Lecture #4

OUTLINE

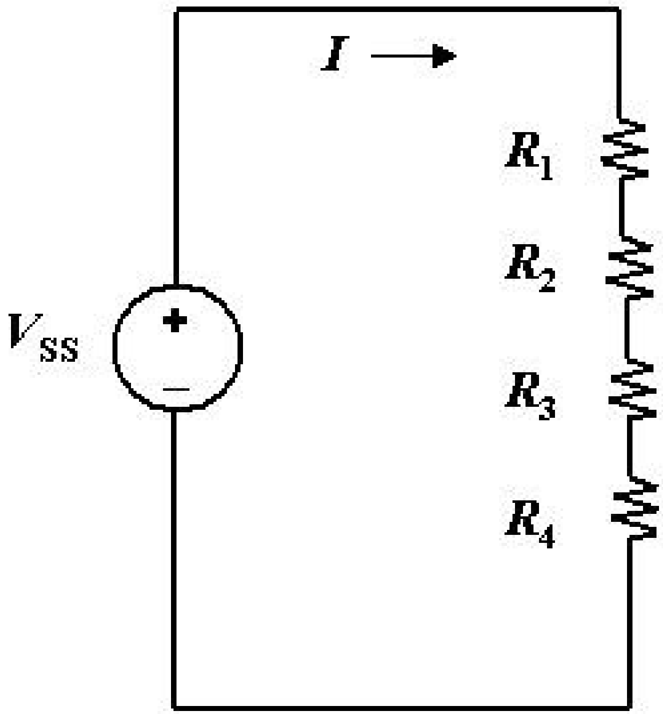
- Resistors in series
  - equivalent resistance
  - voltage-divider circuit
  - measuring current
- Resistors in parallel
  - equivalent resistance
  - current-divider circuit
  - measuring voltage
- Examples
- Node Analysis

Reading

Chapter 2

Resistors in Series

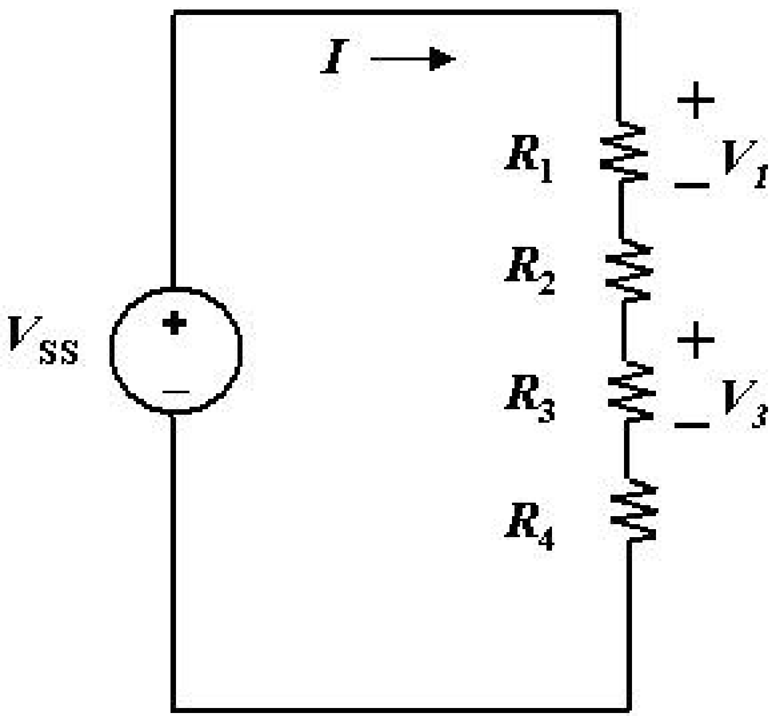
Consider a circuit with multiple resistors connected in series. Find their “equivalent resistance”.



- KCL tells us that the same current (I) flows through every resistor
- KVL tells us

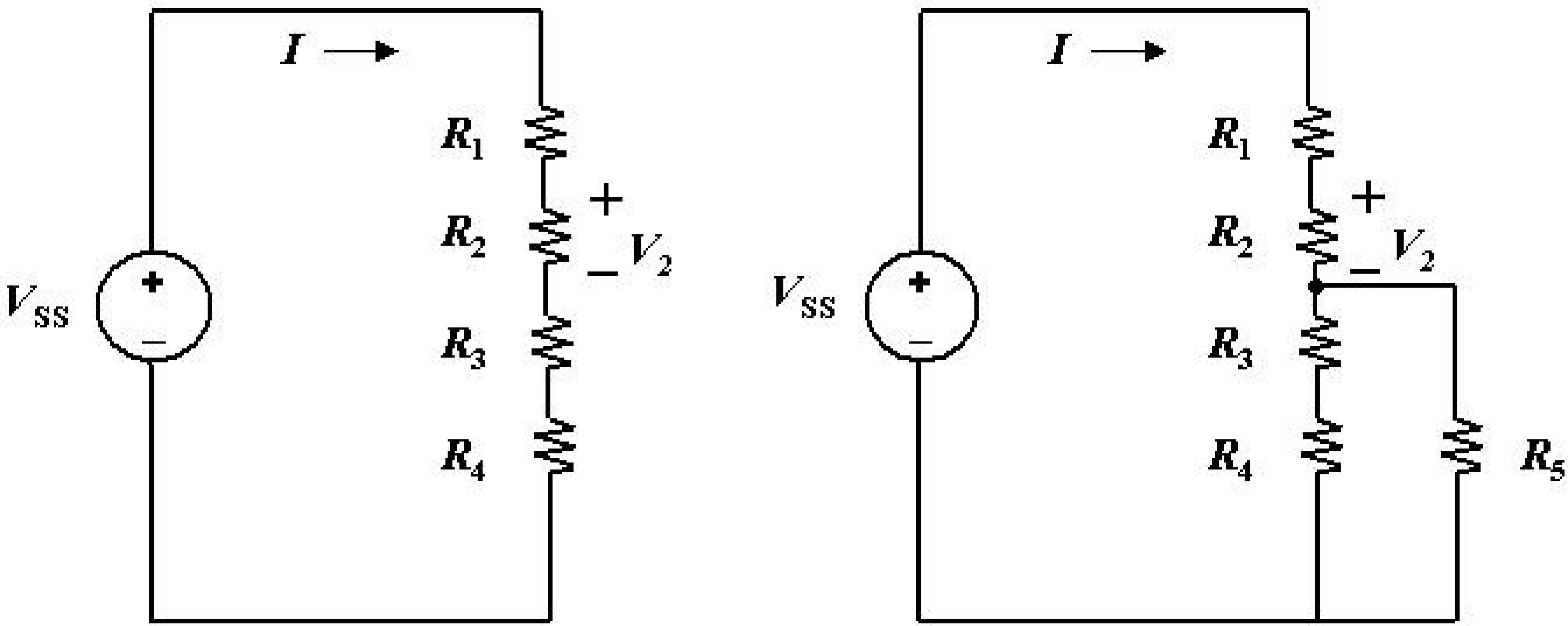
Equivalent resistance of resistors in series is the sum

Voltage Divider



$$I = V_{SS} / (R_1 + R_2 + R_3 + R_4)$$

When can the Voltage Divider Formula be Used?



$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

$$V_2 \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

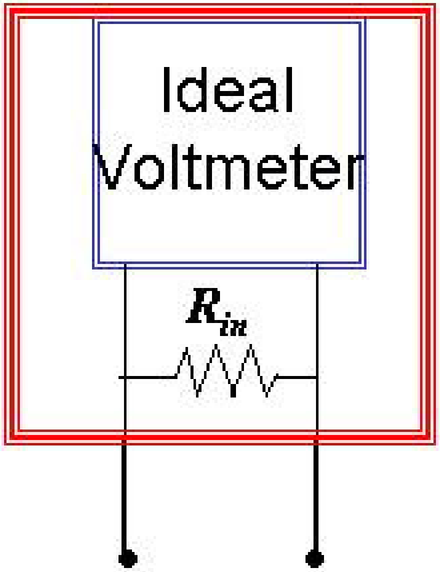
Correct, if nothing else is connected to nodes

because R_5 removes condition of resistors in series

Measuring Voltage

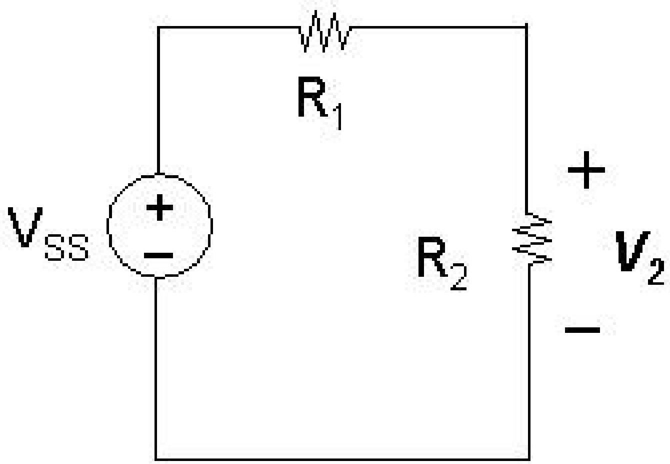
To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.

Voltmeters are characterized by their "voltmeter input resistance" (R_{in}). Ideally, this should be very high (typical value 10 MΩ)



