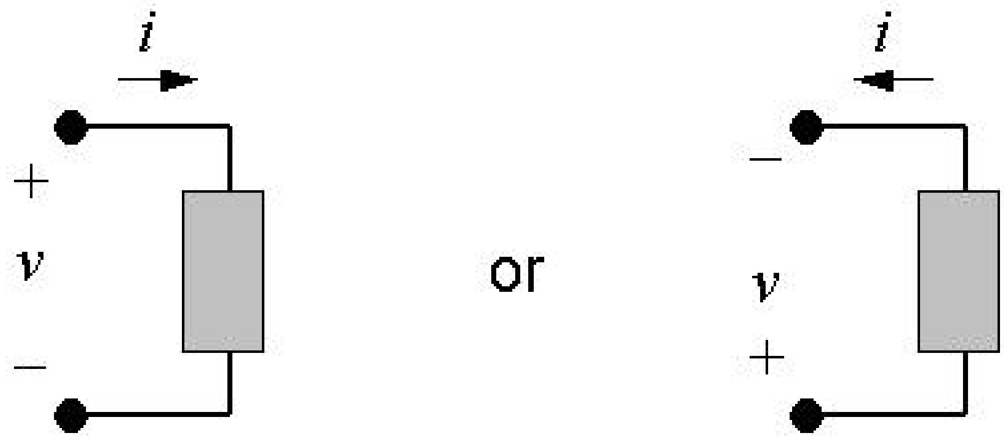


- **If $p > 0$, power is being delivered to the box.**
- **If $p < 0$, power is being extracted from the box.**

Power

If an element is absorbing power (*i.e.* if $p > 0$), positive charge is flowing from higher potential to lower potential.

$p = vi$ if the "passive sign convention" is used:

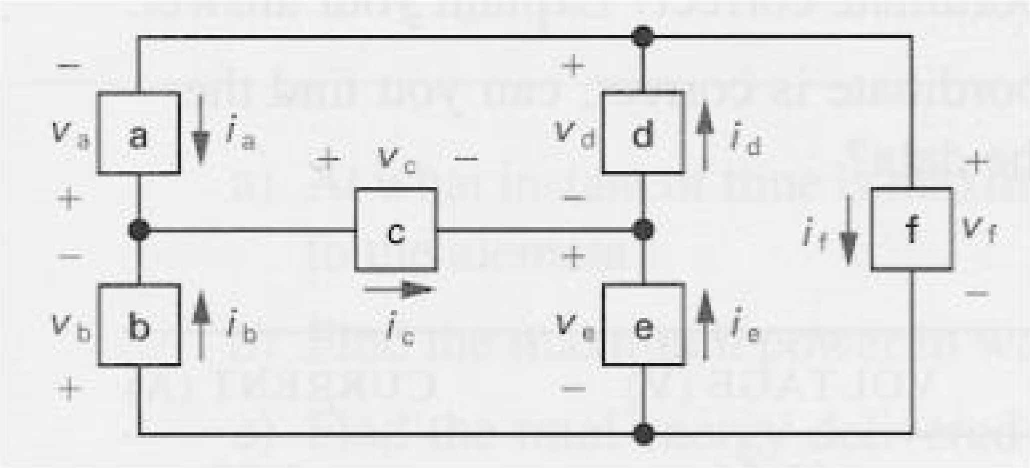


How can a circuit element absorb power?

By converting electrical energy into heat (resistors in toasters), light (light bulbs), or acoustic energy (speakers); by storing energy (charging a battery).

Power Calculation Example

Find the power absorbed by each element:



Conservation of energy
→ total power delivered equals total power absorbed

ELEMENT	VOLTAGE (V)	CURRENT (A)	*vi* (W)	*p* (W)
a	−18	−51	918	
b	−18	45	-810	
c	2	−6	-12	
d	20	−20	-400	
e	16	−14	-224	
f	36	31	1116	

Circuit Elements

- 5 ideal basic circuit elements:
  - voltage source
  - current source
  - resistor
  - inductor
  - capacitor

voltage source, current source: ***active elements***, capable of generating electric energy

resistor, inductor, capacitor: ***passive elements***, incapable of generating electric energy

- Many practical systems can be modeled with just sources and resistors
- The basic analytical techniques for solving circuits with inductors and capacitors are the same as those for resistive circuits

Electrical Sources

- An ***electrical source*** is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

