Announcements

- Visit the class website to see updated TA section assignments
- Tuesday discussion section moved to Wednesday
- HW #1 due Friday, jan. 30, in hw box in 240 Cory

http://www-inst.eecs.berkeley.edu/~ee40

Lecture #3

OUTLINE

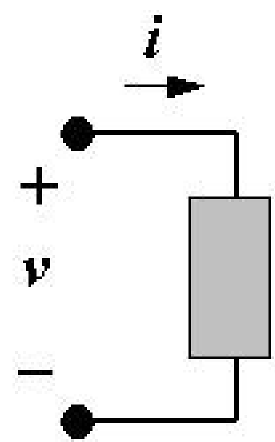
- Circuit element *I-V* characteristics
- Construction of a circuit model
- Kirchhoff's laws – a closer look

Reading

(Chapter 1, begin Ch. 2)

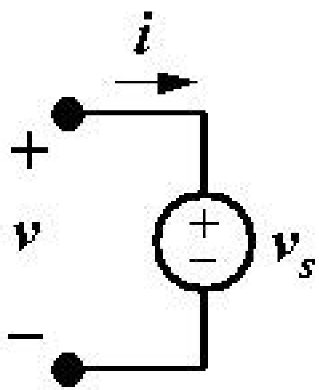
Current *vs.* Voltage (*I-V*) Characteristic

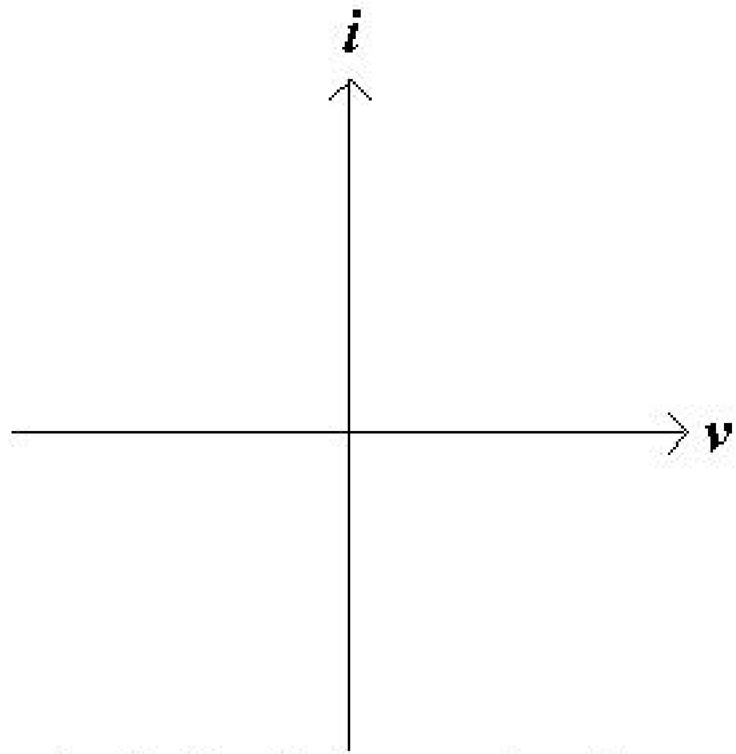
- Voltage sources, current sources, and resistors can be described by plotting the current (i) as a function of the voltage (v)



- Later, we will see that the *I-V* characteristic of any circuit consisting only of sources and resistors is a straight line.

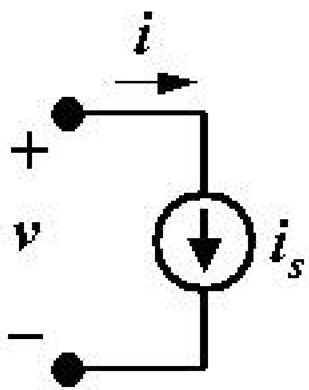
I-V Characteristic of Ideal Voltage Source

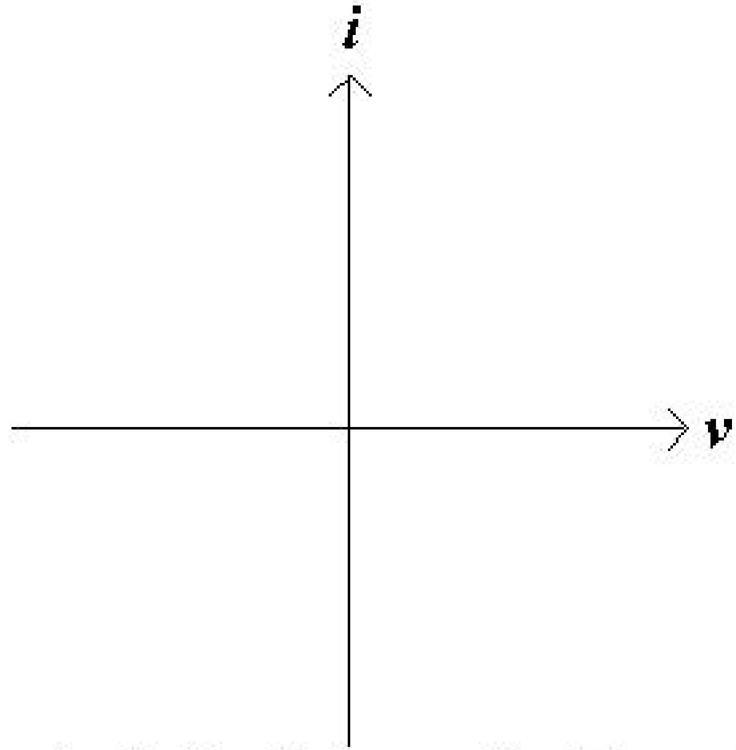




1. Plot the *I-V* characteristic for $v_s > 0$. For what values of i does the source absorb power? For what values of i does the source release power?
2. Repeat (1) for $v_s < 0$.
3. What is the *I-V* characteristic for an ideal wire?

