HW12 Solutions

1. (10 points)
   (a) In the saturation region, \( V_{DS} \geq V_{GS} - V_{ir} = 2V \)
   In the triode region, \( V_{DS} \leq V_{GS} - V_{ir} = 2V \)
   Also check \( V_{GS} \geq V_{ir} \) so it’s not operated in the cutoff region
   (b) In the saturation region,
   \[ i_D = K(V_{GS} - V_{ir})^2 = 0.5(V_{GS} - 2)^2 \]

2. (10 points)
   When \( V_{GS} = V_{DS} = 5V \), the transistor is operated in the saturation region.
   \[ i_D = \left( \frac{W}{L} \right) K P \frac{1}{2} (V_{GS} - V_{ir})^2 \]
   Substitute all the values into the equation and we can solve \( W/L = 1.25 \).
   For \( L = 2\mu m \), we need \( W = 2.5\mu m \).

3. (10 points)
   The load-line equation is \( V_{DD} = R_D i_D + V_{DS} \), and the plots are:
4. (10 points)
(a) The 1.7Mohm and 300kohm resistors act as a voltage divider that gives a dc voltage $V_{GSQ} = 3V$. The capacitor is treated as a short for the ac signal, so we have,
$$V_{GS}(t) = 3 + \sin(2000\pi t)$$

(b) –(d)
Three operation regions:
Cutoff: $V_{GS} \leq V_{i0}$
$$i_D = 0$$
Triode: $V_{GS} \geq V_{i0}, V_{DS} \leq V_{GS} - V_{i0}$
$$i_D = K[2(v_{GS} - V_{i0})v_{DS} - v_{DS}^2]$$
Saturation: $V_{GS} \geq V_{i0}, V_{DS} \geq V_{GS} - V_{i0}$
$$i_D = K(V_{GS} - V_{i0})^2$$

The load-line equation is $V_{DD} = 2k*i_D + V_{DS}$

From the load-line, we find $V_{DSQ} = 16V$, $V_{DS\text{ max}} = 19V$, and $V_{DS\text{ min}} = 11V$.

5. (10 points)
(a) Assume the FET is operated in the saturation region,
$$I_{DQ} = K(V_{GSQ} - V_{i0})^2 \quad (1)$$
$$V_G = 20\frac{1M}{1M + 1M} = 10V \quad \text{from voltage divider.}$$

Using KVL, we get
$$V_{GSQ} + I_{DQ} \times 1k\Omega = 10V \quad (2)$$

Solving equations (1) & (2), $I_{DQ} = 4mA$, $V_{GSQ} = 6V$, and $V_{DSQ} = 20V - I_{DQ}(1k + 1k) = 12V$
Verification: \( V_{DSQ} \geq V_{GSQ} + V_{to} \), so it’s in the saturation region.

(b) Assume the FET is operated in the saturation region. Similar to part (a), we can solve

\[
I_{DQ} = 6.234\ mA, \quad V_{GSQ} = 3.765\ V, \quad \text{and} \quad V_{DSQ} = 20\ V - I_{DQ}(1k + 1k) = 7.53\ V
\]

Verification: \( V_{DSQ} \geq V_{GSQ} + V_{to} \), so it’s in the saturation region.

6. (10 points)
Write \( V_{DD} = R_D I_{DQ} + V_{DSQ} + R_S I_{DQ} \). Substituting values and solving we obtain \( R_S = 3k\Omega \). Next we have \( K = \frac{1}{2} K P(W/L) = 0.2\ mA/V^2 \). Assuming the NMOS operates in saturation, we have \( I_{DQ} = K (V_{GSQ} - V_{to})^2 \). Substituting values and solving we obtain \( V_{GSQ} = -1.236\ V \) and \( V_{GSQ} = 3.236\ V \). The correct root is \( V_{GSQ} = 3.236\ V \). Then we have \( V_G = V_{GSQ} + R_S i_{D} = 6.236\ V \). However we also have \( V_G = V_{DD} R_2/(R_1 + R_2) \). Substituting values and solving we obtain \( R_2 = 1.082M\Omega \)

7. (10 points)
We have \( V_G = V_{GSQ} = 10 \ R_2/(R_1 + R_2) = 2.5\ V \). Then we have \( I_{DQ} = K (V_{GSQ} - V_{to})^2 = 0.5625\ mA \).
\( V_{DSQ} = V_{DD} - R_D I_{DQ} = 4.375\ V \).

8. (10 points)
For constant drain current in the saturation region, we have \( r_d = \infty \).

9. (10 points)

\[
g_m = \frac{\partial i_D}{\partial V_{GS}}\bigg|_{Q_{po\ int}} = 9V_{GS}^2\bigg|_{Q_{po\ int}} = 9mS
\]

\[
1/r_d = \frac{\partial i_D}{\partial V_{DS}}\bigg|_{Q_{po\ int}} = 0.1mS\bigg|_{Q_{po\ int}} = 0.1mS
\]

\[
r_d = 10k\Omega
\]
10. (10 points)

From Figure P12.41 at an operating point defined by $V_{GSQ} = 2.5\, V$ and $V_{DSQ} = 6\, V$, we have

\[
g_m = \frac{\Delta i_D}{\Delta V_{GS}} = \frac{(6.4 - 1.5) mA}{1 V} = 4.9\, mS
\]

\[
1/r_d = \frac{\Delta i_D}{\Delta V_{GS}} \approx \frac{(4.0 - 3.1) mA}{(8 - 4) V} = 0.225 \times 10^{-3}
\]

Taking the reciprocal, we find $r_d = 4.44\, k\Omega$