Lecture #6

OUTLINE

- Complete Mesh Analysis Example(s)
- Superposition
- Thévenin and Norton equivalent circuits
- Maximum Power Transfer

Reading

Chapter 2

Superposition

A ***linear circuit*** is constructed only of linear elements (linear resistors, linear dependent sources) and independent sources.

Principle of Superposition:

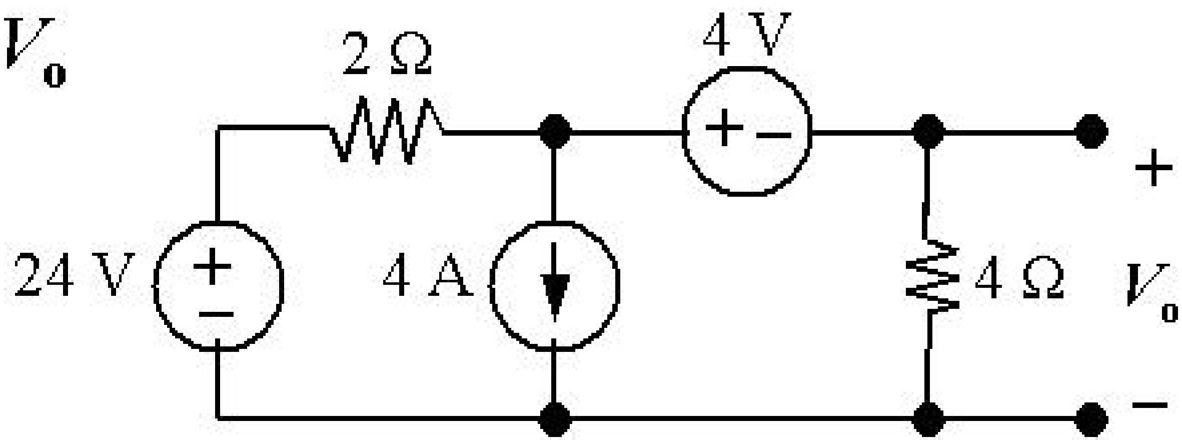
- In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

Procedure:

1. Determine contribution due to an independent source
   - Set all other sources to 0
2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

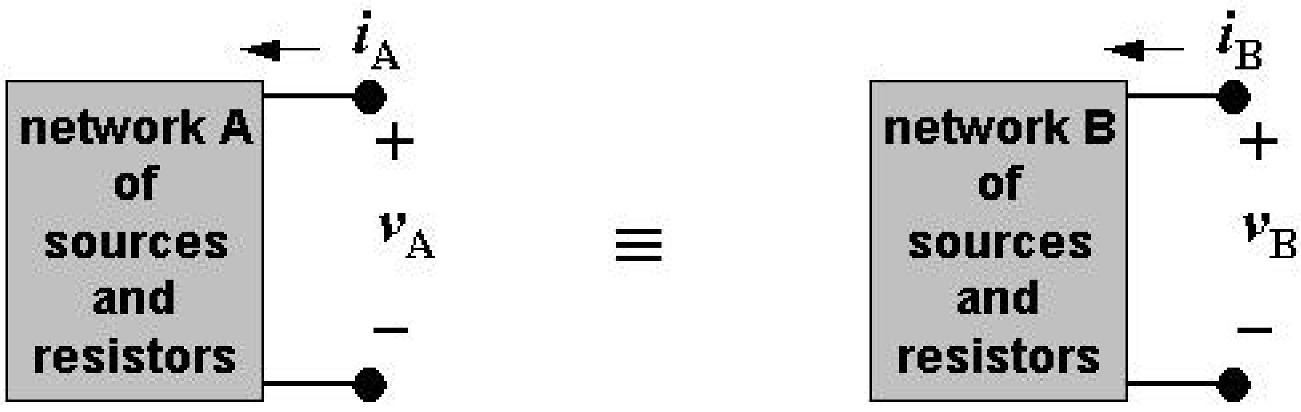
Superposition Example

- Find V_o



Equivalent Circuit Concept

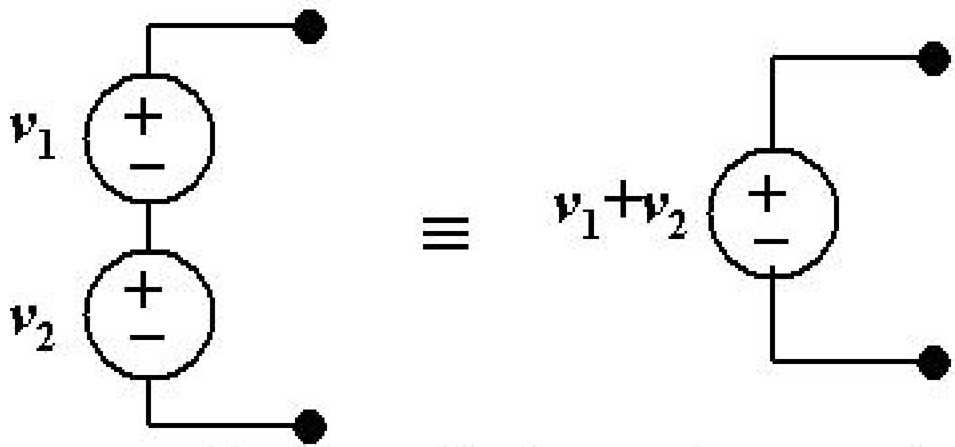
- A network of voltage sources, current sources, and resistors can be replaced by an ***equivalent circuit*** which has identical terminal properties (*I-V* characteristics) without affecting the operation of the rest of the circuit.