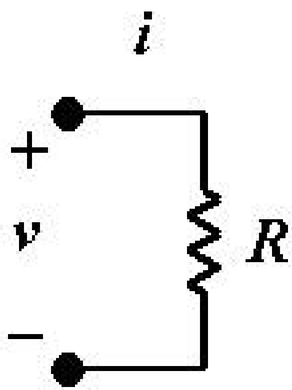
I-V Characteristic of Ideal Current Source

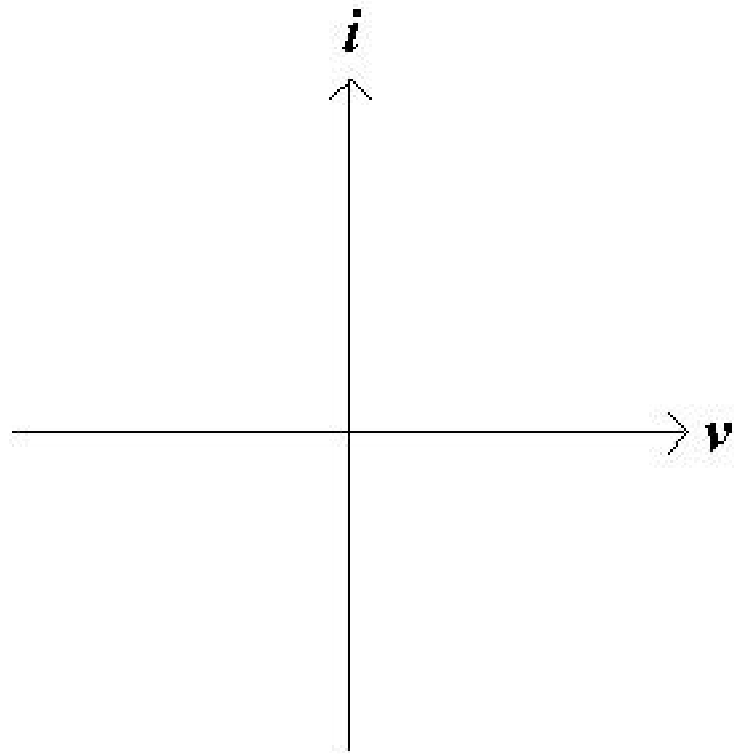




1. Plot the *I-V* characteristic for $i_s > 0$. For what values of v does the source absorb power? For what values of v does the source release power?
2. Repeat (1) for $i_s < 0$.
3. What is the *I-V* characteristic for an open circuit?

I-V Characteristic of Ideal Resistor





1. Plot the *I-V* characteristic for R = 1 kΩ. What is the slope?

"Lumped Element" Circuit Modeling

(Model = representation of a real system which simplifies analysis)

- In circuit analysis, important characteristics are grouped together in "lumps" (separate circuit elements) connected by perfect conductors ("**wires**")
- An electrical system can be modeled by an **electric circuit** (combination of paths, each containing 1 or more **circuit elements**) if

$$\lambda = c/f >> \text{physical dimensions of system}$$

Distance travelled by a particle travelling at the speed of light in one period

Example: f = 60 Hz

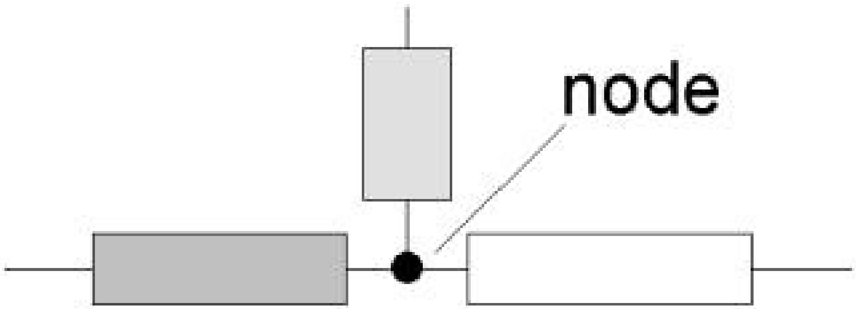
$$\lambda = 3 \times 10^8 \text{ m/s} / 60 = 5 \times 10^6 \text{ m}$$

Construction of a Circuit Model

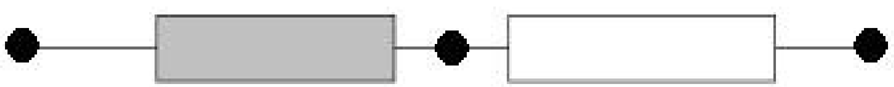
- The electrical behavior of each physical component is of primary interest.
- We need to account for undesired as well as desired electrical effects.
- Simplifying assumptions should be made wherever reasonable.

Terminology: Nodes and Branches

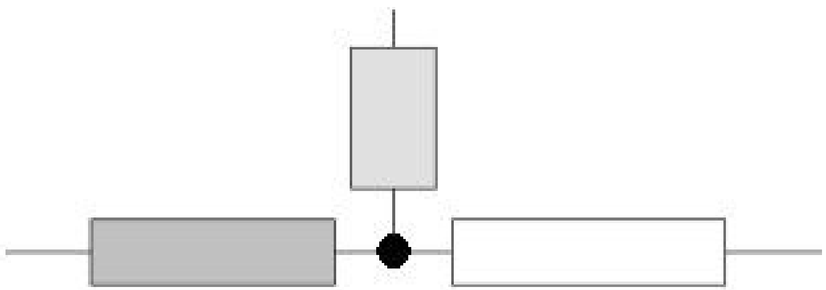
Node: A point where two or more circuit elements are connected – entire wire



Branch: A path that connects two nodes



A single branch

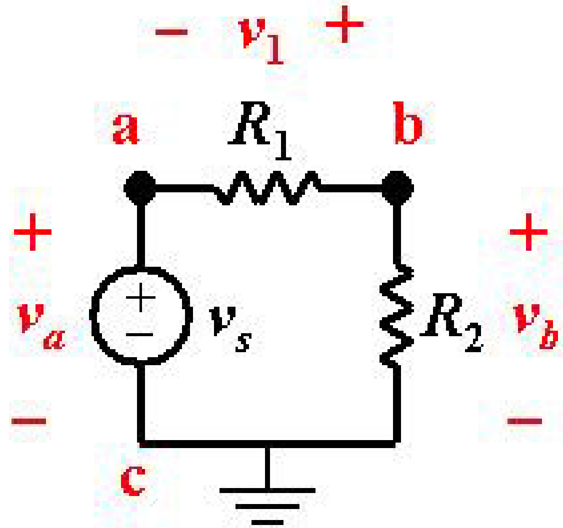


NOT a single branch

Notation: Node and Branch Voltages

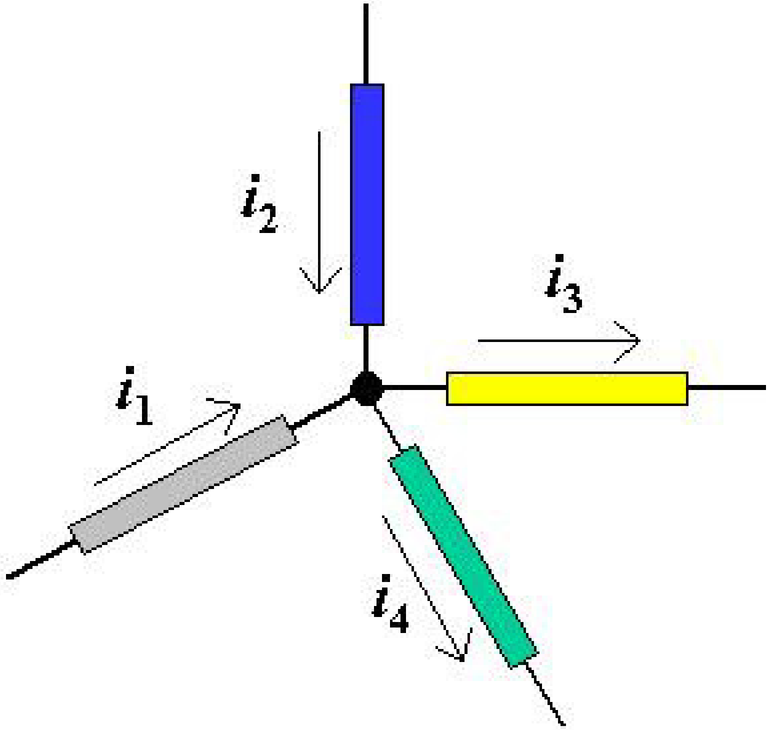
- Use one node as the reference (the "common" or "ground" node) – label it with a symbol
- The voltage drop from node **x** to the reference node is called the **node voltage v_x**.
- The voltage across a circuit element is defined as the difference between the node voltages at its terminals

Example:



Using Kirchhoff's Current Law (KCL)

Consider a node connecting several branches:



- Use **reference directions** to determine whether currents are "entering" or "leaving" the node – with no concern about actual current directions

Formulations of Kirchhoff's Current Law

(Charge stored **in node** is zero.)

<u>Formulation 1</u>:

Sum of currents entering node
= sum of currents leaving node

<u>Formulation 2</u>:

Algebraic sum of currents entering node = 0

- Currents leaving are included with a minus sign.

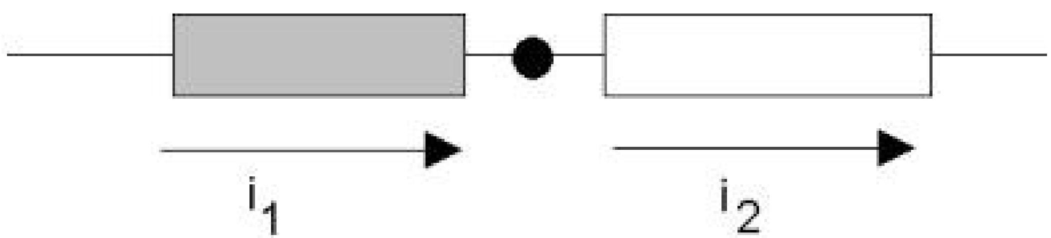
<u>Formulation 3</u>:

Algebraic sum of currents leaving node = 0

- Currents entering are included with a minus sign.

A Major Implication of KCL

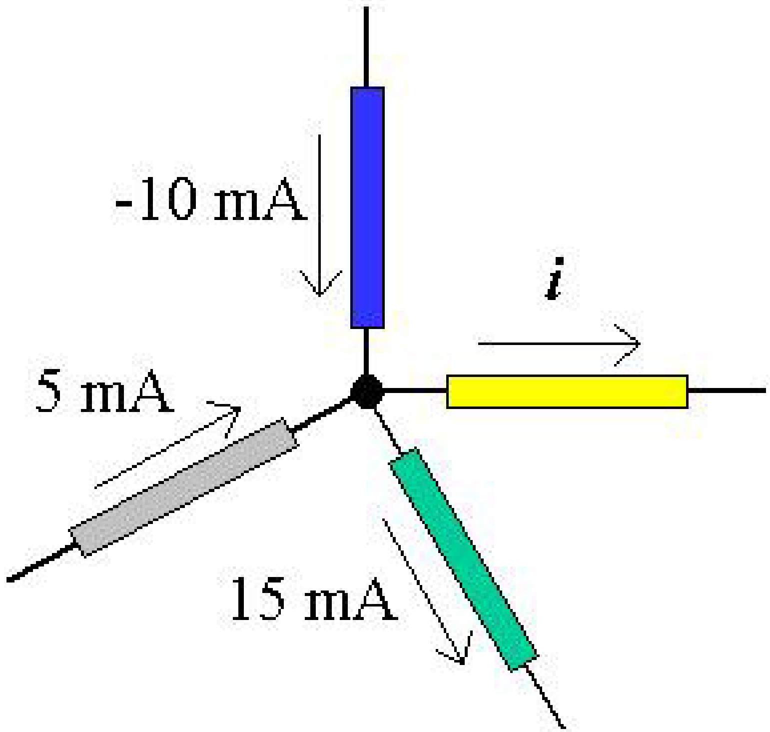
- KCL tells us that **all of the elements in a single branch carry the same current**.
- We say these elements are connected ***in series***.



Current entering node = Current leaving node

$$i_1 = i_2$$

KCL Example



Currents entering the node:

Currents leaving the node:

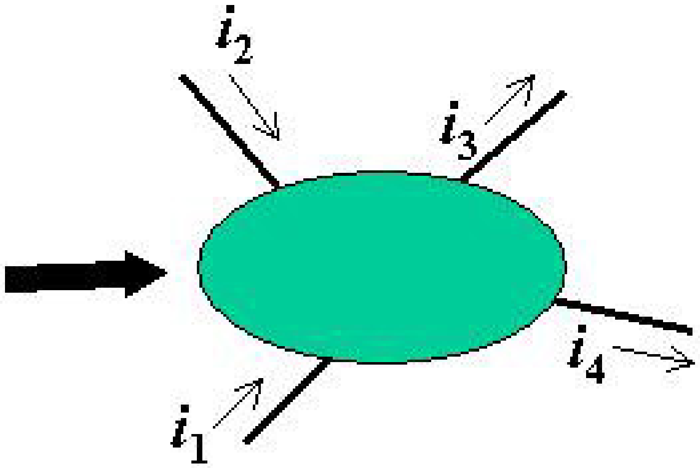
<u>3 formulations of KCL</u>:

1.
2.
3.

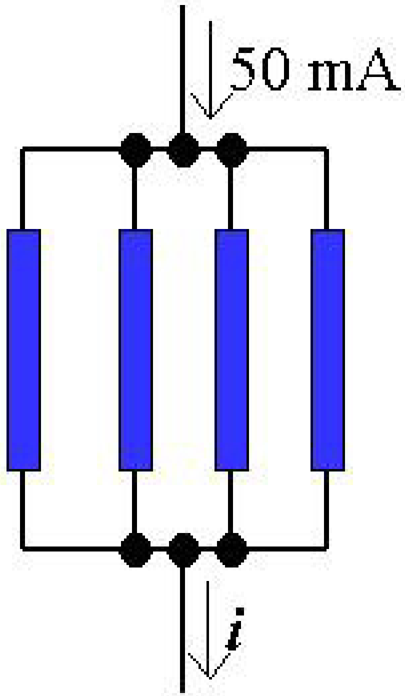
Generalization of KCL

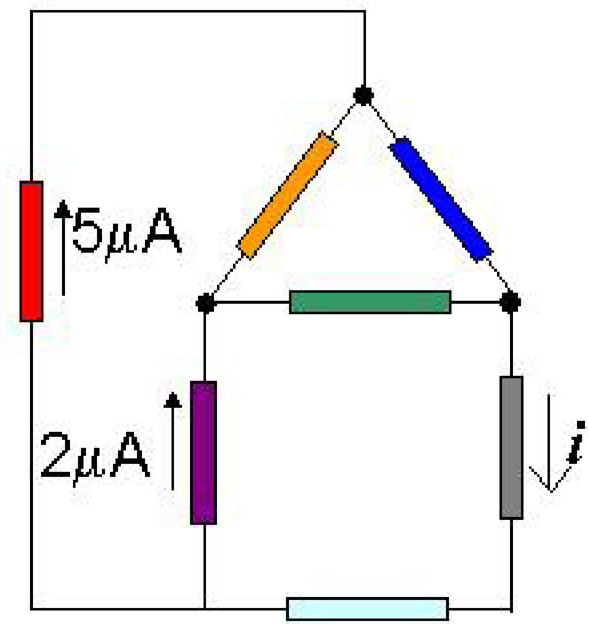
- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a "black box"



Generalized KCL Examples





Using Kirchhoff's Voltage Law (KVL)

Consider a branch which forms part of a loop:

- Use **reference polarities** to determine whether a voltage is dropped – with no concern about actual voltage polarities

Formulations of Kirchhoff's Voltage Law

(Conservation of energy)

Formulation 1:

Sum of voltage drops around loop
= sum of voltage rises around loop

Formulation 2:

Algebraic sum of voltage drops around loop = 0

- Voltage rises are included with a minus sign.

(Handy trick: Look at the first sign you encounter on each element when tracing the loop.)

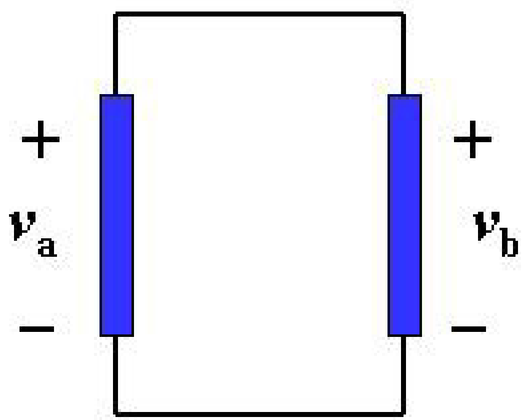
Formulation 3:

Algebraic sum of voltage rises around loop = 0

- Voltage drops are included with a minus sign.

A Major Implication of KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage**.
- We say these elements are connected **in parallel**.

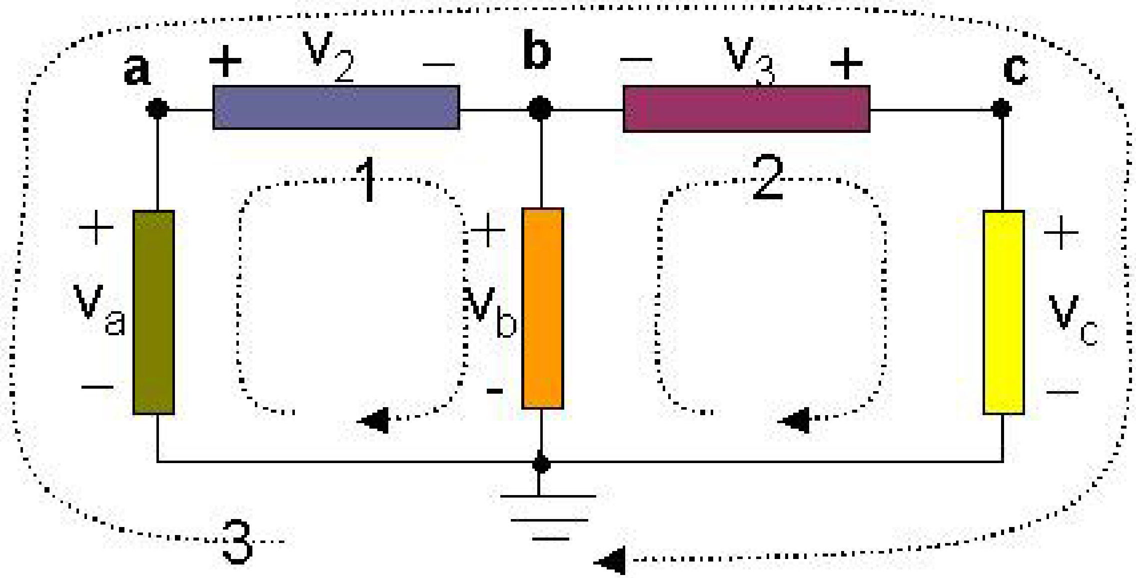


Applying KVL in the clockwise direction, starting at the top:

$$v_b - v_a = 0 \rightarrow v_b = v_a$$

KVL Example

Three closed paths:



Path 1:

Path 2:

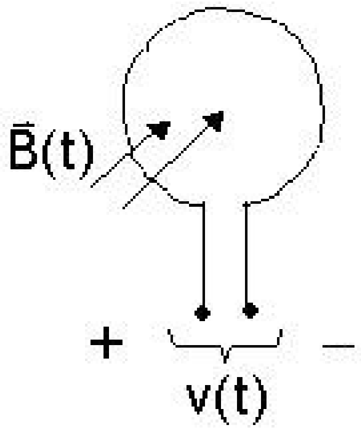
Path 3:

An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop

  Otherwise, there would be an induced voltage (Faraday's Law)

- Note: Antennas are designed to "pick up" electromagnetic waves; "regular circuits" often do so undesirably.