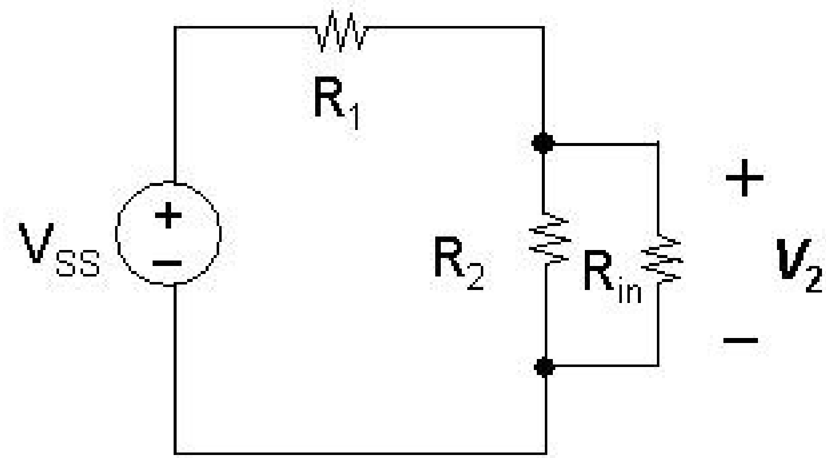
Effect of Voltmeter

undisturbed circuit



$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right]$$

circuit with voltmeter inserted



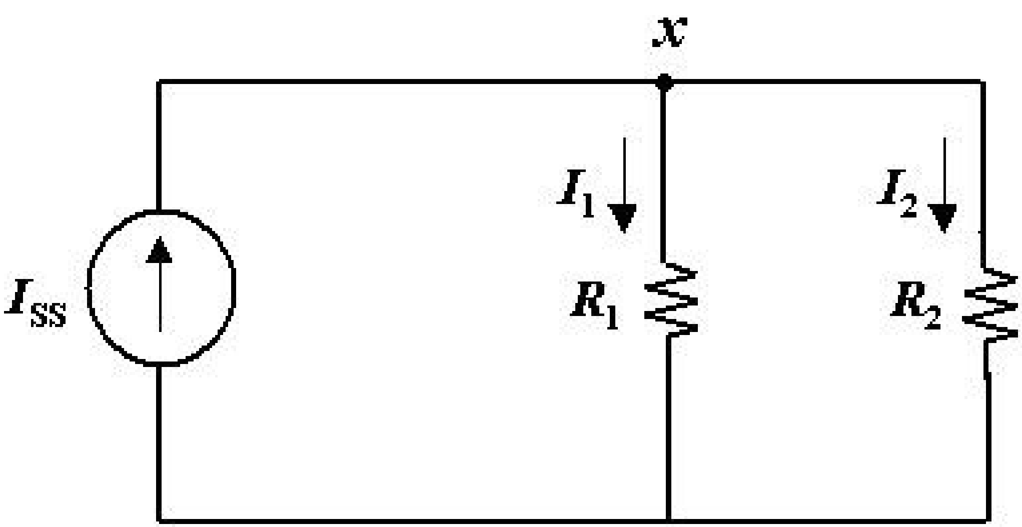
$$V_2' = V_{SS}\left[\frac{R_2 \parallel R_{in}}{R_2 \parallel R_{in} + R_1}\right]$$

Example: $V_{SS} = 10\,V, R_2 = 100\,K, R_1 = 900\,K \Rightarrow V_2 = 1V$

If $R_{in} = 10M, V_2' = 0.991V,$

Resistors in Parallel

Consider a circuit with two resistors connected in parallel. Find their "equivalent resistance".



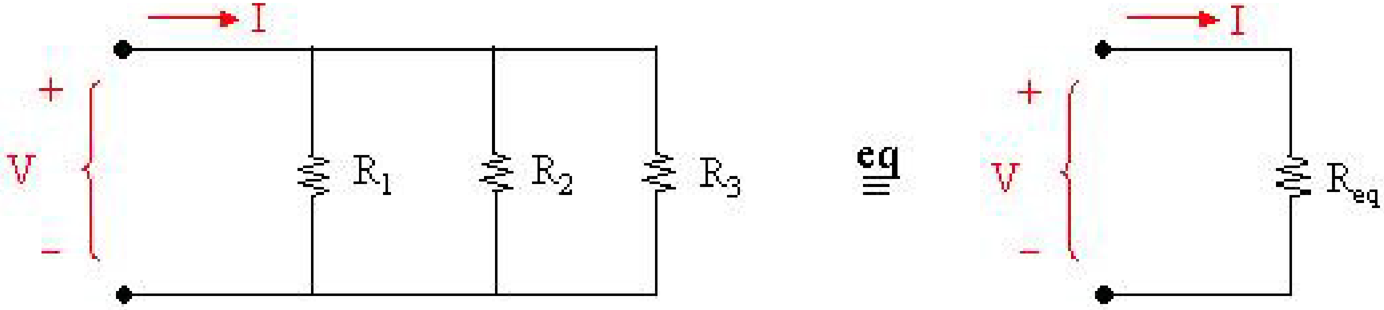
- KVL tells us that the same voltage is dropped across each resistor

$$V_x = I_1 R_1 = I_2 R_2$$

- KCL tells us

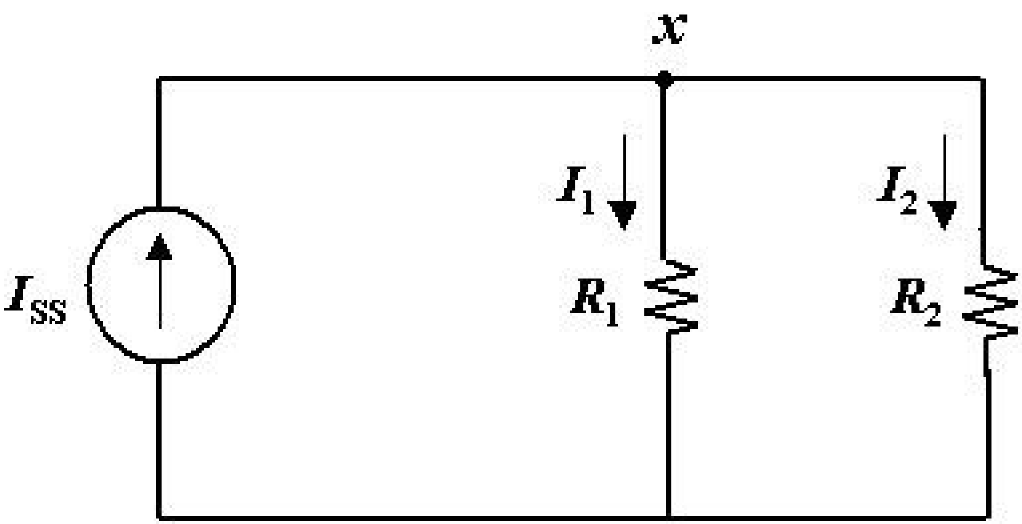
General Formula for Parallel Resistors

What single resistance R_{eq} is equivalent to three resistors in parallel?



Equivalent <u>conductance</u> of resistors in parallel is the sum

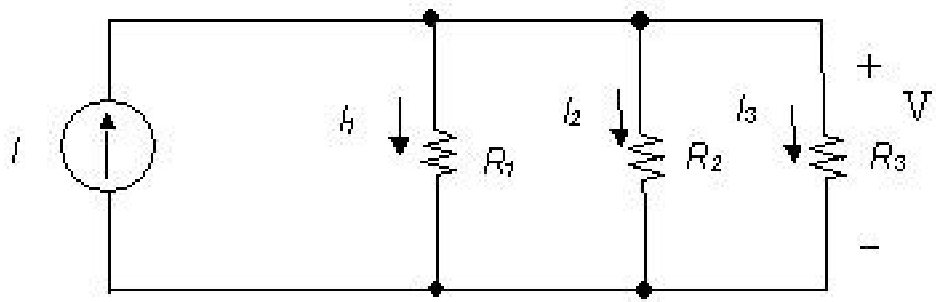
Current Divider



$$V_x = I_1 R_1 = I_{SS} R_{eq}$$

Generalized Current Divider Formula

Consider a current divider circuit with >2 resistors in parallel:

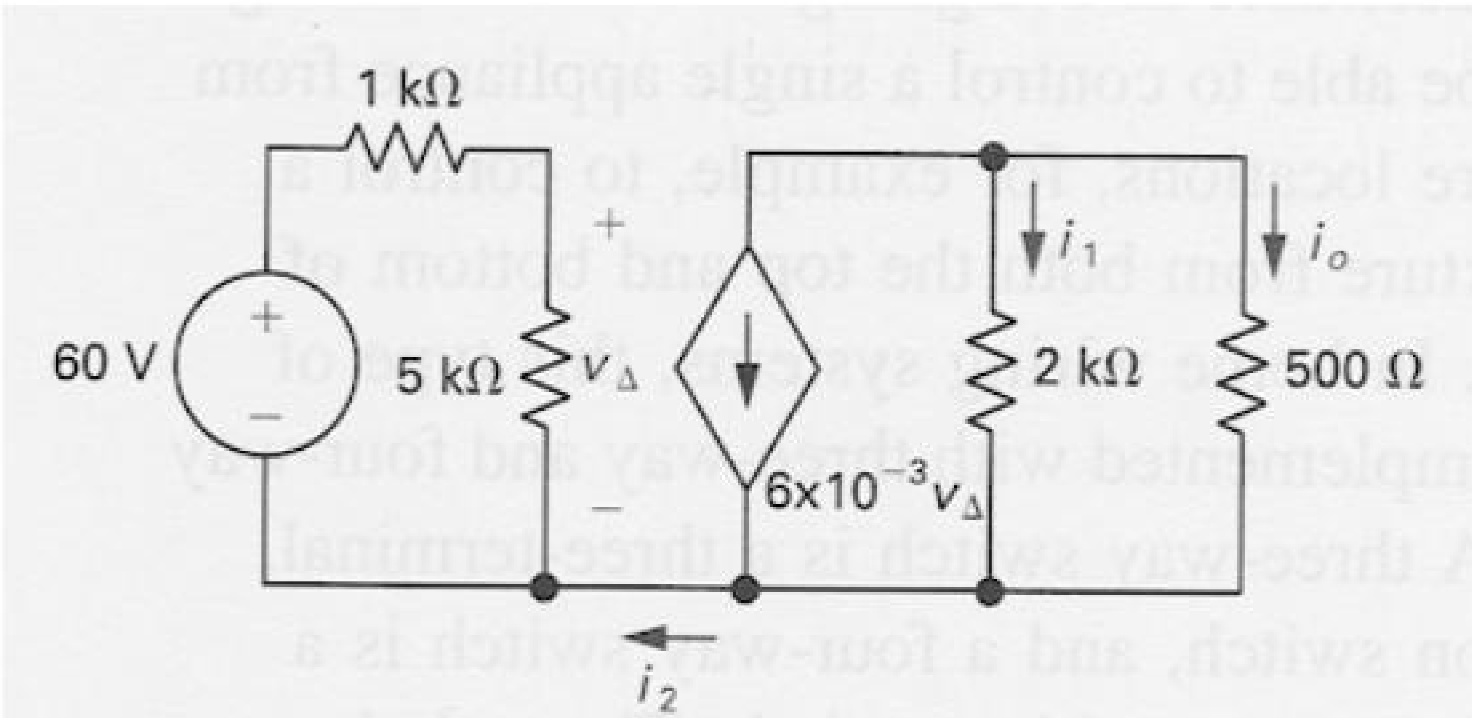


$$V = \frac{I}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)}$$

$$I_3 = \frac{V}{R_3} = I\left[\frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3}\right]$$

Circuit w/ Dependent Source Example

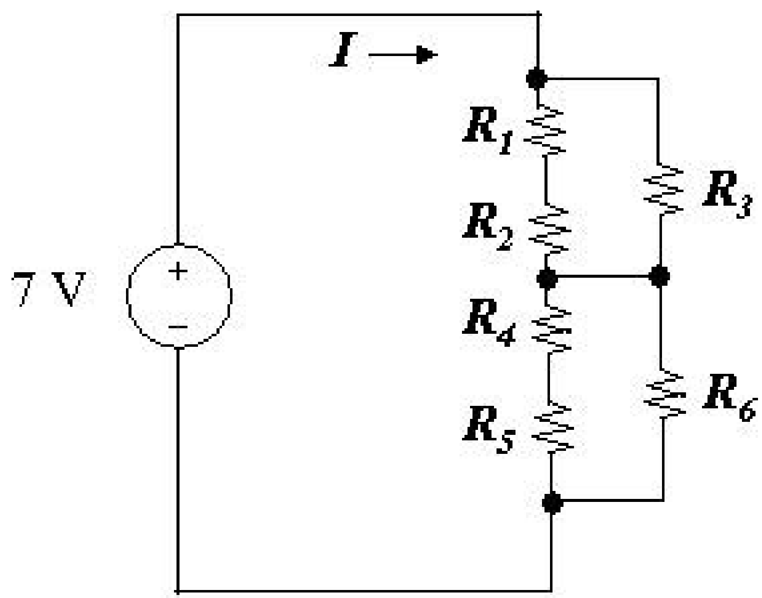
Find i_2, i_1 and i_o



Using Equivalent Resistances

Simplify a circuit before applying KCL and/or KVL:

Example: Find I



$R_1 = R_2 = 3\ \text{k}\Omega$

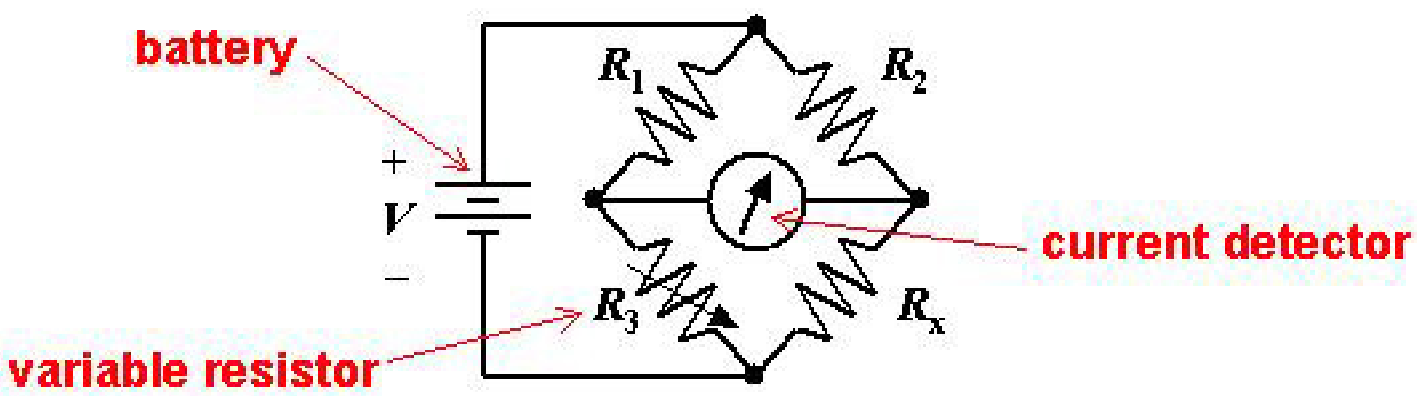
$R_3 = 6\ \text{k}\Omega$

$R_4 = R_5 = 5\ \text{k}\Omega$

$R_6 = 10\ \text{k}\Omega$

The Wheatstone Bridge

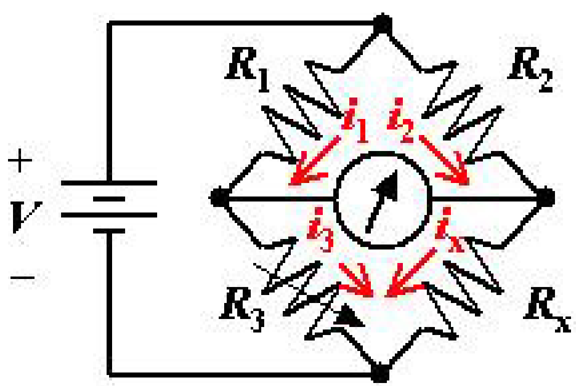
- Circuit used to precisely measure resistances in the range from 1 Ω to 1 MΩ, with ±0.1% accuracy
  - R_1 and R_2 are resistors with known values
  - R_3 is a variable resistor (typically 1 to 11,000Ω)
  - R_x is the resistor whose value is to be measured