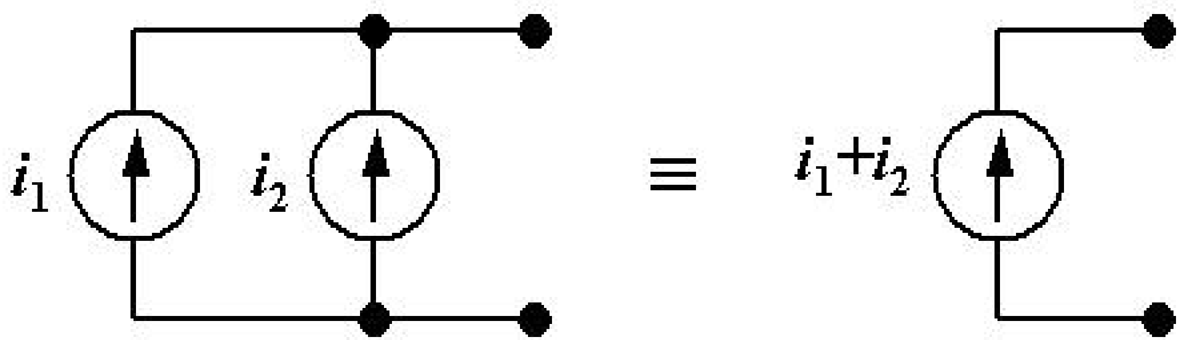
$$i_A(v_A) = i_B(v_B)$$

Source Combinations

- Voltage sources in series can be replaced by an equivalent voltage source:

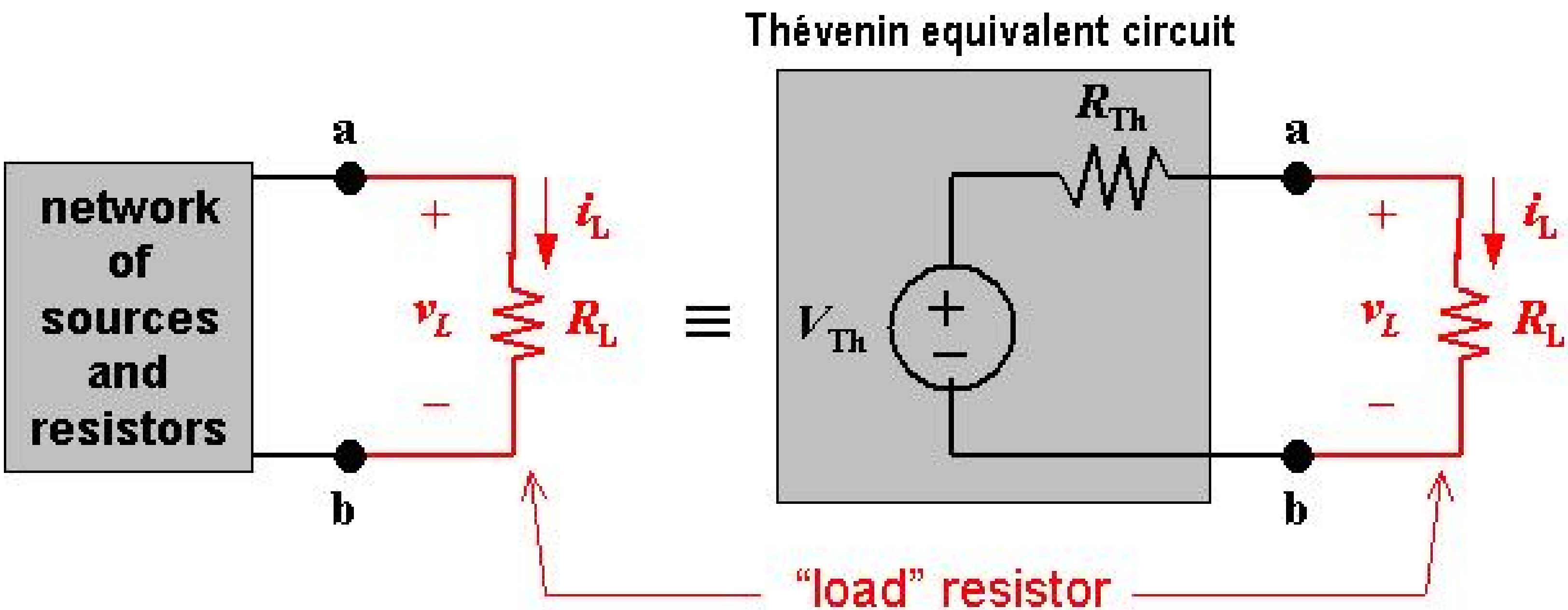


- Current sources in parallel can be replaced by an equivalent current source:



Thévenin Equivalent Circuit

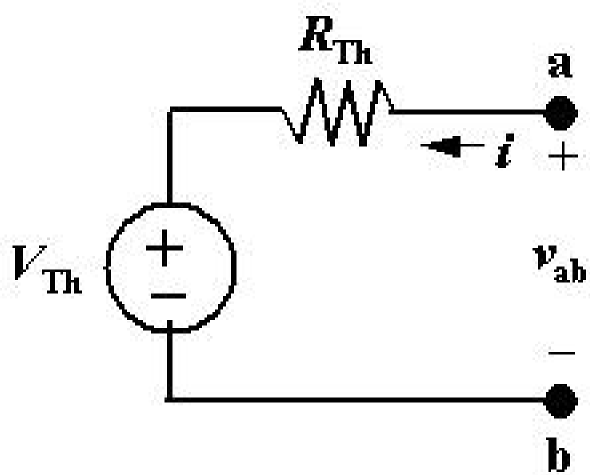
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent voltage source in series with a resistor without affecting the operation of the rest of the circuit.

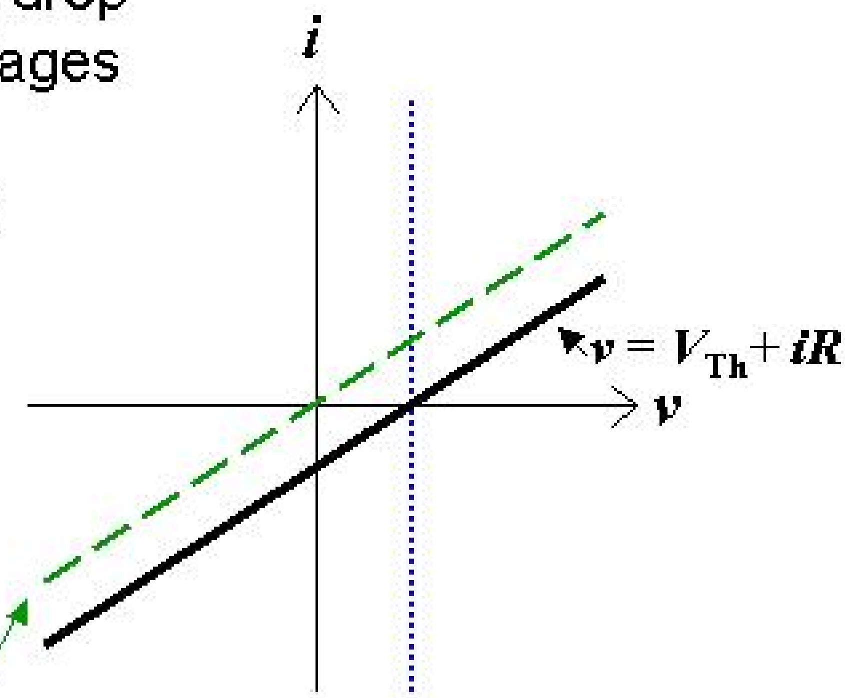


I-V Characteristic of Thévenin Equivalent

- **The *I-V* characteristic for the series combination of elements is obtained by adding their voltage drops:**

For a given current $\boldsymbol{i}$, the voltage drop v_{ab} is equal to the sum of the voltages dropped across the source (V_{Th}) and the across the resistor (iR_{Th})





I-V characteristic of resistor: $v = iR$

I-V characteristic of voltage source: $v = V_{Th}$

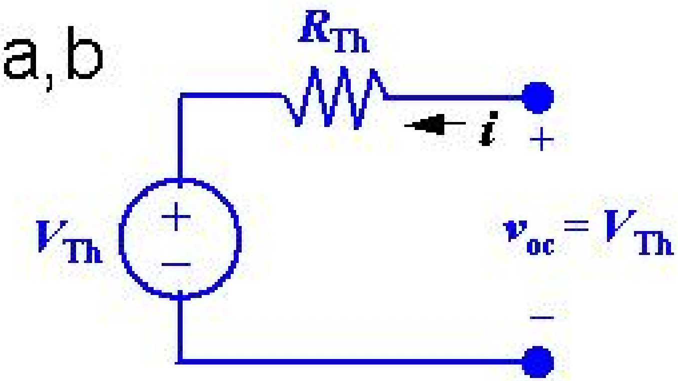
Finding V_{Th} and R_{Th}

Only two points are needed to define a line.

