EE40 Spring 2008 Homework 1 Solution

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NOTE: EACH PROBLEM IS WORTH 10 POINTS

Problem 1 Solution (P 1.21)

(a) Since we have an active reference configuration, we compute the power as

$$p = -v_a \cdot i_a = 30\ W$$

Since this is positive, power is being absorbed by the element.

(b) Since we have an active reference configuration, we compute the power as

$$p = v_b \cdot i_b = 30\ W$$

Since this is positive, power is being absorbed by the element.

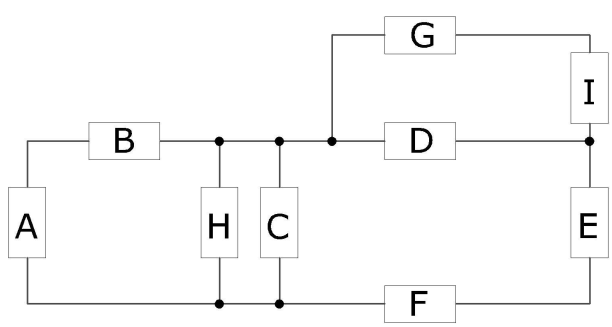
(c) We compute the power as

$$p = v_{DE} \cdot i_{DE} = -60\ W \text{ OR}$$
$$p = -v_{DE} \cdot i_{ED} = -60\ W$$

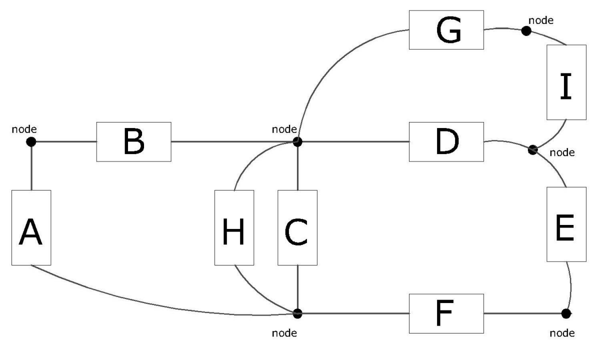
Since this is negative, power is being delivered by the element.

Problem 2 Solution

The figure from the problem is shown below



The solution is shown below



A node is a point at which two or more current elements are joined together. All points connected by ideal conductors are considered to be a single node. There are 6 nodes in this circuit as indicated in the figure above.

Problem 3 Solution

- Writing KCL at the node where B,D,E and G connect, we have

$$i_g + i_e = i_b + i_d$$

Hence

$$\begin{aligned} i_d &= i_g + i_e - i_b \\ &= 4 - 2 - 2 \\ &= 0\ A \end{aligned}$$

- Writing KCL at the node where F,G and H connect, we have

$$\begin{aligned} i_f &= i_g + i_h \\ &= 4 - 2 \\ &= 2\ A \end{aligned}$$

- Writing KCL at the node where A,D and F connect, we have

$$i_a + i_d = i_f$$

Hence

$$\begin{aligned} i_a &= i_f - i_d \\ &= 2 - 0 \\ &= 2\ A \end{aligned}$$

- Finally, we have

$$i_c + i_h = i_e$$

So

$$\begin{aligned} i_c &= i_e - i_h \\ &= -2 + 2 \\ &= 0\ A \end{aligned}$$

Problem 4 Solutions (P 1.41)

(a) A and B are in parallel
(b) By KVL, $v_a + v_b = 0\ V$
(c) By KVL, $v_b = -v_a = -2\ V$
Also by KVL in the loop defined by B,C and

$$v_b + v_d + v_c = 0$$

Hence, $v_c = -v_b - v_d = 2 + 5 = 7\ V$

Problem 5 Solutions

- By KVL in the loop defined by A,D and B

$$v_b + v_a - v_d = 0$$

Hence, $v_b = v_d - v_a = 5 - 3 = 2\ V$

- By KVL in the loop defined by D,F and G

$$v_d + v_f - v_g = 0$$

Hence, $v_g = v_d + v_f = 5 - 1 = 4\ V$

- By KVL in the loop defined by E,C,A and D

$$v_e + v_c + v_a - v_d = 0$$

Hence, $v_e = v_d - v_c - v_a = 5 + 3 - 3 = 5\ V$

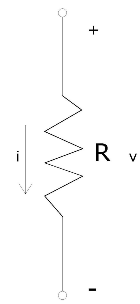
- Finally, by KVL in the loop defined by C,A,F and H

$$v_c + v_a + v_f + v_h = 0$$

Hence, $v_h = -v_c - v_a - v_f = 3 - 3 + 1 = 1\ V$

Problem 6 Solution

Let R denote the resistance, v the voltage across it and i the current through it in the passive reference configuration. Refer to the figure shown below.



We have $v = 50\ V$ and the power delivered is $P = 200\ W$. Hence, $i = \frac{200}{50} = 4\ A$. Therefore, $R = \frac{v}{i} = \frac{50}{4} = 12.5\ \Omega$.
If the voltage is increased to $v = 75\ V$, then the current increases to $i = \frac{75}{R} = \frac{75}{12.5} = 6\ A$. That is, the current increases by 50 %.
The power delivered increases to $75 * 6 = 450\ W$ (increases by 125 %).

Problem 7 Solution

The instantaneous power delivered to the resistance is:

$$p(t) = \frac{v^2(t)}{R} = \frac{(10\sin(4\pi t))^2}{10} = 10\sin^2(4\pi t)$$

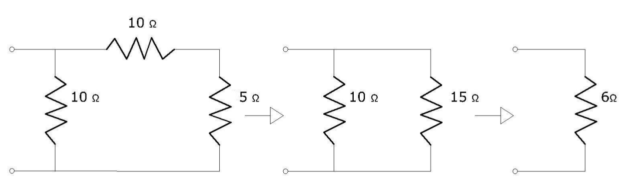
To find the total energy between $t = 0$ and $t = 10s$, we need to compute the integral as shown below:

$$\int_0^{10} 10\sin^2(4\pi t)\,dt$$

Now, $\int_0^1 \sin^2(4\pi t)\,dt = 5$, the energy is $50\ J$

Problem 8 Solution (P 1.65)

The total resistance across the terminals of the voltage source can be determined by a series-parallel combination because $\frac{1}{10} + \frac{1}{15} = \frac{25}{150} = \frac{1}{6}$. Refer to figure below.



The voltage across the terminals of the current source is therefore $6I_x\ V$ (assuming I_x is given in amps). By the voltage divider formula, given the voltage drop of $15\ V$ across the 5Ω resistor we have $15 = \frac{5}{5+10}6I_x = \frac{30I_x}{15} = 2I_x$.
Thus $I_x = 7.5A$

Problem 9 Solution (P 1.67)

First, we have $i_0 = \sqrt{\frac{P_0}{8}} = 1\ A$

Applying Ohm's law and KVL to the right hand loop, we have

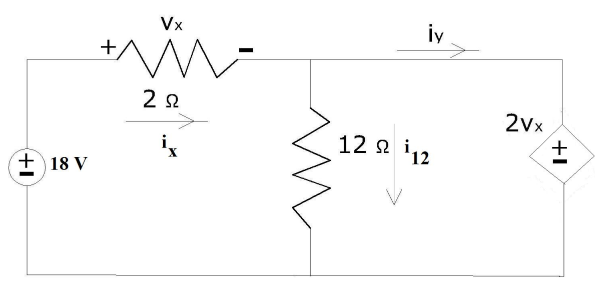
$$1000 v_{in} = 2 i_0 + 8 i_0$$

This means $v_{in} = 10\ mV$. Then, $i_{in} = \frac{v_{in}}{10^4} = 1\ \mu A$

Finally, we have $V_x = 5000 i_{in} + 10000 i_{in} = 15\ mV$.

Problem 10 Solution (P 1.71 + Power Calculation)

Refer to the figure below



Applying KVL around the periphery of the circuit, we have $-18 + v_x + 2v_x = 0$ which yields $v_x = 6\ V$. Then we have $v_{12} = 2v_x = 12\ V$. Using Ohm's law we obtain $i_{12} = \frac{v_{12}}{12} = 1\ A$ and $i_x = \frac{v_x}{2} = 3\ A$. Then KCL applied to the node at the top of the 12Ω resistor gives $i_x = i_{12} + i_y$ which yields $i_y = 2\ A$.

With the sign conventions as shown in the figure, the power delivered to the 18 V source is:

$$-(18)(i_y + \frac{v_x}{6}) = -18 \cdot 3 = -54\ W$$

The power delivered to the 2Ω resistance is

$$v_x(i_y + \frac{v_x}{6}) = 6 \cdot 3 = 18\ W$$

The power delivered to the 12Ω resistance is

$$2v_x \frac{v_x}{6} = 12\ W$$

The power delivered to the controlled voltage source is

$$2v_x i_y = 12 \cdot 2 = 24\ W$$

Note that $-54 + 18 + 12 + 24 = 0\ W$ as expected.