UNIVERSITY OF CALIFORNIA AT BERKELEY EECS Department

# EECS 40/42/100, Spring 2007
# Prof. Chang-Hasnain

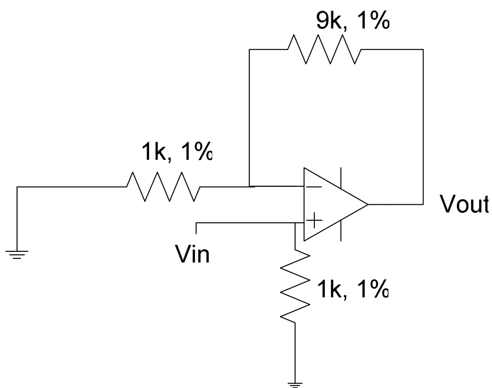
## Homework #9 (Note: EE42/100 is out of 85 pts, EE40 is out of 100 pts)

Due at 6 pm in 240 Cory on Wednesday, 04/04/07
Total Points: 100

- Put (1) your name and (2) **discussion section number** on your homework.
- You need to put down all the derivation steps to obtain full credits of the problems. Numerical answers alone will at best receive low percentage partial credits.
- No late submission will be accepted expect those with prior approval from Prof. Chang-Hasnain.
- *Note: Power gain is defined as the ratio between power to a load and power from an input source.

1. Hambley, P14.33 **[5 points]**

Because of the variation requirements, we must use 1% resistors. To satisfy the 1k input impedance requirement we place a 1k resistor at the input. To satisfy a gain of 10, we require a ratio of 9 in the feedback line resistances.




In this configuration, maximum output variation in the lower direction is:

$$\text{Gain} = 1 + \frac{9k(0.99)}{1k(1.01)} = 9.82$$

The maximum in the upper direction is:

$$\text{Gain} = 1 + \frac{9k(1.01)}{1k(0.99)} = 10.18$$

Both of these values satisfy the 3% variation requirement.

*Students may have different amplifier structures (such as cascaded inverting amplifiers)
*1pt for drawing your amplifier structure labeled with values
*2pts for satisfying the gain relationship
*1pt for satisfying the gain variation requirement
*1pt for satisfying the input resistance variation requirement

2. Hambley, P14.39 **[10 pts]**
   a) Applying nodal analysis at the output node, and KVL around the outer loop: [5 pts]

$$\frac{V_s - V_o}{R_{in}} + \frac{A_{ol}V_1 - V_o}{R_o} = 0, V_1 = V_s - V_o$$

$$\frac{V_s}{R_{in}} + \frac{A_{ol}V_s}{R_o} = \frac{V_o}{R_{in}} + \frac{V_o}{R_o} + \frac{A_{ol}V_o}{R_o}$$

$$V_s\left(\frac{R_o + A_{ol}R_{in}}{R_{in}R_o}\right) = V_o\left(\frac{R_o + R_{in}(1 + A_{ol})}{R_{in}R_o}\right)$$

$$\frac{V_o}{V_s} = \left(\frac{R_o + R_{in}A_{ol}}{R_o + R_{in}(1 + A_{ol})}\right) = \left(\frac{25\Omega + 1M\Omega(10^5)}{25\Omega + 1M\Omega(1 + 10^5)}\right) = 0.99999 \approx 1$$

   b) By Ohm's Law: [2 pts]

$$i_s = \frac{V_s - V_o}{R_{in}} = \frac{V_s - V_s(A_{CL})}{R_{in}} = V_s\left(\frac{1 - A_{CL}}{R_{in}}\right),$$

where $A_{CL}$ is the closed loop gain computed in (a)

$$Z_{in} = \frac{V_s}{i_s} = \left(\frac{R_{in}}{1 - A_{CL}}\right) = \left(\frac{1M\Omega}{1 - 0.99999}\right) = 100G\Omega$$

This is similar to the "infinite input impedance" assumed for ideal op-amps.

   c) To compute output impedance, we short the input source, and apply a test voltage at the output. [3 pts]

$$i_{test} = \frac{V_{test}}{R_{in}} + \frac{V_{test} - A_{ol}V_1}{R_o} = \frac{V_{test}}{R_{in}} + \frac{V_{test} + A_{ol}V_{test}}{R_o} = V_{test}\left(\frac{R_o + R_{in}(1 + A_{ol})}{R_{in}R_o}\right)$$

since $V_{test} = -V_1$

$$Z_{out} = \frac{V_{test}}{i_{test}} = \left(\frac{R_{in}R_o}{R_o + R_{in}(1 + A_{ol})}\right) = \left(\frac{1M\Omega(25\Omega)}{25\Omega + 1M\Omega(1 + 10^5)}\right) = 2.5 \times 10^{-4}\,\Omega$$

This is consistent with the "0 output impedance" assumed for ideal op-amps.

3. Hambley, P14.52 **[5 pts]**
   a) Finding the bandwidth we can use the formula: [2 pts]
   $$f_{FP} = \frac{SR}{2\pi V_{om}} = \frac{1V/\mu s}{2\pi(10V)} = 15.9kHz$$
   b) Given f=5kHz and R=100Ω [1 pt]
   Checking current limitation:
   $100\Omega(25mA) = 2.5V$ → the output current limit supports at most 2.5V amp.
   Checking slew rate limitation:
   $$V_{\max} = \frac{SR}{2\pi f} = \frac{1V/\mu s}{2\pi(5kHz)} = 31.8V$$
   Thus the maximum output is limited by current and is 2.5V amplitude.
   c) Given f=5kHz and R=10kΩ [1 pt]
   Checking current limitation:
   $10k\Omega(25mA) = 250V$ → the output current limit allows for 10V amplitude.
   Thus the maximum output is limited by supply voltages and is 10V amp.
   d) Given f=100kHz and R=10kΩ [1 pt]
   Checking slew rate limitation:
   $V_{\max} = \frac{SR}{2\pi f} = \frac{1V/\mu s}{2\pi(100kHz)} = 1.6V$ → the slew rate limit allows for 1.6V amp.
   Thus the maximum output is limited by slew rate and is 1.6V amplitude.

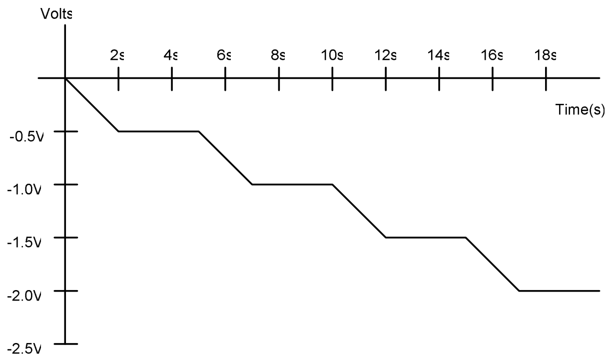
4. Hambley, P14.63 **[5 pts]**

   If the input is 5V, then by Ohm's Law $i = \frac{5V}{10k\Omega} = 0.5mA$

   Then, with each pulse we have by Q = CV that the voltage drops by

   $V = \frac{Q}{C} = \frac{(-0.5mA)(2ms)}{2\mu F} = -0.5V$

   Thus, -10V output indicates 20 pulses have been counted. [2 pts] [3 pts-graph]

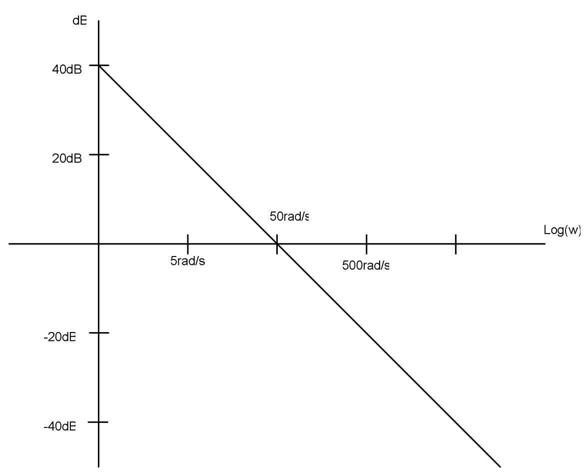



5. Hambley, P14.67 **[5 pts]**

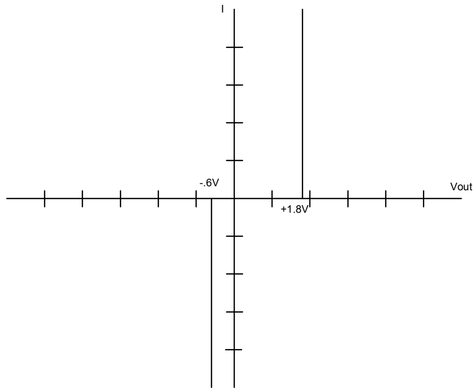
   We can use complex impedances along with the summing point constraint to determine the transfer function. Note that the structure of the circuit is the basic inverting amplifier, so we can reuse previously derived equations.

   $\frac{V_{out}}{V_{in}} = -\frac{Z_C}{Z_R} = -\frac{1}{j\omega RC} = -\frac{1}{j\omega(10k\Omega)(2\mu F)} = -\frac{1}{j\omega(0.02s)}$ [2 pts]

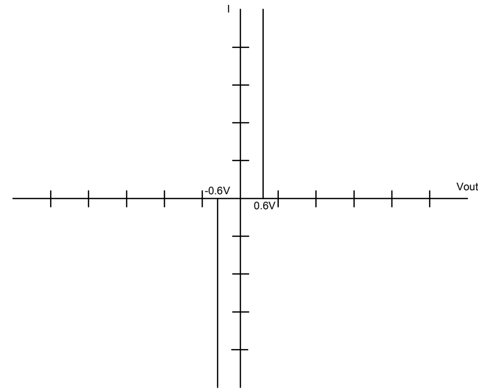
   This has the following magnitude Bode plot: [3 pts]



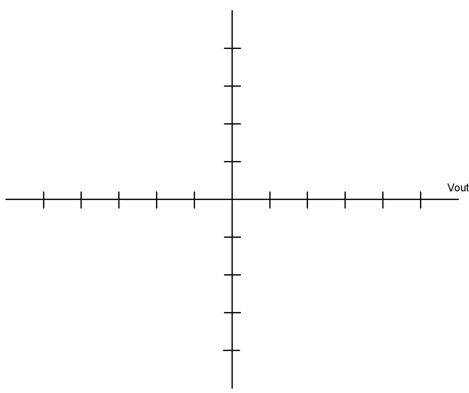
6. Hambley, P10.7 [10 pts – 2.5pts each]
   Although these diodes are “standard small signal diodes” we make a large signal approximation for ease of graphing.



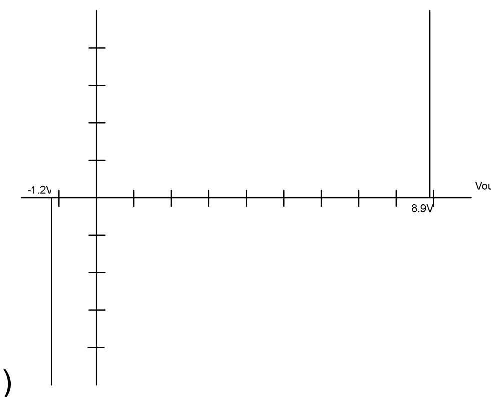

a)




b)

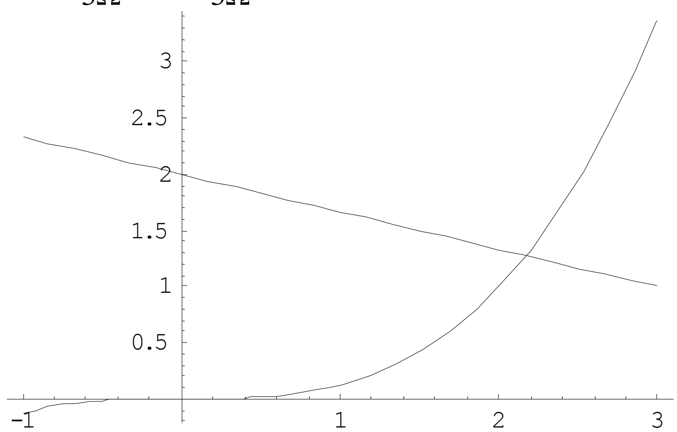



c)




d)

7. Hambley, P10.16 **[5 pts]**
   To determine the intersection point, we need an expression for I in terms of $V_s$ for the linear circuit.
   $i = \frac{6V - V_s}{3\Omega} = -\frac{1}{3\Omega}V_s + 2A$ [2 pts]




[1 pt graph]

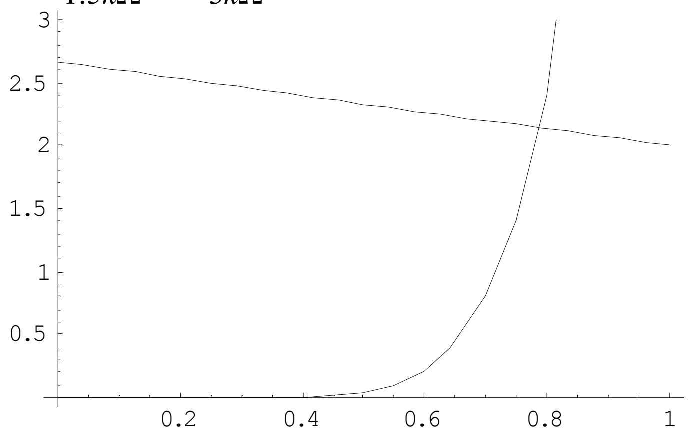
**Figure 1: Plot of current (A) vs. voltage (V)**

At the intersection point, $\boxed{i \approx 1.3A, V \approx 2.2V}$ [1 pt each for V & I]

8. Hambley, P10.20 **[15 pts – 5 pts each]**

a) Constructing an expression relating I to V for the linear circuit, we obtain

$$i = \frac{4V - V}{1.5k\Omega} = -\frac{2}{3k\Omega}V + 2.67mA$$ [2 pts]




[1 pt graph]

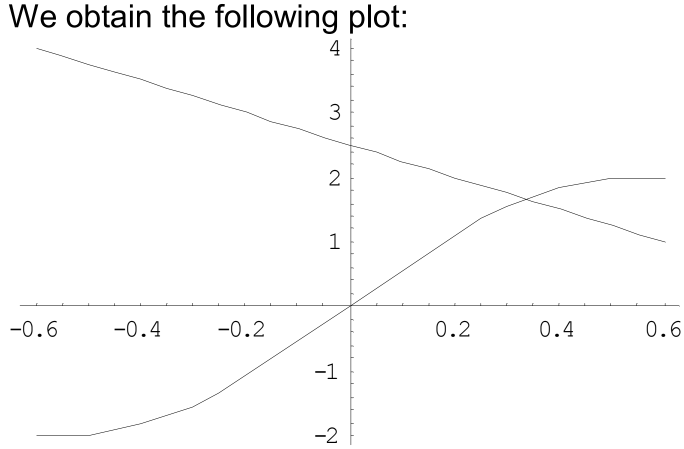
**Figure 2: Plot of i(mA) versus v(volts) for 10.20a)**

From this plot, $i \approx 2.2mA, v \approx 0.78V$ [2 pts]

b) Determining the Thevenin equivalent, by inspection the short circuit current is 2.5mA. In addition, the Thevenin resistance is 400Ω.

A representative function of I vs. V is: $i = -\frac{1}{400\Omega}V_x + 2.5mA$ [2 pts]

We obtain the following plot:

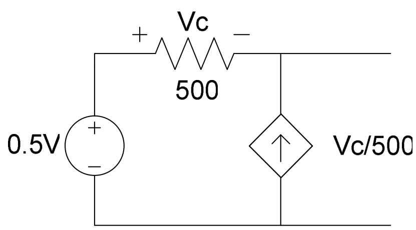



[1 pt graph]

**Figure 3: Plot of i(mA) vs v(volts) for 10.20b)**

From the plot, we obtain $i \approx 1.7mA\ , V_b \approx (5mA - 1.6mA)200\Omega = 0.66V$ [2 pts]

c) Finding the Thevenin equivalent for the linear circuit,
Computing $V_{Th}$ [1 pt]




$$V_{Th} = 0.5V - V_c = 0.5V + \left(\frac{V_c}{500\Omega}\right)500\Omega = 0.5V + V_c \rightarrow$$

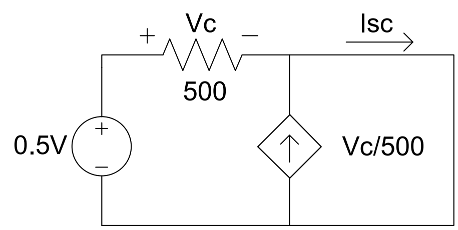
$$V_c = 0V, V_{Th} = 0.5V$$

The first equality comes from KVL, the second from Ohm's Law

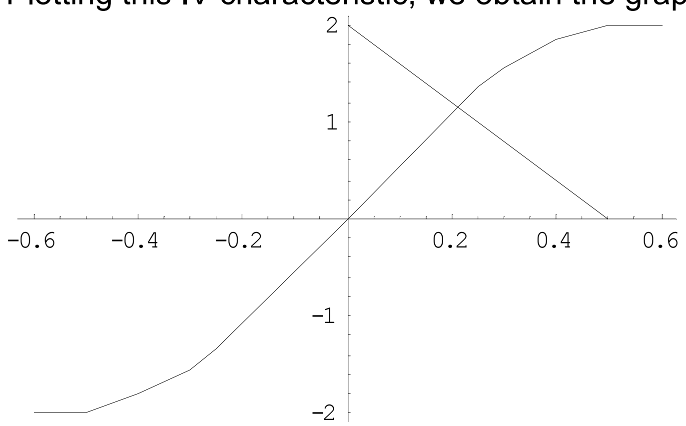
Computing $I_{SC}$, [1 pt]

$$i_{sc} = \frac{0.5V}{500\Omega} + \frac{0.5V}{500\Omega} = 2mA,$$

since $V_C$=0.5V with the short circuit in place.




Plotting this IV characteristic, we obtain the graph:




[1 pt graph]

**Figure 4: Plot of i(mA) vs v(volts) for 10.20c)**

By inspection, $\boxed{i \approx 1.2mA}$, and $V_x \approx 0.2V$, giving $\boxed{V_c \approx 0.5V - 0.2V = 0.3V}$.
[2 pts]

9. Hambley, P10.21 **[10 pts]**
   a) When the diode and X are in series, for a constant current the resultant voltage is the sum of voltages (add the two graphs horizontally).

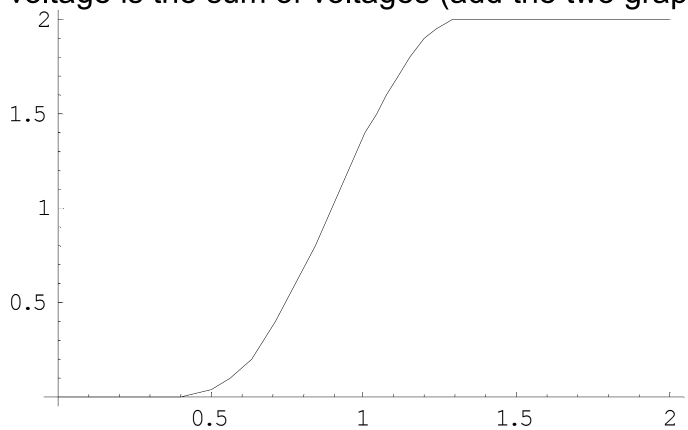



**Figure 5: Plot of i(mA) vs V(volts) for 10.21a)**

   b) When the diode and X are in parallel, for a constant voltage the resultant current is the sum of currents (add the two graphs vertically).

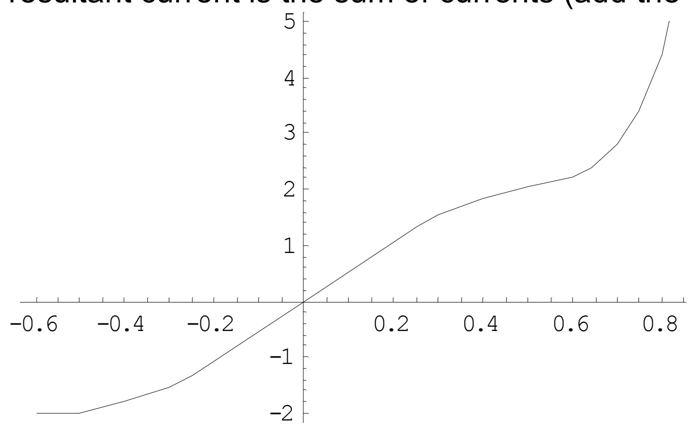



**Figure 6: Plot of i(mA) vs V(volts) for 10.21b)**

10. Hambley, P10.33 **[10 pts]**

a) Assume D1 on and D2 off. [3 pts]
Then, the voltage at the (+) end of D2 is given by voltage divider as

$$15V\left(\frac{5\Omega}{15\Omega}\right) = 5V$$

The voltage at the (-) end of D2 is given by voltage divider as

$$15V\left(\frac{10\Omega}{15\Omega}\right) = 10V$$

This is consistent with D2 being off. In this case, V=10V, I=0A

b) Assume D1 is on, D2 is off. [3 pts]
Then V=6V, which is consistent with D2 off. I=6V/1k=6mA.

c) Assume D1 is on, D2 is on. [4 pts]
The (+) side of V is 15V, the (-) side is -15V, so V=30V.
I is given as the sum of 2 currents.

$$I = \frac{30V}{2.2k\Omega} + \frac{30V}{1.5k\Omega} = 33.6mA.$$

11. Hambley, P10.34a. **[5 pts]**

Assume D1 is on, D2 is on and D3 is off.
At the first and second junctions, we have V=0V. We thus get 15mA flowing out through D1, and no current in the second resistor, hence I=0mA.
Current also flows into the circuit through D2 in the direction of the -15V source.
This 15V is divided between 2 equal resistors, so V=-7.5V.
Note this is consistent with D3 being off as its (+) is -7.5V and its (-) is 0V.
→ I=0mA, V=-7.5V

**12. [15 Points]**

a) Suppose the capacitor is initially charged as -4.5V → this puts the (-) terminal at 9V. Because this is the highest voltage in the system we can assume it drives the output to 0. (We assume that $V_{(-)}>V_{(+)}$).

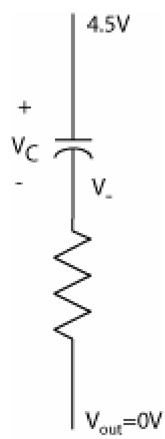
By a simple voltage division $V_+$=(0V+4.5)/2=2.25V<9V, consistent with the open-loop behavior of the op-amp.

**[2 points]**

b) Using the formula for $V_+$ from before, $V_+$=2.25V throughout this time interval.

**[1 point]**

To find $V_-(t)$, we note that, since no current flows into the opamp's (–) terminal, and since $V_{out}$ and the 4.5V are fixed voltages, we can redraw the relevant part of the circuit as:




We can see now that V- = 4.5V-Vc. Now we just need Vc.

Note that Vc is initially -4.5 (from part a), and that in the steady state (when the capacitor acts like an open circuit), Vc=+4.5V. Thus, we can express Vc using the generic series RC charging expression:

$V_C = V_{final} + (V_{initial} - V_{final})e^{-t/RC}$.

Plugging in the given values, and using $V_{initial}$=-4.5V, $V_{final}$=+4.5V, we get:

$V_C = 4.5V - 9Ve^{-t/(1ms)}$ and therefore:

$V_-(t) = 9Ve^{-t/(1ms)}$ **[3 points]**

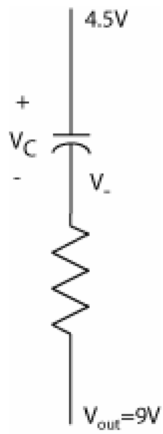
c) Just solve the equation from part b for V-=V+=2.25V, to get t=1.386ms.

**[1 point]**

d) Now, for this time interval, Vout has switched to 9V, so we have V+=6.75V

**[1 point]**

And:




Once again, V- = 4.5V-Vc.

Note that Vc is initially +2.25V (from part b), and that in the steady state (when the capacitor acts like an open circuit), Vc=-4.5V. Thus, we can express Vc using the generic series RC charging expression:

$V_C = V_{final} + (V_{initial} - V_{final})e^{-\tau/RC}$.

Plugging in the given values, and using $V_{initial}$=+2.25V, $V_{final}$=-4.5V, we get:

$V_C = -4.5V + 6.75e^{-\tau/(1ms)}$ and therefore:

$V_- = 9V - 6.75Ve^{-\tau/(1ms)}$

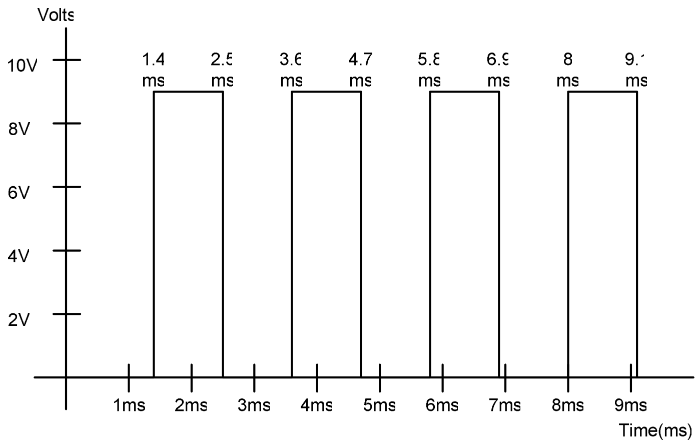
In these equations, we use τ = t - t1 to express the offset from t=0.

**[3 points]**

e) Just solve the equation from part d for V-=V+=6.75, to get τ =1.1ms. Add this τ to t1 to get t2=2.5ms.

**[1 point]**

f) Note that for all subsequent periods after t1, the charging/discharging is symmetric (because V+ goes from +2.25 to -2.25), so after the first the signal has a regular switching interval of 1.1ms: **[2 Points]**




g) The circuit is an oscillator, with period = 2.2ms. **[1 Point]**