EE 40: Introduction to Microelectronic Circuits
Spring 2008: HW 10 Solutions
(due 05/12, 5 pm)

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May 12, 2008

Problem 12.5 Since $v_{GS} = 4 > 1 = v_{to}$, the n-channel MOSFET is not in the cutoff region. Thus, at most it must be in either the triode region or in the saturation region.

For $v_{GS} \geq v_{to}$, the condition on v_{DS} determining the boundary between the triode and the saturation regions is given by:

$$v_{GS} - v_{DS}^* = v_{to}$$

This is equation 12.5 of the text. In this example, the equation reduces to:

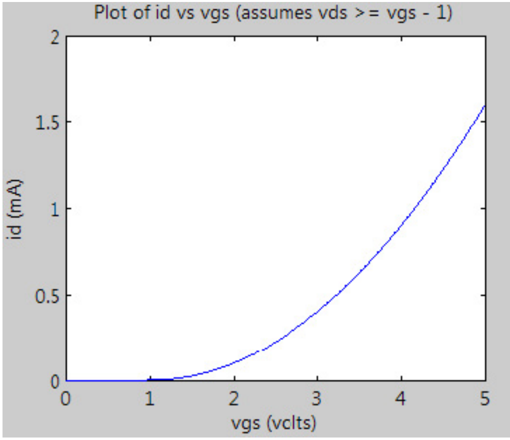
$$v_{DS}^* = v_{GS} - v_{to} = 3\ V$$

Thus if $v_{DS} < 3\ V$, the MOSFET is in the triode region, while if $v_{DS} > 3\ V$, it is in the saturation region.

In the saturation regions, the drain current i_D does not depend on v_{DS} (as long as it is big enough to place the MOSFET in the saturation region) and is given in terms of v_{GS} by:

$$i_D = K(v_{GS} - v_{to})^2$$

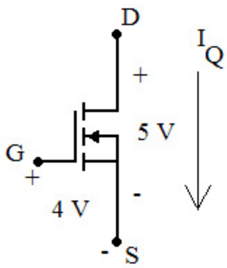
This is equation 12.4 of the text. In this example, this reduces to $i_D = 10^{-1}(v_{GS} - 1)^2\ mA$, plotted as:



Problem 12.8

Part (a)

This is an n-channel MOSFET. The body has been tied to the source. From the figure, we have $v_{GS} = 4\ V$ and $v_{DS} = 5\ V$. Also, the parameters $|V_{to}|$ and k are given. Since this is an n-channel MOSFET we know that $V_{to} > 0$.



Here, $v_{to} = 1V$. Since $v_{GS} = 4 > 1 = v_{to}$, the MOSFET is not in the cutoff region. The boundary between triode and saturation regions is defined by:

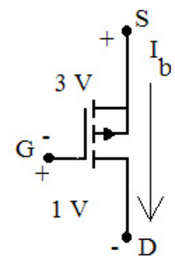
$$v_{GS} - v_{DS}^* = v_{to}$$

i.e..by $v_{DS}^* = v_{GS} - v_{to} = 3V$. Since $v_{DS} = 5 > 3 = v_{DS}^*$ the MOSFET is in the saturation regions. The drain current (here denoted by I_Q) is then given by:

$$\begin{aligned} I_Q &= K(v_{GS} - v_{to})^2 \\ &= 0.1(4-1)^2 \\ &= 0.9\ mA. \end{aligned}$$

Part (b)

This is a p-channel MOSFET (which explains why the terminals are labelled as shown, given that the total drop from the top terminal to the bottom terminal is positive). The body has been tied to the source. Here, we have: $v_{GS} = -3V$