Finding the value of R_x

- Adjust R_3 until there is no current in the detector

Then, $$R_x = \frac{R_2}{R_1} R_3$$



Typically, R_2 / R_1 can be varied from 0.001 to 1000 in decimal steps

Derivation:

KCL => $i_1 = i_3$ and $i_2 = i_x$

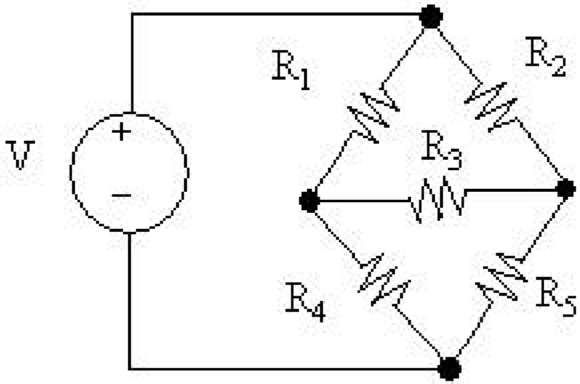
KVL => $i_3 R_3 = i_x R_x$ and $i_1 R_1 = i_2 R_2$

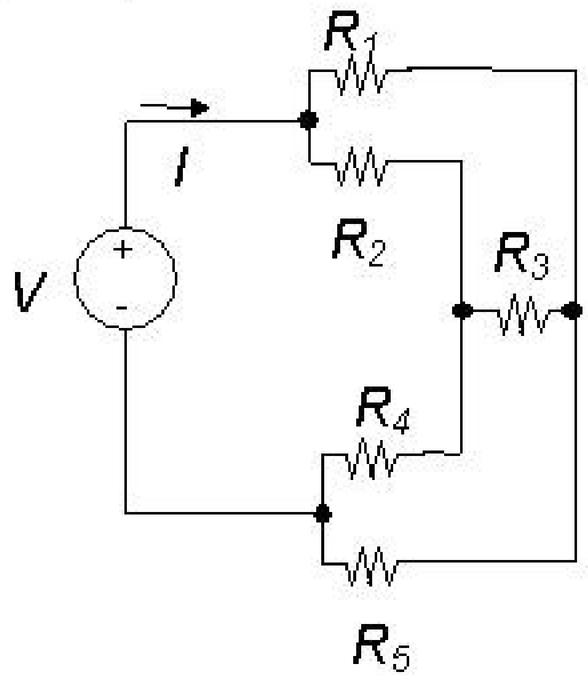
$i_1 R_3 = i_2 R_x$

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

Identifying Series and Parallel Combinations

Some circuits *must* be analyzed (not amenable to simple inspection)





Special cases:

$R_3 = 0$ **OR** $R_3 = \infty$

Resistive Circuits: Summary

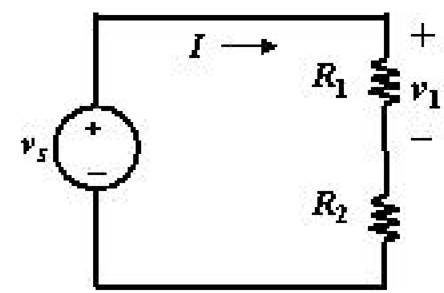
- Equivalent resistance of *k* resistors in series:

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k$$

- Equivalent resistance of *k* resistors in parallel:

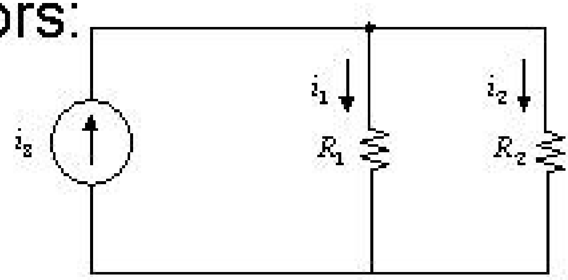
$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}$$

- Voltage divided between 2 series resistors:



$$v_1 = \frac{R_1}{R_1 + R_2} v_s$$

- Current divided between 2 parallel resistors:



$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$

Node-Voltage Circuit Analysis Method

1. **Choose a reference node** ("ground")

   Look for the one with the most connections!