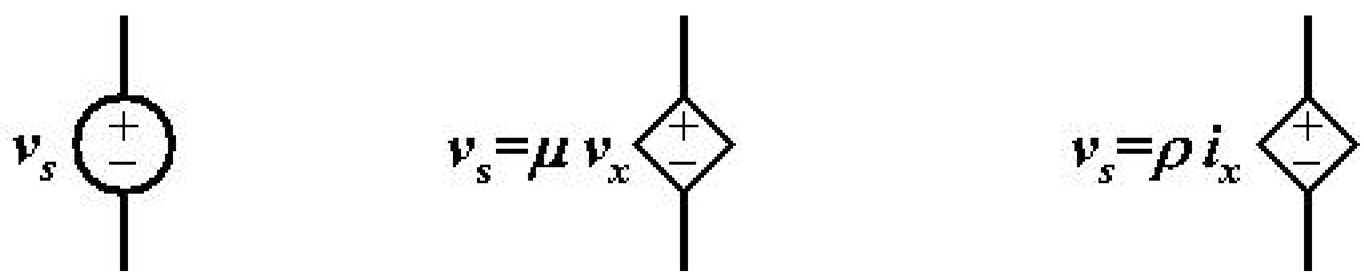
  Examples:
  - battery: chemical ⟺ electric
  - dynamo (generator/motor): mechanical ⟺ electric

→ Electrical sources can either deliver or absorb power

Ideal Voltage Source

- Circuit element that maintains a prescribed voltage across its terminals, **regardless of the current flowing in those terminals**.
  - Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Circuit symbols:

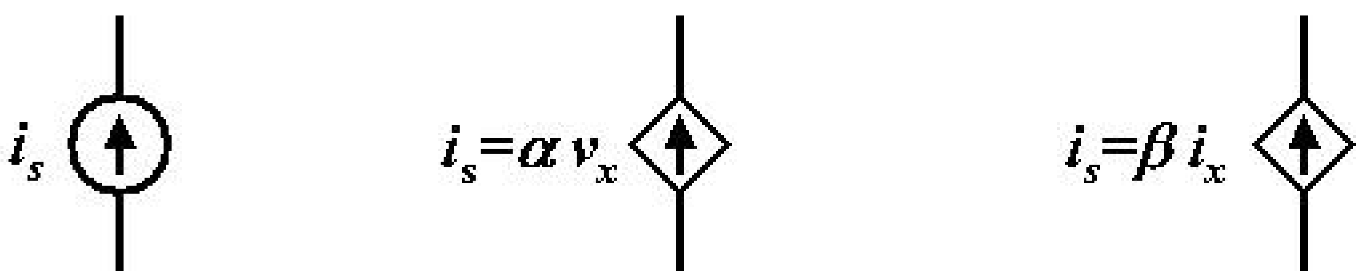


independent **voltage-controlled** **current-controlled**

Ideal Current Source

- Circuit element that maintains a prescribed current through its terminals, **regardless of the voltage across those terminals**.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Circuit symbols:



independent **voltage-controlled** **current-controlled**

Electrical Resistance

- ***Resistance:*** Electric field is proportional to current density, within a resistive material. Thus, voltage is proportional to current. The circuit element used to model this behavior is the ***resistor***.

<u>Circuit symbol</u>:

<u>Units</u>: Volts per Ampere ≡ ohms (Ω)

- The current flowing in the resistor is proportional to the voltage across the resistor:

$$v = i\,R$$

(Ohm's Law)

where v = voltage (V), i = current (A), and R = resistance (Ω)

Electrical Conductance

- **Conductance** is the reciprocal of resistance.

Symbol: G

Units: **siemens (S) or mhos (℧)**

Example:

Consider an 8 Ω resistor. *What is its conductance?*

Short Circuit and Open Circuit

<u>Wire</u> ("short circuit"):

- $R = 0$ → **no voltage difference exists**
  (all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

<u>Air</u> ("open circuit"):

- $R = \infty$ → **no current flows**
- Voltage difference can exist,
  as determined by the circuit

Circuit Nodes and Loops

- A ***node*** is a point where two or more circuit elements are connected.
- A ***loop*** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once

Example:

Kirchhoff's Laws

- **<u>Kirchhoff's Current Law (KCL)</u>:**
  - The algebraic sum of all the currents entering any node in a circuit equals zero.

- **<u>Kirchhoff's Voltage Law (KVL)</u>:**
  - The algebraic sum of all the voltages around any loop in a circuit equals zero.

Example: Power Absorbed by a Resistor

$$p = vi = (iR)i = i^2R$$

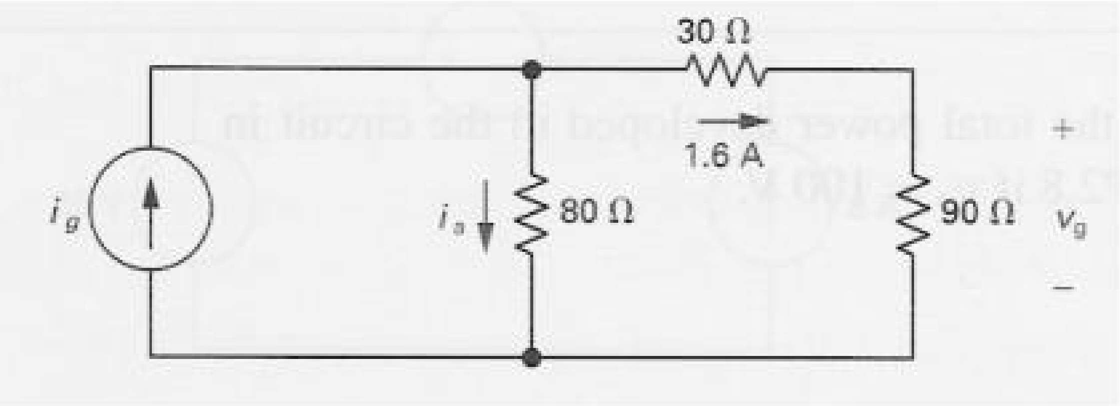
$$p = vi = v(v/R) = v^2/R$$

Note that $p > 0$ always, for a resistor.

Example:

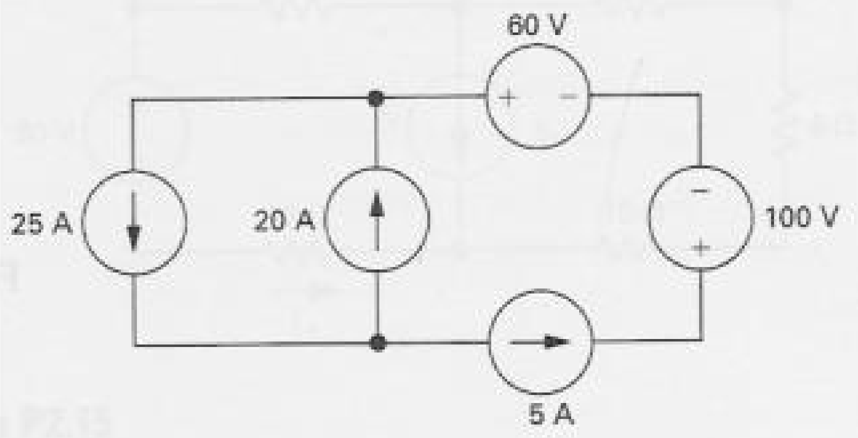
a) Calculate the voltage v_g and current i_a.

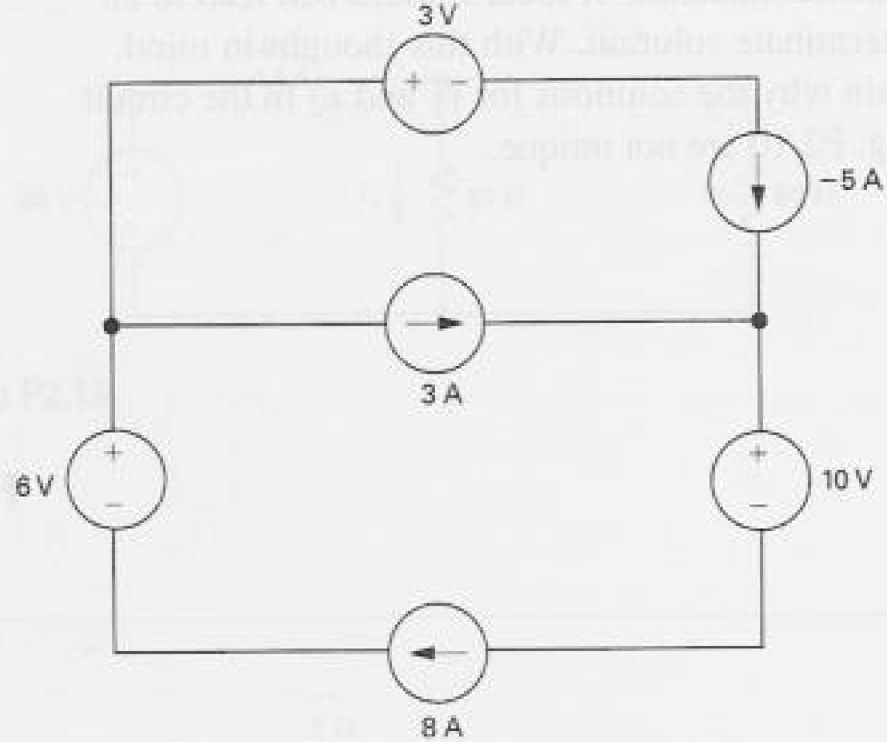
b) Determine the power dissipated in the 80Ω resistor.



More Examples

- Are these interconnections permissible?





Summary

- ***Current*** = rate of charge flow
- ***Voltage*** = energy per unit charge created by charge separation
- ***Power*** = energy per unit time
- ***Ideal Basic Circuit Elements***
  - 2-terminal component that cannot be sub-divided
  - described mathematically in terms of its terminal voltage and current
  - An ***ideal voltage source*** maintains a prescribed voltage regardless of the current in the device.
  - An ***ideal current source*** maintains a prescribed current regardless of the voltage across the device.
  - A ***resistor*** constrains its voltage and current to be proportional to each other:

$$v = iR \quad \text{(Ohm's law)}$$

Summary (cont'd)

- ***Passive sign convention***
  - Reference direction for current through the element is in the direction of the reference voltage drop across the element