Avoid these loops!

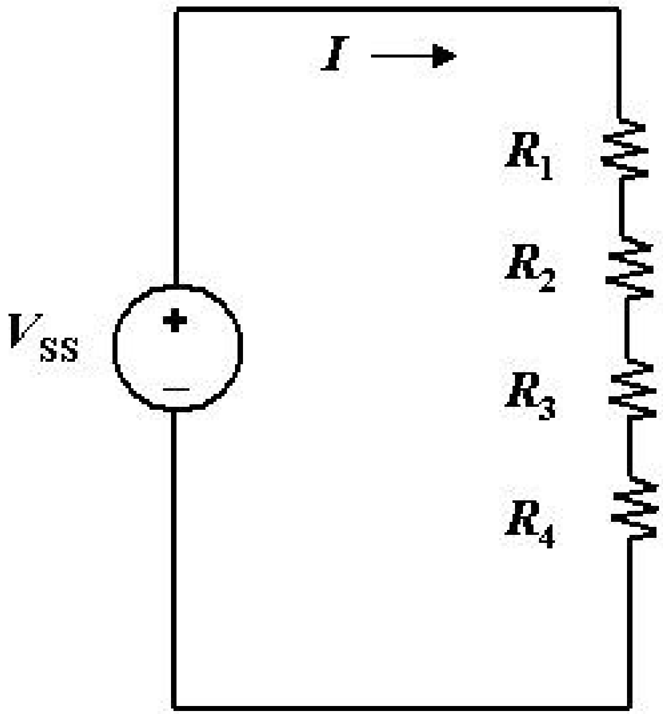
How do we deal with antennas (EECS 117A)?

Include a voltage source as the circuit representation of the induced voltage or "noise".

(Use a lumped model rather than a distributed (wave) model.)

Resistors in Series

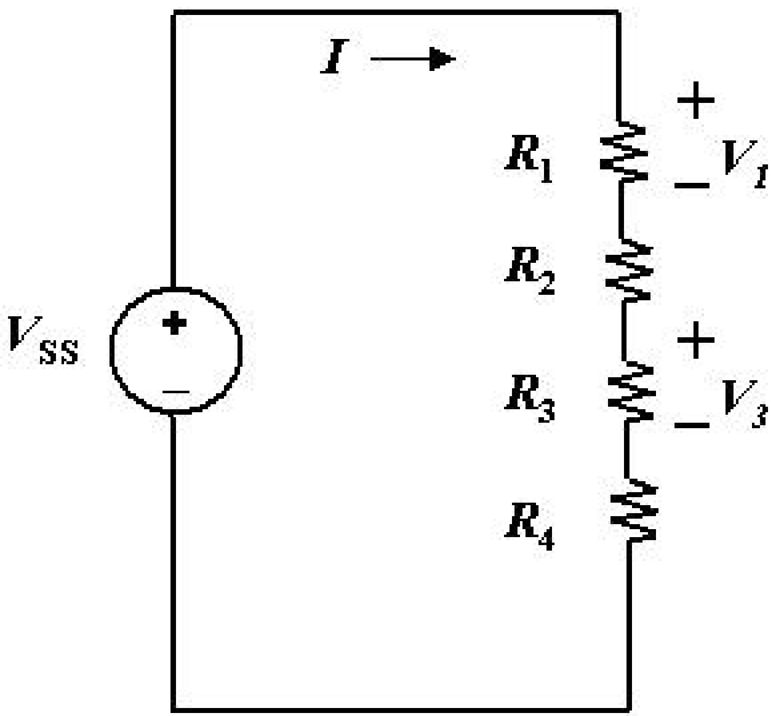
Consider a circuit with multiple resistors connected in series. Find their "equivalent resistance".



- KCL tells us that the same current (I) flows through every resistor
- KVL tells us

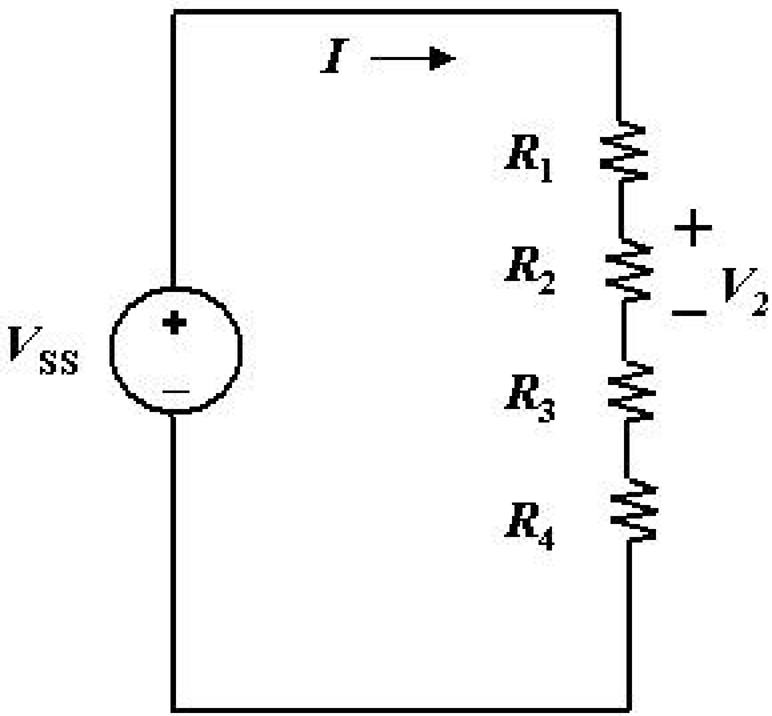
Equivalent resistance of resistors in series is the sum

Voltage Divider



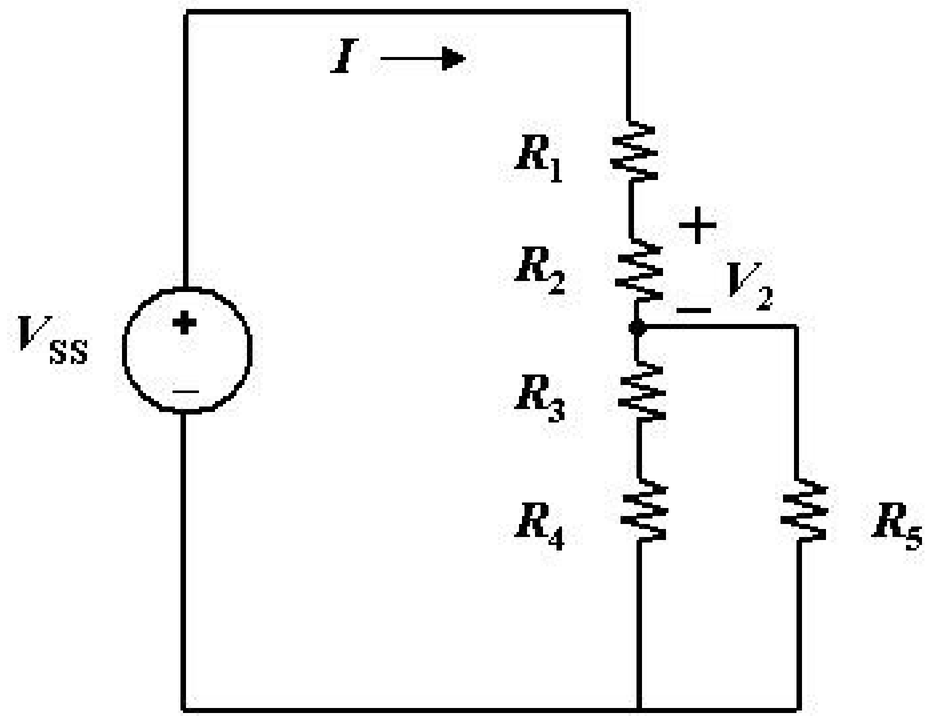
$$I = V_{SS} / (R_1 + R_2 + R_3 + R_4)$$

When can the Voltage Divider Formula be Used?



$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Correct, if nothing else is connected to nodes

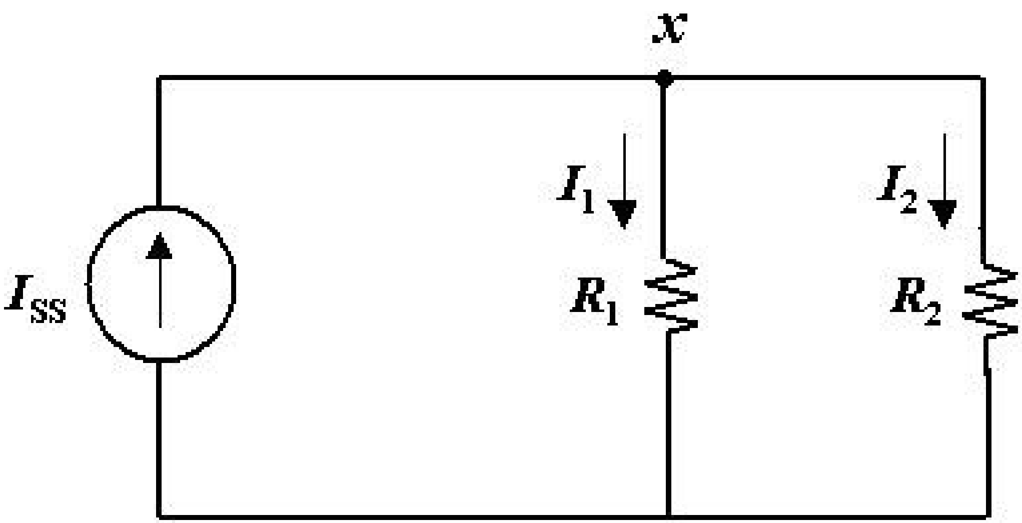


$$V_2 \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

because R_5 removes condition of resistors in series

Resistors in Parallel

Consider a circuit with two resistors connected in parallel. Find their "equivalent resistance".



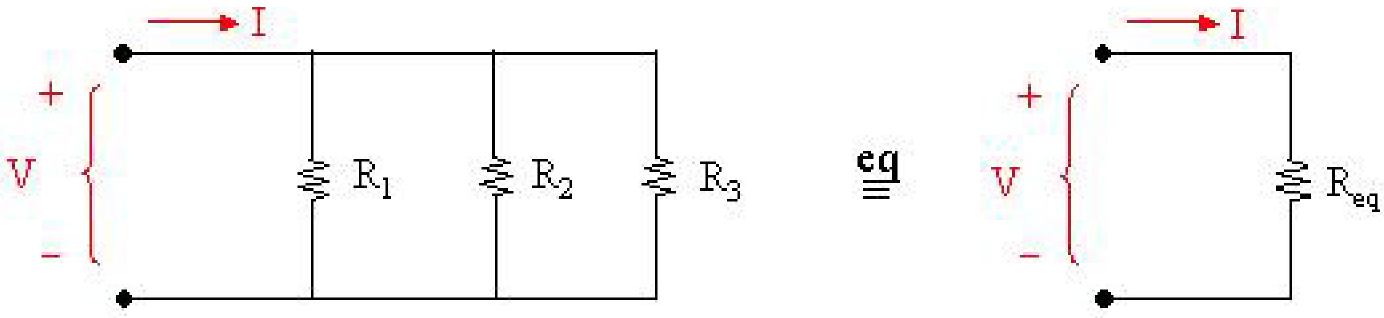
- KVL tells us that the same voltage is dropped across each resistor

$$V_x = I_1 R_1 = I_2 R_2$$

- KCL tells us

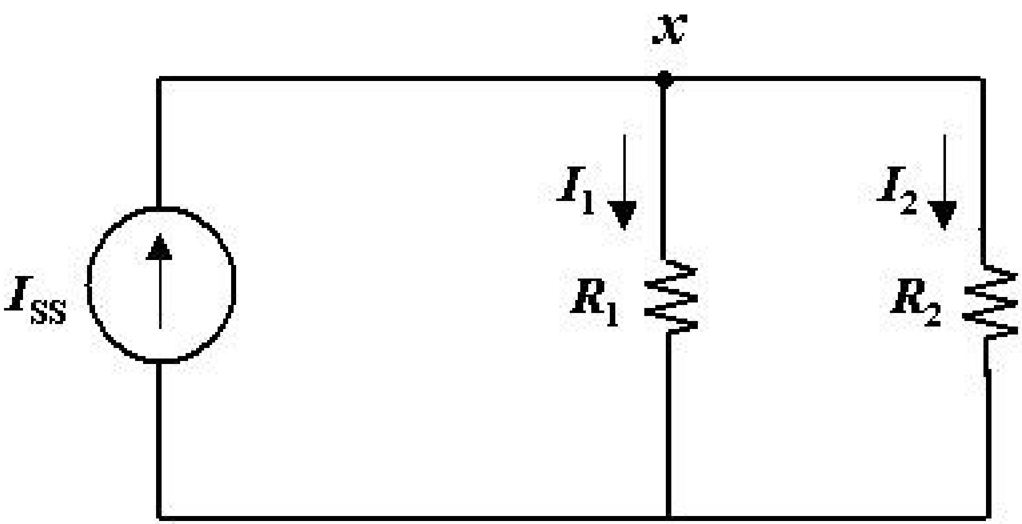
General Formula for Parallel Resistors

What single resistance R_{eq} is equivalent to three resistors in parallel?



Equivalent conductance of resistors in parallel is the sum

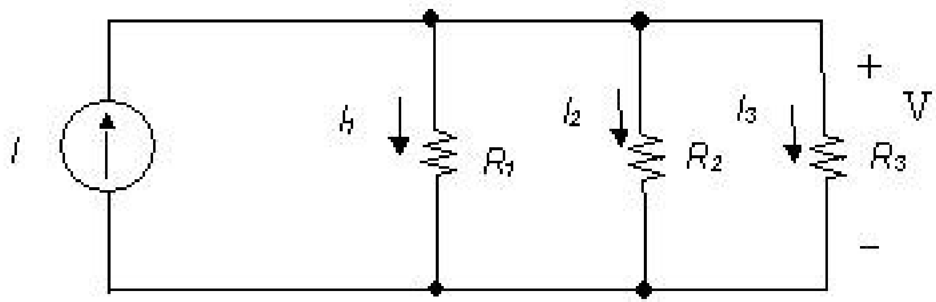
Current Divider



$$V_x = I_1 R_1 = I_{SS} R_{eq}$$

Generalized Current Divider Formula

Consider a current divider circuit with >2 resistors in parallel:



$$V = \frac{I}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)}$$

$$I_3 = \frac{V}{R_3} = I\left[\frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3}\right]$$

Summary

- An ***ideal voltage source*** maintains a prescribed voltage regardless of the current in the device.
- An ***ideal current source*** maintains a prescribed current regardless of the voltage across the device.
- A ***resistor*** constrains its voltage and current to be proportional to each other:

$$v = iR \quad \text{(Ohm's law)}$$

- ***Kirchhoff's current law*** states that the algebraic sum of all currents at any node in a circuit equals zero.
- ***Kirchhoff's voltage law*** states that the algebraic sum of all voltages around any closed path in a circuit equals zero.

Summary (cont'd)

- Resistors in Series – Voltage Divider
- Conductances in Parallel – Current Divider