Choose two convenient points:

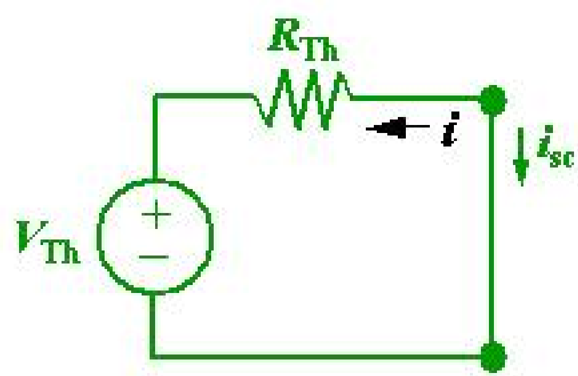
1. Open circuit across terminals a,b

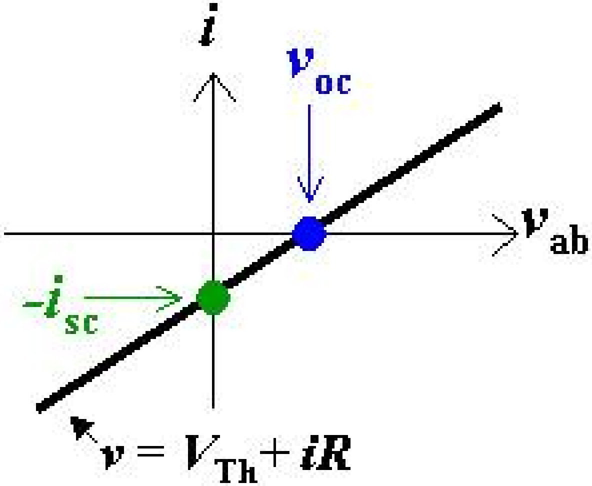
$$i = 0,\ v_{ab} \equiv v_{oc}$$



2. Short circuit across terminals a,b

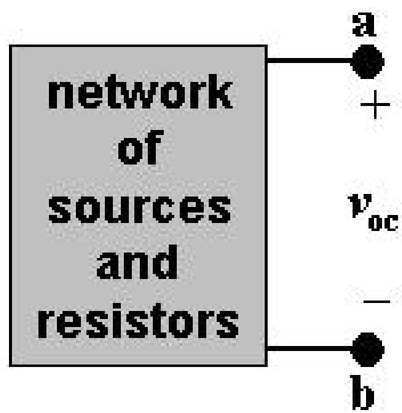
$$v_{ab} = 0,\ i \equiv -i_{sc} = -V_{Th}/R_{Th}$$



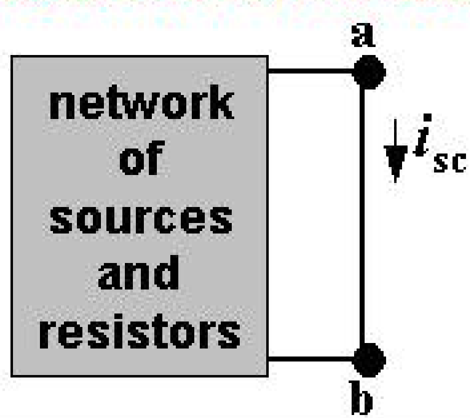


Calculating a Thévenin Equivalent

1. Calculate the ***open-circuit voltage***, v_{oc}



2. Calculate the ***short-circuit current***, i_{sc}
   - Note that i_{sc} is in the direction of the open-circuit voltage drop across the terminals a,b !

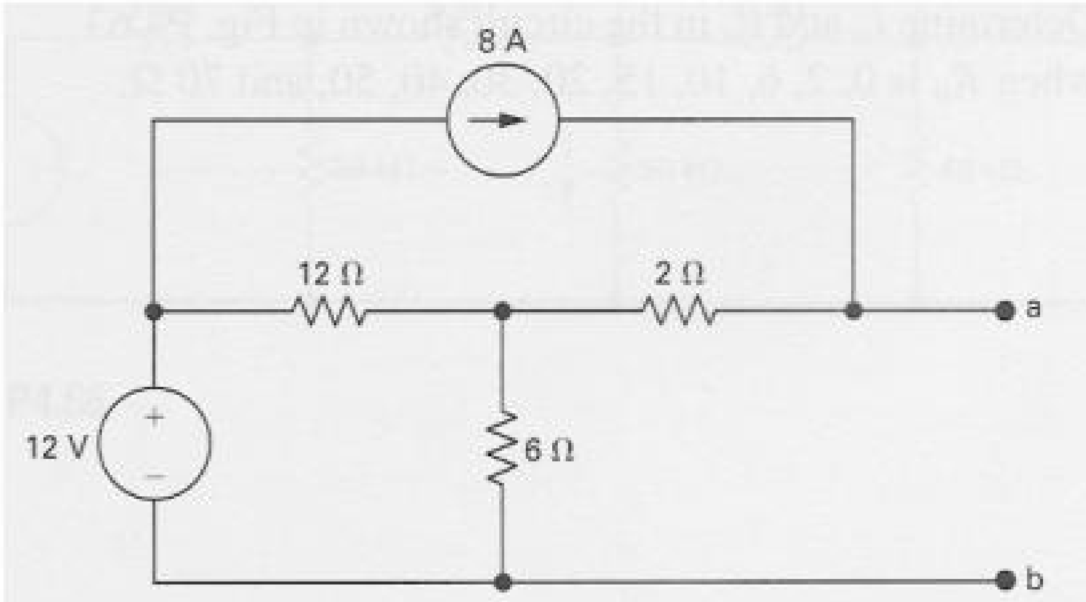


$$V_{Th} = v_{oc}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}}$$

Thévenin Equivalent Example

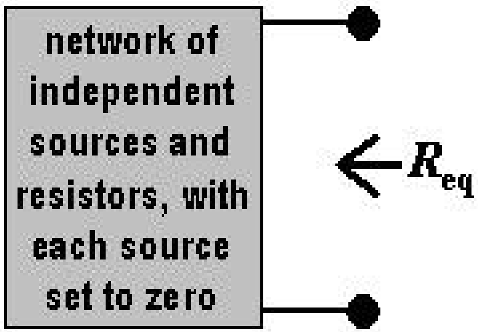
Find the Thevenin equivalent with respect to the terminals a,b:



Alternative Method of Calculating R_{Th}

For a network containing only independent sources and linear resistors:

1. Set all independent sources to zero
   voltage source → short circuit
   current source → open circuit

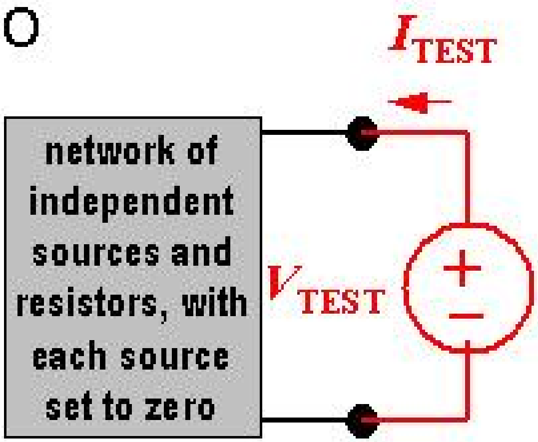


2. Find equivalent resistance R_{eq} between the terminals by inspection

$$R_{\text{Th}} = R_{eq}$$

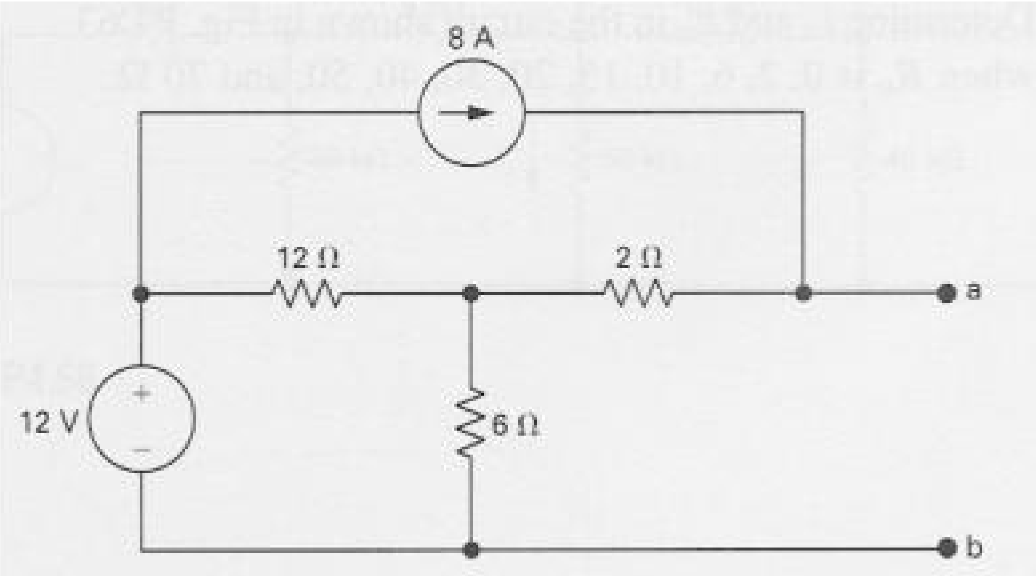
Or, set all independent sources to zero

1. Apply a test voltage source V_{TEST}
2. Calculate I_{TEST}



$$R_{\text{Th}} = \frac{V_{\text{TEST}}}{I_{\text{TEST}}}$$

R_{Th} Calculation Example #1



Set all independent sources to 0:

Comments on Dependent Sources

A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.

(device model, used to model behavior of transistors & amplifiers)

To specify a dependent source, we must identify:

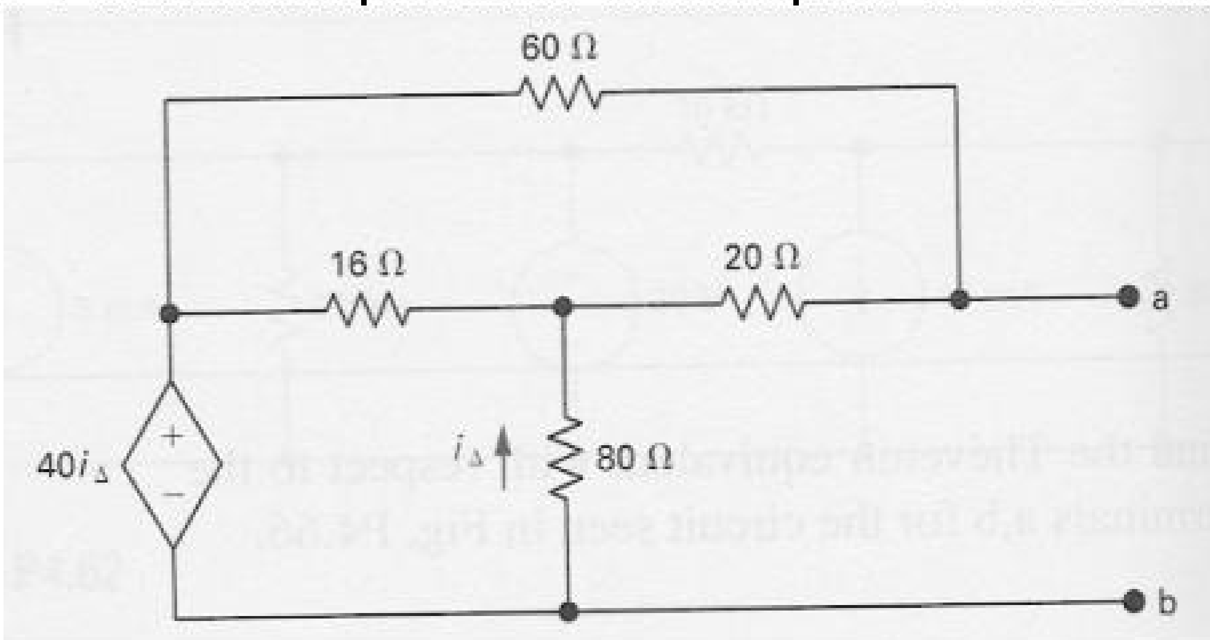
1. the controlling voltage or current (must be calculated, in general)
2. the relationship between the controlling voltage or current and the supplied voltage or current
3. the reference direction for the supplied voltage or current

The relationship between the dependent source and its reference cannot be broken!

– Dependent sources cannot be turned off for various purposes (*e.g.* to find the Thévenin resistance, or in analysis using Superposition).

R_{Th} Calculation Example #2

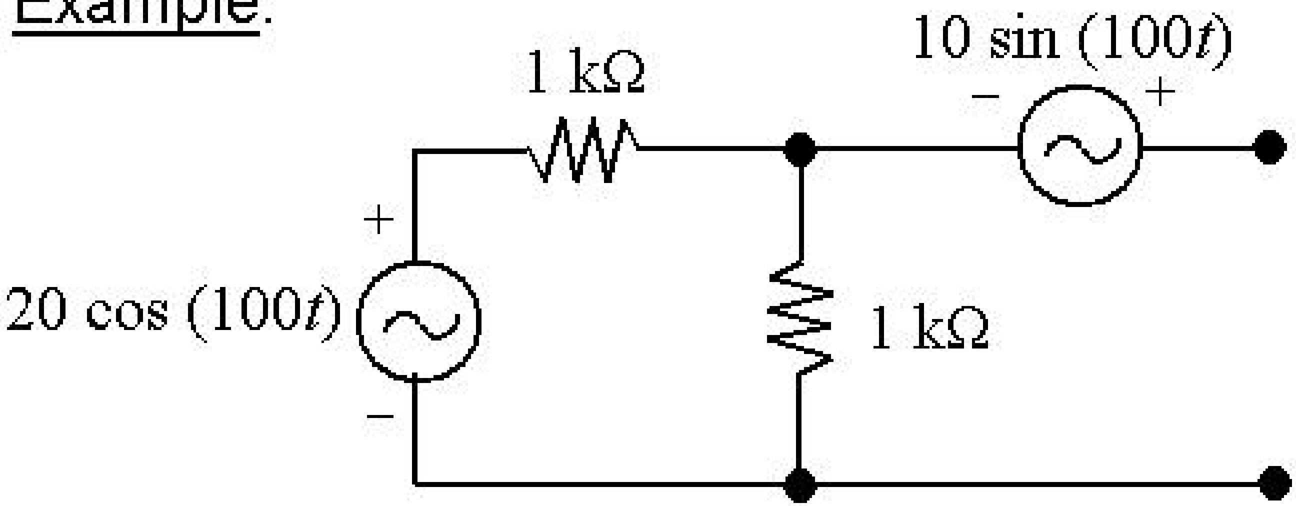
Find the Thevenin equivalent with respect to the terminals a,b:



Networks Containing Time-Varying Sources

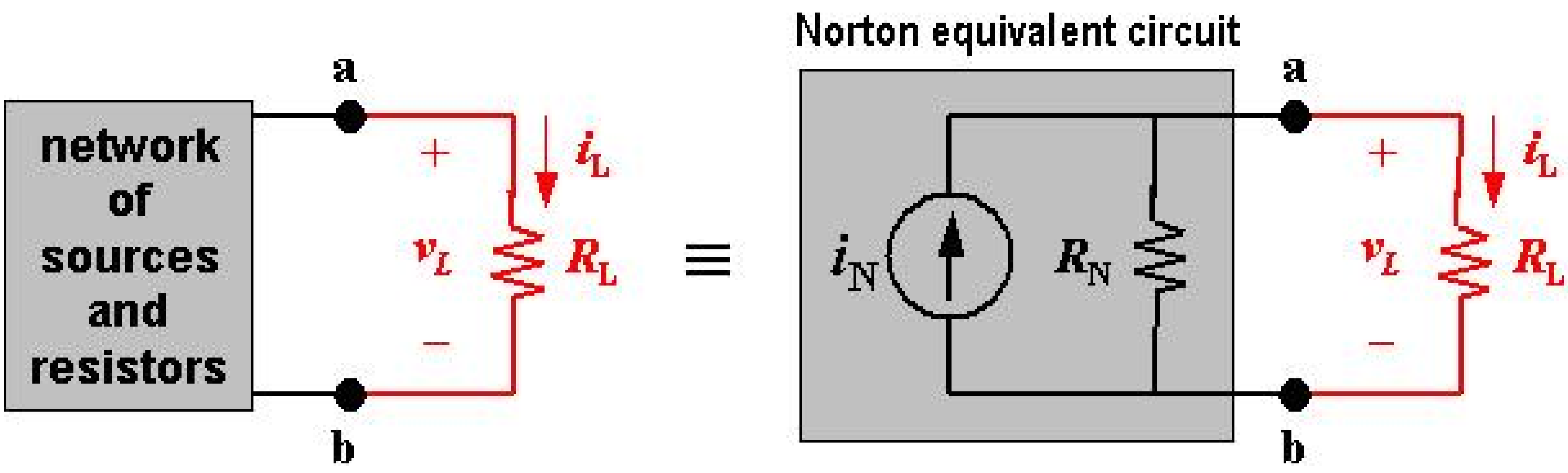
Care must be taken in summing time-varying sources!

Example:



Norton Equivalent Circuit

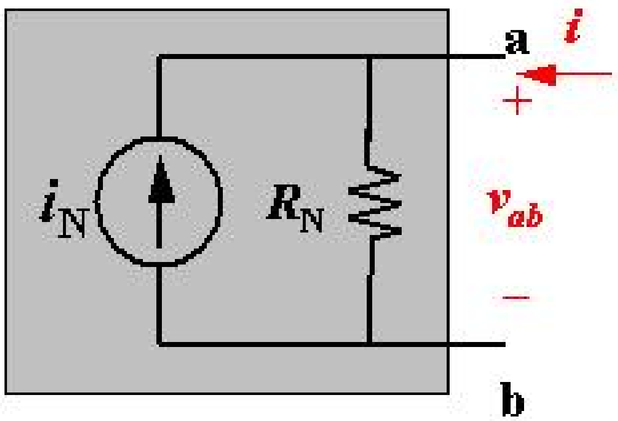
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent current source in parallel with a resistor without affecting the operation of the rest of the circuit.

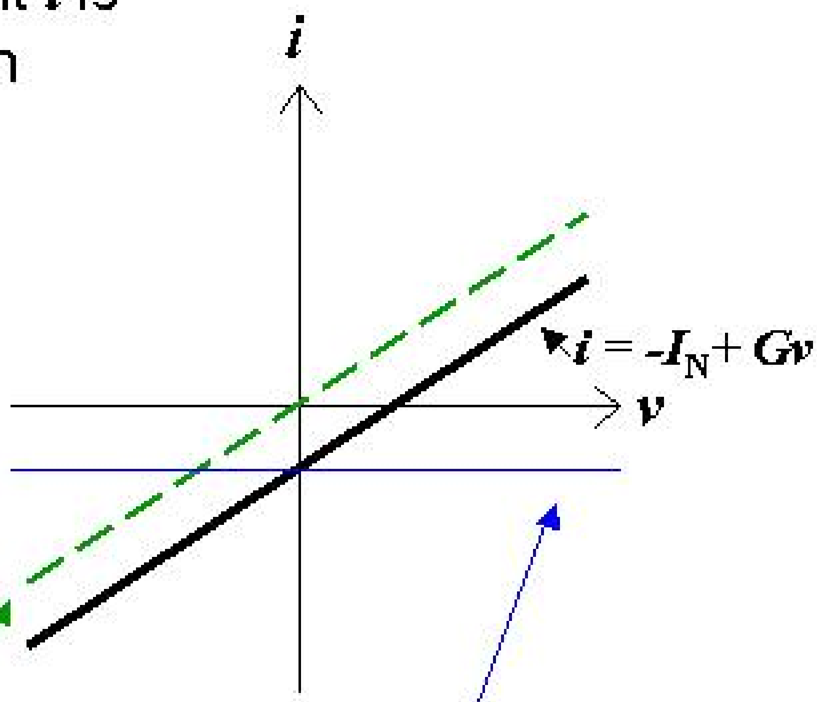


I-V Characteristic of Norton Equivalent

- **The *I-V* characteristic for the parallel combination of elements is obtained by adding their currents:**

For a given voltage v_{ab}, the current i is equal to the sum of the currents in each of the two branches:





I-V characteristic of resistor: $i=Gv$

I-V characteristic of current source: $i = -I_N$

Finding I_N and $R_N = R_{Th}$

Analogous to calculation of Thevenin Eq. Ckt:

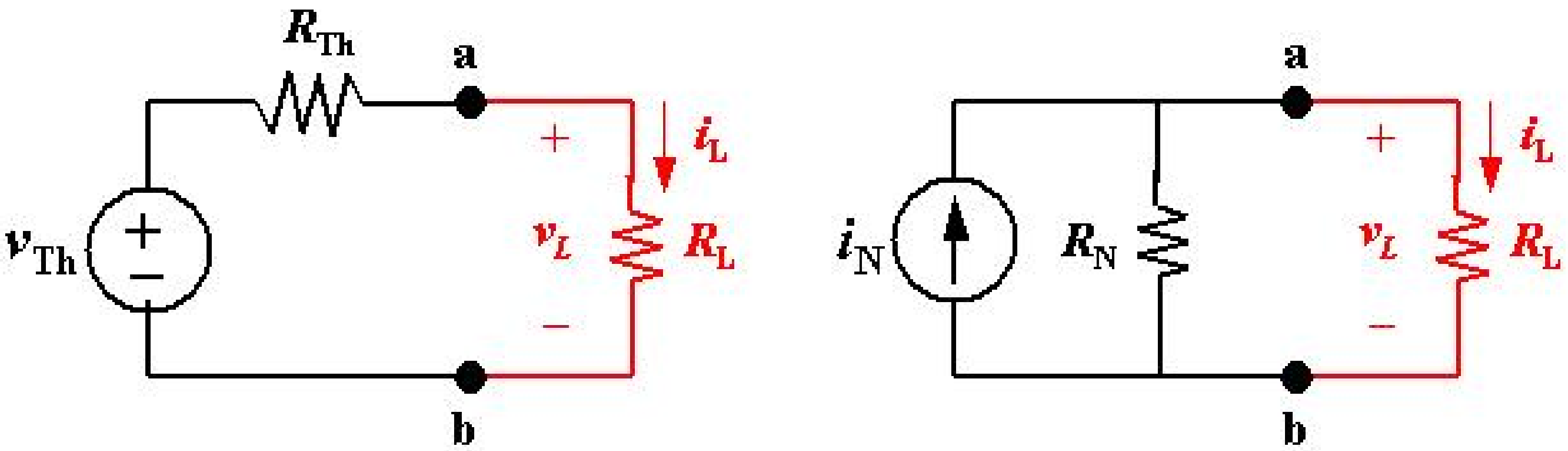
1) Find o.c voltage and s.c. current

$$I_N \equiv i_{sc} = V_{Th}/R_{Th}$$

2) Or, find s.c. current and Norton (Thev) resistance

Finding I_N and R_N

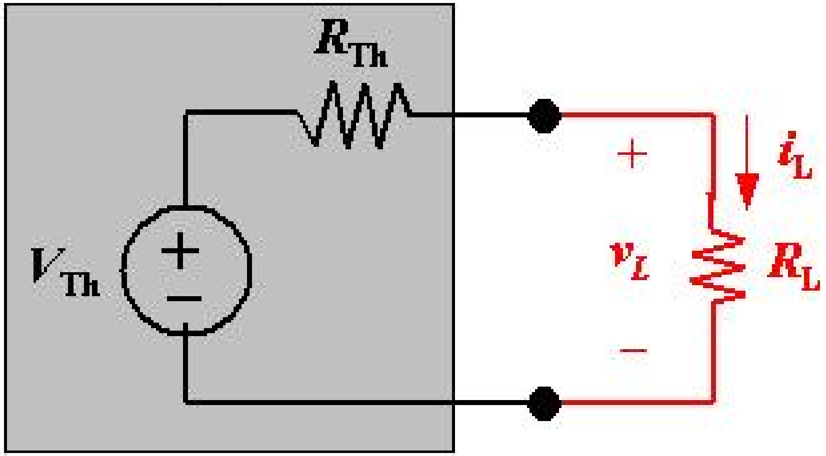
- We can derive the Norton equivalent circuit from a Thévenin equivalent circuit simply by making a source transformation:



$$R_N = R_{Th} = \frac{v_{oc}}{i_{sc}}; \quad i_N = \frac{v_{Th}}{R_{Th}} = i_{sc}$$

Maximum Power Transfer Theorem

Thévenin equivalent circuit



Power absorbed by load resistor:

$$p = i_L^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

To find the value of R_L for which p is maximum, set $\frac{dp}{dR_L}$ to 0:

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$

$$\Rightarrow (R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L) = 0$$

$$\Rightarrow \boxed{R_{Th} = R_L}$$

A resistive load receives maximum power from a circuit if the load resistance equals the Thévenin resistance of the circuit.