2. **Define unknown node voltages**

   those which are not fixed by voltage sources

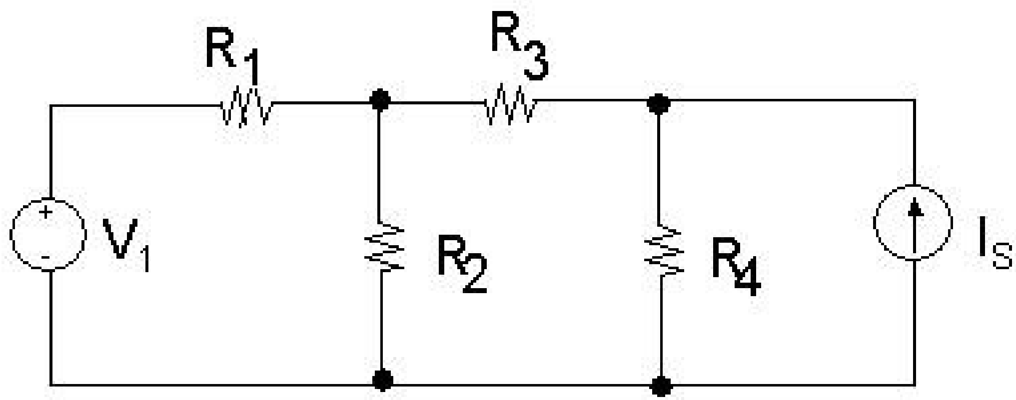
3. **Write KCL at each unknown node**, expressing current in terms of the node voltages (using the *I*-*V* relationships of branch elements)

   Special cases: floating voltage sources

4. **Solve the set of independent equations**

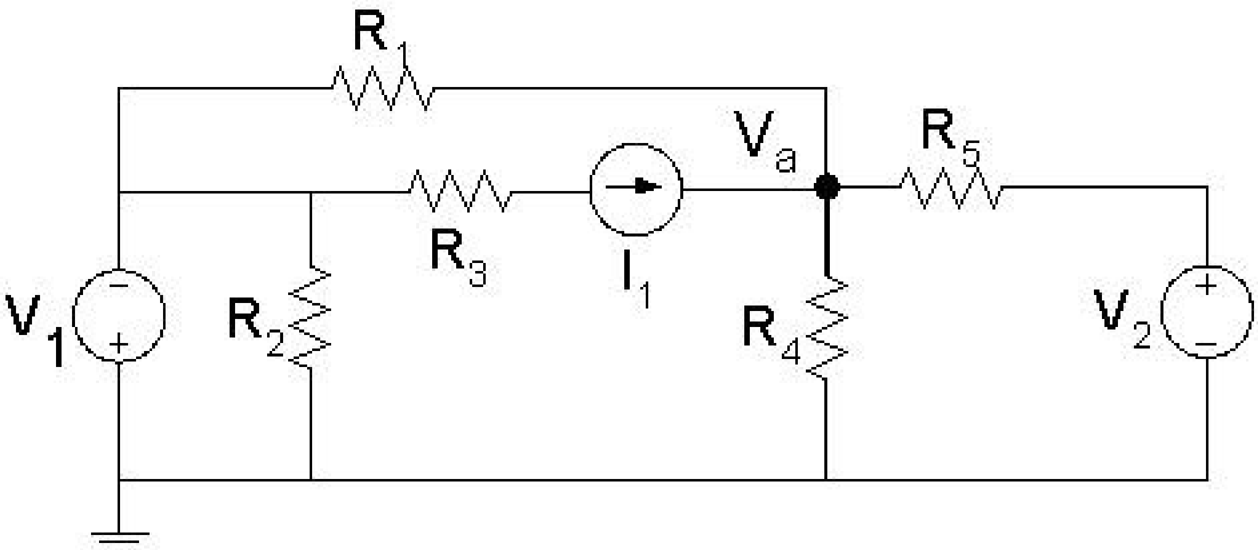
   N equations for *N* unknown node voltages

Nodal Analysis: Example #1



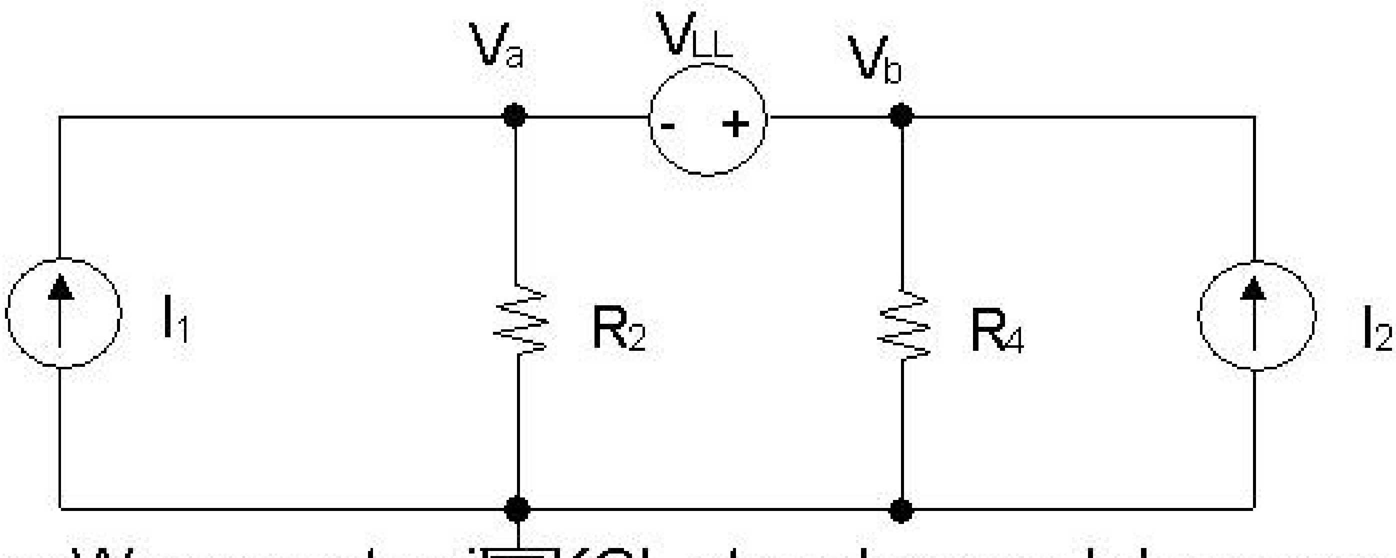
1. Choose a reference node.
2. Define the node voltages (except reference node and the one set by the voltage source).
3. Apply KCL at the nodes with unknown voltage.
4. Solve for unknown node voltages.

Nodal Analysis: Example #2



Nodal Analysis w/ "Floating Voltage Source"

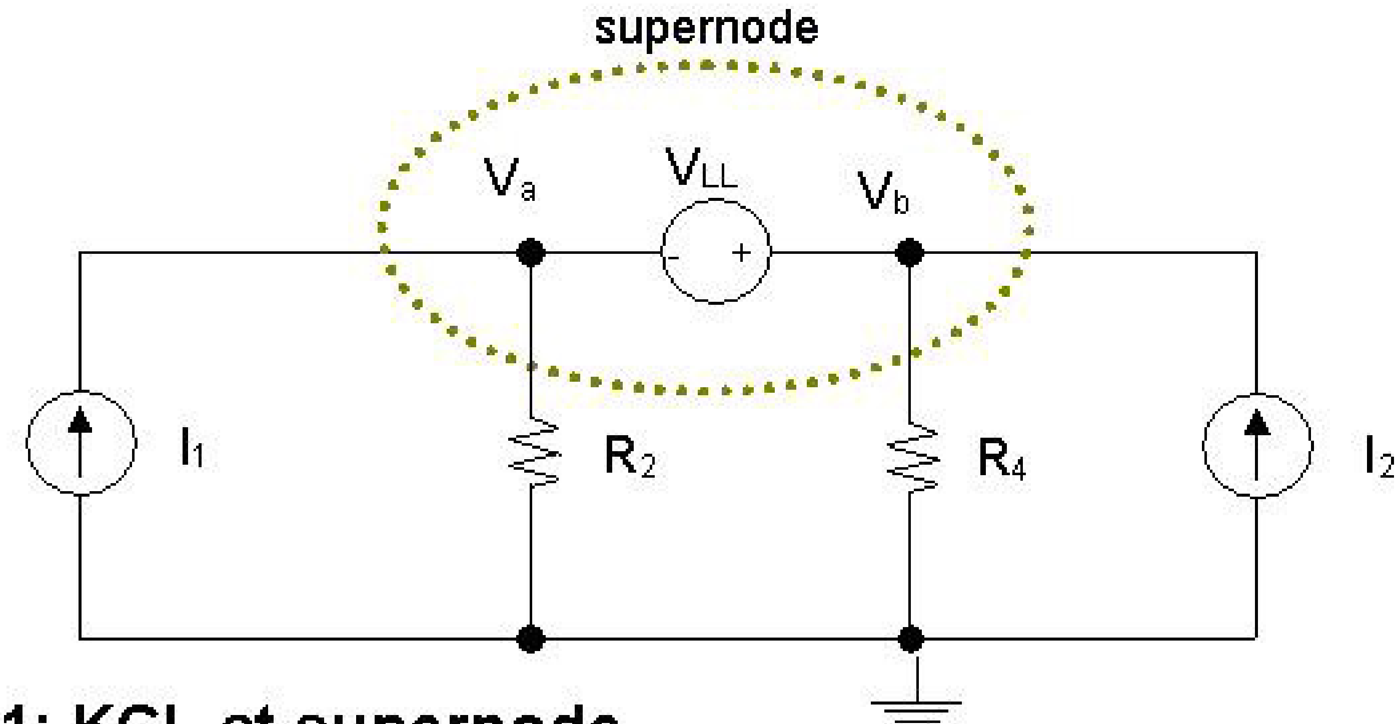
A "floating" voltage source is one for which neither side is connected to the reference node, e.g. V_{LL} in the circuit below:



Problem: We cannot write KCL at nodes a or b because there is no way to express the current through the voltage source in terms of V_a-V_b.

Solution: Define a "supernode" – that chunk of the circuit containing nodes a and b. Express KCL for this supernode. Incorporate voltage source constraint into KCL equation.

Nodal Analysis: Example #3



Eq'n 1: KCL at supernode

Substitute property of voltage source: