HW11 Solutions

1.

Because the doping concentration is much larger than the intrinsic carrier concentration, the electron concentration is equal to the doping concentration.

$$n_0 = N_d = 5\times10^{15}\,cm^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{\sqrt{N_c N_V e^{\frac{-Eg}{kT}}}}{n_0} 1.18\times10^{11}\,cm^{-3}$$

Where Eg is the bandgap of Ge, k is Boltzmann constant, and T is temperature in Kelvin.

2.

$$x_{p0} = 0.2W = 0.2(x_{p0} + x_{n0})$$

$$\frac{x_{p0}}{x_{n0}} = \frac{1}{4} = \frac{N_d}{N_a}$$

The relation between doping concentrations and built-in potential is,

$$V_{bi} = \frac{kT}{q}\ln(\frac{N_a N_d}{n_i^2}) = 0.026\times\ln(\frac{4N_d^2}{(1.86\times10^6)^2}) = 1.2V$$

$$N_d = 9.79\times10^{15}\,cm^{-3}$$

$$N_a = 4N_d = 3.915\times10^{16}\,cm^{-3}$$

Where q is the charge of an electron, k is Boltzmann constant, and T is temperature in Kelvin.

From Na and Nd, we can get depletion region width,

$$W = \left[\frac{2\varepsilon_s V_{bi}}{q}(\frac{N_a + N_d}{N_a N_d})\right]^{1/2} = 4.3\times10^{-4}\,m = 430\mu m$$

$$x_{p0} = 0.2W = 86\mu m$$

$$x_{n0} = 0.8W = 344\mu m$$

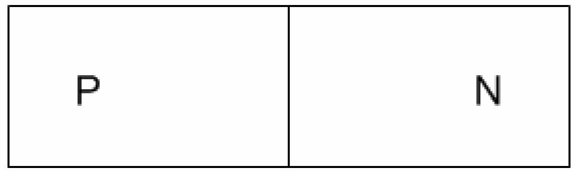
The formula for maximum electric field is,

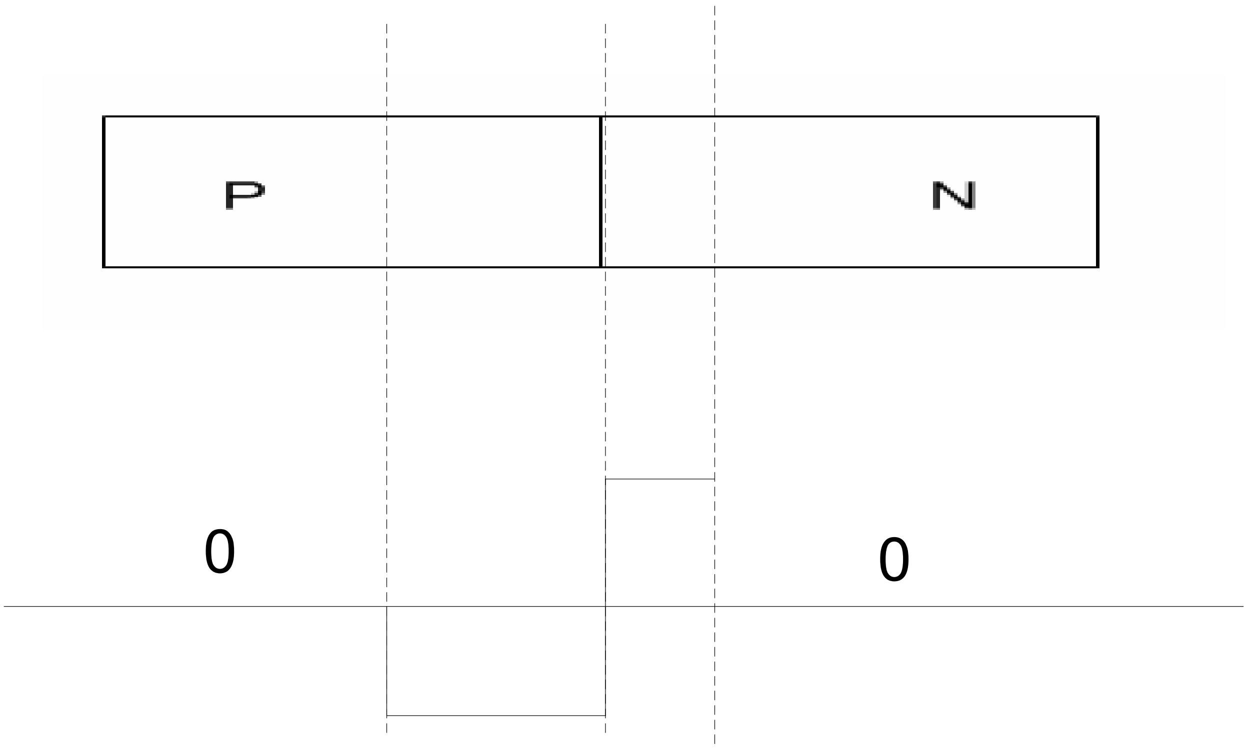
$$E_{\max} = \frac{-qN_d x_{n0}}{\varepsilon_s} = -4648\frac{V}{m}$$

3. This problem will explore the relationship between the charge density, electrical field, and voltage potential in the depletion region.

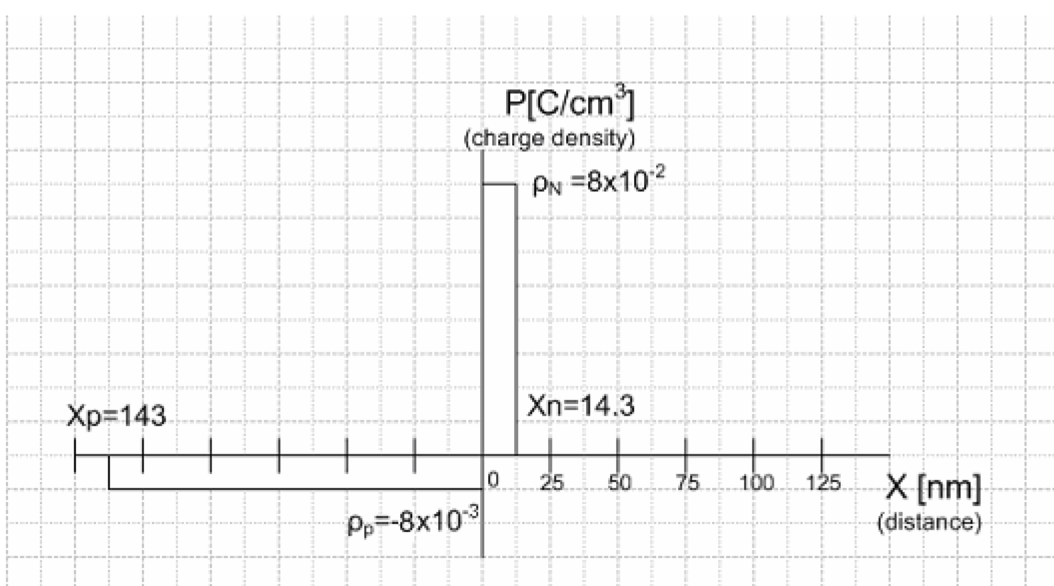
   a) What is the *approximation* in the Depletion Approximation?
   The free carriers within the depletion region are completely depleted. The depletion region stops abruptly at the boundaries with quasi-neutral region.

b) Give the following PN Junction, sketch the depletion region and label the charges seen in each region.





c) The charge profile in the PN Junction is shown below.



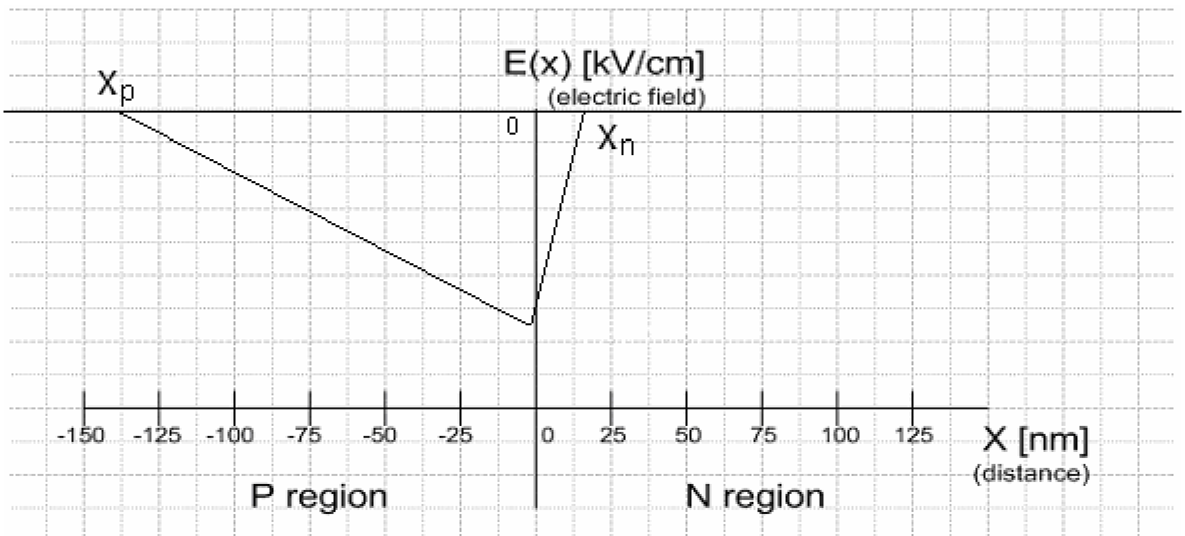
Given this charge profile, use Gauss Law to find and sketch the Electric Field within the depletion region as a function of x, where x = 0 represents the origin of the x-axis. Xn and Xp represent the width of the depletion region in the P and N regions, respectively. Pn and Pp represents the charge density of the depletion region in the P and N regions, respectively. Remember that the Electric Field outside of the depletion region is 0.

Note:
Gauss Law:
$\frac{dE}{dx} = \frac{\mathrm{P}(x)}{\varepsilon}$, where P(x) is the charge density and ε is 11.7x8.85e-14 F/cm.

Integrate the above equation from left to right, with the boundary conditions E=0, we have

$$E(x) = \begin{cases} 0 & x < x_p \\ \frac{\rho_P (x - x_p)}{\varepsilon} & x_p < x < 0 \\ \frac{\rho_p (-x_p)}{\varepsilon} + \frac{\rho_n x}{\varepsilon} & 0 < x < x_n \\ 0 & x > x_n \end{cases}$$

The value at x=0 is 1.1E5 V/cm → 1.1E2 kV/cm.



d) Having solved for the electric field E(x), why is the depletion region in the P region much wider than that in the N region of the PN Junction?
N-doping concentration is much higher than P-doping. In order to keep the charge neutrality, depletion region width in P must be much bigger than the one in N region.

e) Given the Electric Field found above, find the Voltage across the depletion region in terms of x. Assume Vo @ -150nm = 0V

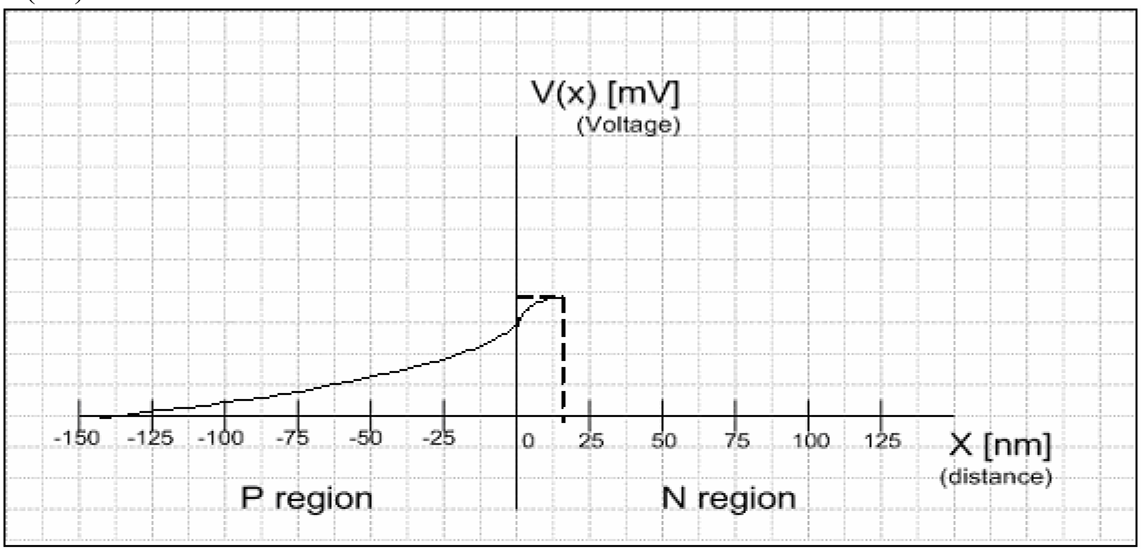
Note: $V(x) = Vo + \int_0^x - E(x)dx$

The integration can be carried out by simply using the formula for area of a triangle.

$$V(x) = \frac{1}{2}(x - x_p)\frac{\rho_P(x - x_p)}{\varepsilon} \qquad (x_p < x < 0)$$

$$V(x) = \frac{1}{2}(-x_p)\frac{\rho_P(-x_p)}{\varepsilon} + \frac{1}{2}(x)\left(\frac{\rho_p(-x_p)}{\varepsilon} + \frac{\rho_n x}{\varepsilon}\right) \qquad (0 < x < x_n)$$

The voltage values at X>=Xn and X = 0nm are:
V(0) = 0.787V = 787mV
V(Xn) = 0.865V = 865mV



4. (a) 5.625
   (b) 7.75
   (c) 10.25
   (d) 7.875
   (e) 8.3125
   (f) 21.375

5. (a) Counting in decimal, 778 follows 777
   (b) Counting in octal, 1000 follows 777
   (c) Counting in hexadecimal, 778 follows 777

6. (a) $F = (A + B)\,\overline{C}$

   (b) $F = A + B + (\overline{BC})$

   (c) $F = AB + (\overline{BC}) + D$

7.

A	B	$A + \overline{A}B$	A+B
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

8. (a) $F = AB + (\overline{C} + A)\,\overline{D} = \overline{(\overline{A} + \overline{B})(C\,\overline{A} + D)}$

(b) $F = A(\overline{B} + C) + D = \overline{(\overline{A} + B\overline{C})\,\overline{D}}$

(c) $F = A\,\overline{B}\,C + A(B + C) = \overline{(\overline{A} + B + \overline{C})(\overline{A} + \overline{B}\,\overline{C})}$

(d) $F = (A + B + C)(A + \overline{B} + C)(\overline{A} + B + \overline{C}) = \overline{\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,C}$

(e) $F = ABC + A\,\overline{B}\,C + \overline{A}\,B\,\overline{C} = \overline{(\overline{A} + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(A + \overline{B} + C)}$

9. The truth table is:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Thus we can write the product of sums expression and apply DeMorgan's Laws to obtain:

$$A \oplus B = (A + B)(\overline{A} + \overline{B}) = \overline{\overline{(A + B)} + \overline{(\overline{A} + \overline{B})}}$$

The circuit is: