EE 40, Spring 2007
Final Exam
Prof. Chang-Hasnain
May 17, 2007
Total Time Allotted: 180 minutes
Total Points: 200

1. This is a closed book exam. However, you are allowed to bring three pages (8.5" x 11") single-sided notes.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam.
4. **Remember to put down units.** Points will be taken off for answers without units.

Last (Family) Name: Perfect
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First Name: Peter
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Student ID: 31415927___________ Signature: PP____________

**MAKE SURE YOU ARE IN EE 40!!!**

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1. (50 pts) **MOS Circuit:**

The transistor is an NMOS with $K = 1\text{mA/V}^2$ and $V_{th} = 1\text{V}$. Let $V_{DD} = 10\text{V}$, $R_1 = 700\text{k}$, $R_2 = 300\text{k}$, $R = 70\text{k}$, $R_S = 1\text{k}$, $R_L = 4\text{k}$. Assume the capacitors and inductors are large.

(a) (10 pts) Use DC analysis. **Draw a load-line** on the following $I_{DS}-V_{DS}$ characteristic. (Hint: How do capacitors behave at DC? What about inductors?).

At DC: Capacitor $\rightarrow$ open
Inductor $\rightarrow$ short

The load-line only considers $R_S$:

$$V_{DSQ} = V_{DD} - I_{DSQ}R_S \Rightarrow$$

$$I_{DSQ} = \frac{V_{DD} - V_{DSQ}}{R_S} = \frac{10\text{V} - V_{DSQ}}{\frac{1}{1000}\text{\Omega}} = 1000\text{V} - 1000V_{DSQ}$$

(b) (6 pts) What are the values of $V_{GSO}$, $V_{DSQ}$, and $I_{DSQ}$?

At DC: Capacitor $\rightarrow$ open

By voltage divider, $V_{GSO} = V_G = \frac{R_2}{R_1 + R_2}V_{DD} = \frac{300\text{k\Omega}}{700\text{k\Omega} + 300\text{k\Omega}} \cdot 10\text{V} = 3\text{V}$

From the load line,

$V_{DSQ} = 6\text{V}$

$I_{DSQ} = 4\text{mA}$
(b) (8 pts) What are $g_m$ and $r_d$? (Remember to put down units!)

$$
g_m = \frac{\partial i_D}{\partial v_{GS}} = 2K(V_{GSQ} - V_t)
= 2 \cdot \left(\frac{1mA}{V^2}\right)(3V - 1V) = 4mA/V = 4S
$$
or:

$$
g_m = 2\sqrt{KI_{DSQ}}
= 2 \cdot \sqrt{\left(\frac{1mA}{V^2}\right)4mA} = 4S
$$

$$
r_d = \infty \quad \text{(Slope of ID vs. VDS is 0 at Q-point)}
$$

$$
\frac{1}{r_d} = \frac{\partial i_D}{\partial v_{DS}} \bigg|_{Q \text{-point}} = 0
$$

(c) (8 pts) Draw the small signal model for the circuit. (Hint: How do capacitors/inductors behave at high frequencies? What is DC voltage in AC small signal model?)

AC: Capacitor → short, Inductor → open
$V_{DD}$ → AC ground

(d) (8 pts) Find the voltage gain $A_v$.

Since $R_d$ is infinite,

$$
v_{out} = g_m v_{gs} R_L
$$

$$
v_g = v_{gs} + v_{out}
= \frac{v_{out}}{g_m R_L} + v_{out} = v_{out} \left(1 + \frac{1}{g_m R_L}\right)
$$

$$
\frac{v_{out}}{v_g} = \left(\frac{g_m R_L}{1 + g_m R_L}\right)
$$
\[ v_{out} = g_m v_{gs} R_L \]

\[ v_g = \frac{R_1 \parallel R_2}{R + R_1 \parallel R_2} v_{in} \]

\[
= \frac{70k\Omega / 300k\Omega}{70k\Omega + 700k\Omega / 300k\Omega} v_{in} = \frac{210k\Omega}{70k\Omega + 210k\Omega} v_{in} = \frac{3}{4} v_{in}
\]

\[ A_v = \frac{v_{out}}{v_{in}} = \frac{v_g}{v_{in}} = \frac{3}{4} \left( \frac{g_m R_L}{1 + g_m R_L} \right) = \frac{3}{4} \left( \frac{16}{17} \right) = \frac{12}{17} \]

(e) (5 pts) Find \( R_{in} \), the input resistance.

\[ R_{in} = R + R_1 \parallel R_2 \]

\[
= 70k\Omega + 700k\Omega / 300k\Omega = 70k\Omega + 210k\Omega = 280k\Omega
\]

(f) (5 pts) Find \( R_{out} \), the output resistance.

Zero the source \( v_{in} \). The resistance RL sees, by looking into the source is found by:

Applying test voltage, \( v_t \), at \( v_{out} : v_{gs} = -v_t \)

KCL:

\[ g_m v_{gs} + i_t = 0 \]

\[ i_t = g_m v_t \]

\[ R_{out} = \frac{v_t}{i_t} = \frac{1}{g_m} = \left( 4mS \right)^{-1} = 250\Omega \]
2. (16 pts) **CMOS Logic**

(a) (8 pts) Fill in the truth table and express the function of this logic gate in sum-of-products form.

(b) (8 pts) Fill in the PMOS pull-up network to complete this CMOS logic gate.

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HI input: NMOS on, PMOS off
LO input: NMOS off, PMOS on

Out = (A+BC)’ = A’B’ + A’C’

Truth-table:
3. (20 pts) **Diode circuit:** Use ideal piece-wise diode model with a threshold voltage of 0.6 volts.

![Diode Circuit Diagram]

a) (5 pts) Determine all the $V_{in}$ range(s) for which there is current through the circuit.

$$V_{in} > 0.6 \times 3 = 1.8V.$$

b) (5 pts) What is the value of $V_{out}$ when $V_{in} = 5 V$?

$$V_{out} = \frac{(V_{in} - 1.8)}{2} = 1.6V$$

c) (5 pts) What is the value of $V_{out}$ when $V_{in} = -5 V$?

$$V_{out} = 0$$
d) (5 pts) Draw the $V_{\text{out}}$ on the same graph for $V_{\text{in}} = 5 \cos(2\pi t)$
4. (26 pts) **Op-amp circuits** (Hint: this is a negative feedback case.)

(a) (10 pts) Derive one equation relating \( V_a \) and \( V_c \) to \( V_{in1} \) (Hint: analyze the circuit around op-amp 1).

\[
\frac{-V_{in}}{R} = \frac{V_a}{R} + \frac{V_c}{R}
\]

(b) (10 pts) Derive **two equations** relating \( V_a \), \( V_b \) and \( V_c \) to \( V_{in2} \) (Hint: analyze the circuit around op-amp 2).

\[
\frac{V_c - V_b}{R} = \frac{V_b - V_{in2}}{2R} + \frac{V_b}{R}
\]

\[
\frac{V_b - V_{in2}}{2R} = \frac{V_b - V_a}{R}
\]

(c) (6 pts) Solve \( V_a \), \( V_b \) and \( V_c \) when \( V_{in1} = V_{in2} = 3V \).

- \( 3 = V_a + V_c \) (1pt)
- \( 2V_c - 2V_b = V_b - V_{in2} + 2V_b \Rightarrow 2V_c - 5V_b = -3 \)
- \( V_b - V_{in2} = 2V_b - 2V_a \Rightarrow -3 = V_b - 2V_a \)
- \( V_a = 1V \)
- \( V_b = -1V \)
- \( V_c = -4V \)
5. (50 pts) **Bode Plot Problem**

(a) (i) (10 pts) Show that the transfer function of this circuit $H_1$ is of the form

$$H_1 = \frac{V_{o1}}{V_{in1}} = \frac{j \omega}{1 + j \omega \omega_b}$$

(b) (50 pts total)

(ii) (10 pt) Express $\omega_1$ and $\omega_b$ in terms of $R_1$, $L$ and $R_L$

$$\omega_1 = \frac{R_1}{L}$$

$$\omega_b = \frac{R_1 R_L}{L(R_1 + R_L)}$$

(iii) (10 pts) Given that $R_1 = 10\Omega$, $L = 10\text{mH}$, $R_L = 10k\Omega$. Sketch the Bode magnitude (in dB) plot. **Label the axes, all important points and indicate the slopes**.

(Hint: Make necessary approximation!) break frequency at: 1000rad/s. slope = -20dB/s
(b) (20 pts) Consider the circuit below. \( R_2 = 10 \text{k}\Omega, C = 10 \text{nF} \).

(i) (5 pts) What is the transfer function \( H_2 = V_{o2}/V_{in2} \)?

\[
H = -\frac{1}{j\omega CR_2}
\]

(ii) (10 pts) Sketch the Bode magnitude plot. **Label the axes, all important points and indicate the slopes.**

(iii) (5 pts) What is the input resistance of this circuit?

Input resistance is \( V_{in2}/i_{in2} = R_2 \)
6. (14 pts) **Equivalent Circuit.** Consider the following circuit.

![Circuit Diagram]

(a) (6 pts) Draw the Thévenin equivalent circuit for terminals a-b. What are the Thévenin voltage and resistance?

\[ R_{th} = 11 \text{Ohms} \]
Assume bottom node is ground,
\[ V_{th} = V_a - V_b = 20V - 7.5V = 12.5V \]

(b) (4 pts) Draw the Norton equivalent circuit for terminals a-b. What are the Norton current source and resistance?

\[ R_{th} \text{ is still 11 Ohms} \]
\[ I_{th} = 12.5/11 \text{ A} \]
(c) (4 pts) if we insert an additional resistor $R_L = 2 \, \Omega$ between terminals a and b, calculate the current through $R_L$?

Using the Thevenin equivalent, current through $R_L$ would be $12.5/(11+2) = 12.5/13 \, A$
7. (24 pts) **Semiconductor Physics**

a) (6 pts) Consider a piece silicon doped with Boron at a concentration of 5x10^{17} \text{cm}^{-3}. What is the majority carrier, electron or hole? What are the majority carrier concentration and the minority carrier concentration (n_i for Silicon is 10^{10} \text{cm}^{-3})?

Boron: Group III (acceptor in Si)
Majority carrier: hole, due to high acceptor concentration

\[ p \equiv N_A = 5 \times 10^{17} \text{cm}^{-3} \]

By mass action law,

\[ n = n_i^2 N_A = \left( 10 \times 10^{10} \text{cm}^{-3} \right)^2 \frac{5 \times 10^{17} \text{cm}^{-3}}{200 \text{cm}^{-3}} = 200 \text{cm}^{-3} \]

b) (4 pts) Consider now a p-n junction. You are given the space charge diagram on the right. Express \( x_1 \) in terms of \( x_2, N_A \) and \( N_D \)

For charge conservation,

\[ -x_2 qN_A \frac{qN_D}{\varepsilon} = x_1 qN_D \frac{x_2 qN_D}{\varepsilon} \Rightarrow \]

\[ x_1 = x_2 \frac{N_A}{N_D} \]

c) (8 pts) Assuming \( \varepsilon \) is the permittivity of the material. **Derive** the electric field \( E(x) \) vs. distance \( x \). Given \( E(-\infty) = 0 \).

For \( x_2 < x < 0 \):

\[ E(x) = \int_{-\infty}^{x} \frac{\rho(x)}{\varepsilon} dx' = -\frac{qN_A}{\varepsilon} \left( x - x_2 \right) \]

For \( 0 < x < x_1 \):

\[ E(x) = \int_{-\infty}^{x} \frac{\rho(x)}{\varepsilon} dx' = \int_{0}^{x} \frac{\rho(x)}{\varepsilon} dx' + E(0) = \frac{qN_D}{\varepsilon} x + \frac{qN_A}{\varepsilon} x_2 \]
d) (4 pts) Plot $E(x)$ vs. $x$. **Label** all important points. (Hint: watch out for the sign of $E$ field.)

\[
E(0) = \frac{qN_x}{\varepsilon} x_2
\]

Where $x_2 < 0$

d) (2 pts) **Where** (what $x$ value) is the electric field strongest (largest in magnitude)?

At $x = 0$, $E(x)$ is most negative.