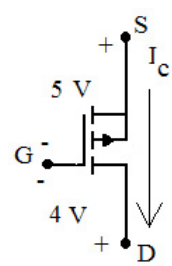
and $v_{DS} = -4V$. Also, we have $v_{to} < 0$ because this is a p-channel MOSFET. Since $v_{GS} = -3 < -1 = v_{to}$, the MOSFET is not in the cutoff region. THe boundary between the triode and saturation regions is given by:

$$v_{GS} - v_{DS}^{*} = v_{to}$$

i.e..by $v_{DS}^{*} = -2\ V$. Since $v_{DS} = -4 < -2 = v_{DS}^{*}$, the MOSFET is in the saturation region. Thus the drain current (here denoted by I_b) is given by:

$$\begin{aligned} I_b &= k(v_{GS} - v_{to})^2 \\ &= 0.1(-2)^2 \\ &= 0.4\ mA \end{aligned}$$

Part (c) This is a p-channel MOSFET. The terminals have been labelled



as shown using the information and nothing that the voltage drop from the top to the bottom terminal is positive. The body has been tied to the source. Here, we have: $v_{GS} = -5\ V$ and $v_{DS} = -1\ V$. Also, we have $v_{to} < 0$. Since $v_{GS} = -5 < -1 = v_{to}$, the MOSFET is not in the cutoff region. The boundary between the triode and saturation regions is given by:

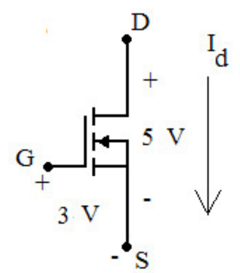
$$v_{DS}^{*} = v_{GS} - v_{to} = -4\ V$$

Since $v_{DS} = -1 > -4 = v_{DS}^{*}$, the MOSFET is in the triode region. The drain current (here denoted by I_c) is then given by:

$$I_c = K(2 * (v_{GS} - v_{to})v_{DS} - v_{DS}^2)$$

$$\begin{aligned} &= 0.1 * (2(-4)(-1) - (-1)^2) \\ &= 0.1 * 7 \\ &= 0.7\ mA \end{aligned}$$

Part (d) This is an n-channel MOSFET. The body has been tied to the



source. We have: $v_{GS} = 3\ V$ and $v_{DS} = 1\ V$. Also, we know that $v_{to} > 0$. Here we have $v_{GS} = 3 > 1 = v_{to}$ so the MOSFET is not in the cutoff region. The boundary between the triode and saturation is given by

$$v_{DS}^* = v_{GS} - v_{to} = 2\ V$$

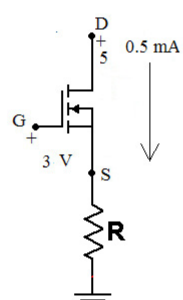
Since

$$v_{DS} = 1 < 2 = v_{DS}^*$$

the MOSFET is in the triode region. We then have the drain current (denoted by I_d) given by:

$$\begin{aligned} I_d &= K(2 * (v_{GS} - v_{to})v_{DS} - v_{DS}^2) \\ &= 0.1(2 * (2.1 - 1)) \\ &= 0.3\ mA \end{aligned}$$

Problem 12.14 This is an n-channel MOSFET. The body has been tied to the



source. The voltages at the gate and drain are referred to ground. Since the drain current is 0.5 mA, the same current goes from source to ground. So the voltage drop across the resistor is $0.5R$, where R is measured in $k\Omega$. From the figure, we then conclude that: $v_{GS} = 3 - 0.5R$ volts and $v_{DS} = 5 - 0.5R$ volts. Since the drain current is strictly positive, the MOSFET is not in the cutoff region. This gives the equation:

$$3 - 0.5R = v_{GS} \geq v_{to} = 1$$

i.e..$3 - 1 \geq 0.5R$. Thus, $R \leq 4k\Omega$. The boundary between the triode and the saturation regions is given by:

$$\begin{aligned} v_{DS}^* &= v_{GS} - v_{to} \\ &= (3 - 0.5R) - 1 \\ &= 2 - 0.5R \end{aligned}$$

Since $v_{DS} = 5 - 0.5R > 2 - 0.5R = v_{DS}^*$, we can conclude that the MOSFET is in the saturation region. You should be somewhat careful here and observe that we also have $v_{DS} \geq 0$, so the drain and source have indeed been properly labelled (actually this is already obvious given the direction of the drain current). The drain current equation in the saturation region can be brought into play. This gives:

$$\begin{aligned} 0.5 - i_D &= K(v_{GS} - v_{to})^2 \\ &= 0.5((3 - 0.5R) - 1)^2 \\ &= 0.5(2 - 0.5R)^2 \end{aligned}$$

From this we conclude that:

$$2 - 0.5R = 1 \; or \; 2 - 0.5R = -1$$

That is,

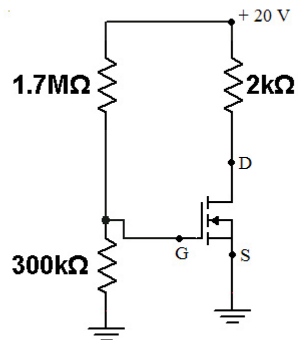
$$R = 2\ k\Omega \; or \; R = 6\ k\Omega$$

Since we already know that $R \leq 4\ k\Omega$, we can conclude that $R = 2\ k\Omega$.

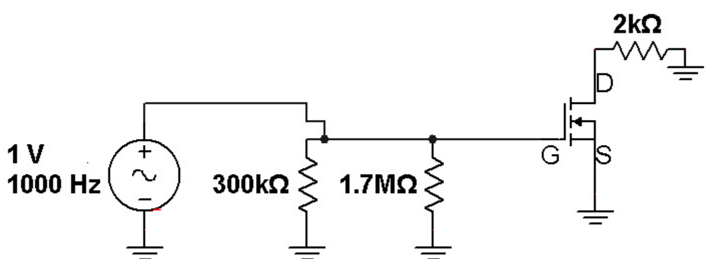
Problem 12.20

Part (a)

The DC equivalent circuit is given below. By the voltage divider principle,



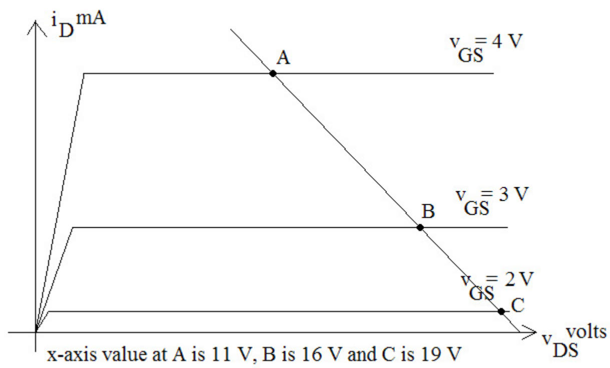
the DC component of v_{GS} is: $\frac{300}{2000} \cdot 20 = 3\ V$. The AC equivalent circuit is shown below. Since no current is drawn at the gate irrespective of what regime



the transistor is operating in, the AC component of v_{GS} is $sin(2000\pi t)$. We conclude that:

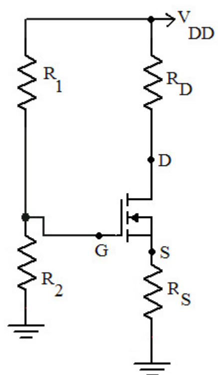
$$v_{GS}(t) = 3 + sin(2000\pi t)\ volts$$

Part (b), (c) and (d)

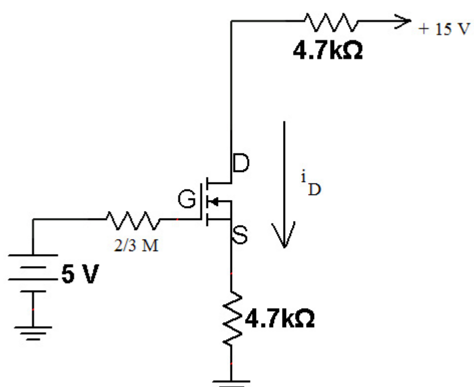


Problem 12.30

The parameters are: $V_{DD} = 15\ V, R_1 = 2\ M\Omega, R_2 = 1\ M\Omega, R_S = 4.7\ k\Omega, R_D =$



$4.7\ k\Omega, v_{to} = 1\ V, k = 0.25\ mA/V^2$. Replacing the gate bias circuit by its Thevenin equivalent, we get the figure shown below. The bias-line (see figure



12.14 of the text) is the constraint relating I_{DQ} to V_{GSQ} given by writing KVL around the gate-source loop and it is:

$$5 = V_{GSQ} + 4.7 * I_{DQ}$$

where V_{GSQ} in volts and I_{DQ} is in mA. This is because no current is drawn at the gate. The drain current in saturation is given in terms of V_{GSR} by:

$$\begin{aligned} I_{DQ} &= K(V_{GSQ} - V_{to})^2 \\ &= 0.25 * (V_{GSQ} - 1)^2 \end{aligned}$$

Plugging in for I_{DQ} in terms of V_{GSQ} from the bias line gives:

$$(5 - V_{GSQ}) = 1.175 * (V_{GSQ} - 1)^2$$

i.e..

$$\frac{47}{40}V_{GSQ}^2 + (1 - \frac{47}{20})V_{GSQ} + (\frac{47}{40} - 5) = 0$$

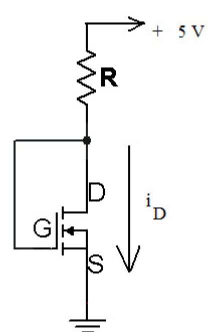
i.e..

$$47V_{GSQ}^2 - 54V_{GSQ} - 153 = 0$$

Solving the above equation, we get $V_{GSQ} = -1.3\ V$ and $V_{GSQ} = 2.5\ V$. We can obviously ignore the negative solution and we thus get $V_{GSQ} = 2.5\ V$. This gives I_{DQ} is approx. $0.56\ mA$. So V_{DQ} is approx. $10 > 2.5 - 1$ confirming that the MOSFET is in saturation.

Problem 12.35

The parameters for the first MOSFET are $v_{to} = 0.5\ V$, $k_p = 100\mu A/V^2$, $\frac{W_1}{L_1} = 1$.



Since no current is drawn at the gate of the second MOSFET irrespective of what regime it is operating in, it does not load the first MOSFET. Thus we can analyze each MOSFET in isolation. From the parameters given, we compute:

$$K = (\frac{W}{L})\frac{k_p}{2} = 50\mu A/V^2 = 0.05mA/V^2$$

If $i_{D1} = 0.2\ mA$, we would have:

$$v_{DS} = v_{GS} = 5 - 0.2R$$

where R is measured in $k\Omega$. Since the drain current is strictly positive, it must be the case that the MOSFET is not in cutoff. Therefore, we must have:

$$5 - 0.2R \geq 0.5$$

i.e..

$$R \leq 22.5\ k\Omega$$

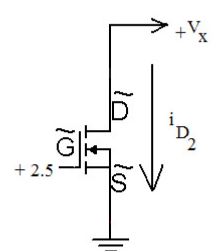
Since $v_{DS} = v_{GS} > v_{GS} - v_{to}$, the MOSFET would be in the saturation region, giving the equation:

$$\begin{aligned} i_{D1} &= K(v_{GS} - v_{to})^2 \\ 0.2 &= 0.05((5 - 0.2R) - 0.5)^2 \\ &= 0.05(4.5 - 0.2R)^2 \end{aligned}$$

This tells us that either $2 = 4.5 - 0.2R$ or $-2 = 4.5 - 0.2R$. That is, $R = 12.5\ k\Omega$ or $R = 32.5\ k\Omega$. Since we already know that $R \leq 22.5\ k\Omega$, it must be that $R = 12.5\ k\Omega$. This then gives:

$$v_{DS} = 5 - 0.2 * 12.5 = 2.5\ V$$

Turning now to the second MOSFET, we can draw the corresponding circuit as shown. Here we have labelled the terminals using tildes to distinguish them



from the terminals of the first MOSFET. We have also observed that the gate to source voltage of the second MOSFET equals the drain to source voltage of the first MOSFET,i.e..2.5 V. For the second MOSFET, we have $v_{to} = 0.5\ V$. Thus the second MOSFET will be in saturation precisely when $v_x \geq 2.5 - 0.5 = 2\ V$. For the second MOSFET, we have:

$$\tilde{K} = (\frac{W_2}{L_2})\frac{k_p}{2} = 0.1mA/V^2$$

This gives:

$$\begin{aligned} i_{D2} &= \tilde{K}(2.5 - 0.5)^2 \\ &= (0.1)4 \\ &= 0.4\ mA \end{aligned}$$

The second transistor appears to be "equivalent" to an ideal current source *if one assumes that it is operating in saturation.*

Problem 12.46

You should first of all realize that even the signal values given are necessarily approximate, since there would have to be some distortion in $i_D(t)$ in a real MOSFET (even given the equations we are working with) when $v_{GS}(t)$ has a purely sinusoidal AC component. The given data corresponds to:

$$v_{GSQ} = 1,\ v_{DSQ} = 4\ and\ I_{DQ} = 2$$

while

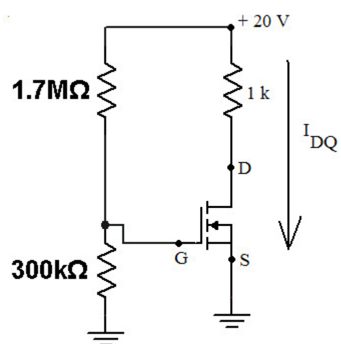
$$v_{gs}(t) = 0.2sin(\omega t),\ v_{ds}(t) = 0\ and\ i_d(t) = 0.1sin(\omega t)$$

Since there is no AC part in the drain to source voltage we cannot estimate r_d. However, we can estimate g_m as:

$$g_m = \frac{0.1}{0.2} = 0.5\ millimhos\ (sounds\ like\ Austin\ Powers)$$

Problem 12.51

The DC equivalent circuit which determines the quiescent point values is shown below. The parameters are $k_p = 0.05\ mA/V^2, W = 600\ \mu m,\ L = 20\ \mu m,\ v_{to} =$



$2\ V\ and\ r_d = \infty$. We have $V_{GSQ} = 3\ V$, from the resistive voltage divider formula. Since,

$$v_{GSQ} = 3 > 2 = v_{to}$$

the transistor is not in cutoff at the quiescent point. The load line to determine the quiescent point value is given by KVL:

$$20 = I_{DQ} * R + V_{DSQ}$$

We compute:

$$K = (\frac{W}{L})\frac{k_p}{2} = 30\frac{0.05}{2} = 0.75\ mA/V^2$$

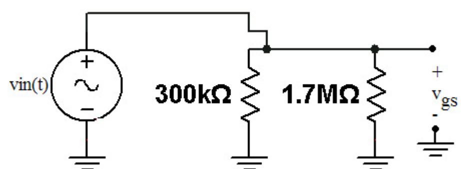
The boundary between the triode region and the saturation region occurs for $V_{GSQ} = 3$ at $V_{DS}^* = V_{GSQ} - v_{to} = 3 - 2 = 1\ V$. The corresponding current on the load line would be $I_D^* = 20 - V_{DS}^* = 19\ mA$. The saturation current at $V_{GSQ} = 3\ V$ is:

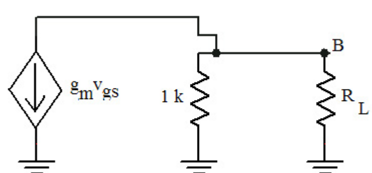
$$K * (V_{GSQ} - v_{to})^2 = 0.75 * 3.2^2 = 0.75\ mA$$

which is less thatn I_D^*. Thus the MOSFET is in saturation at the quiescent point and we have $I_{DQ} = 0.75\ mA$. Further, $V_{DSQ} = 20 - 0.75 = 19.25\ volts$. We may use equation (12.24) on page 590 of the text to write:

$$g_m = 2\sqrt{(KI_{DQ})} = 2\sqrt{(0.75^2)} = 1.5\ mV$$

Part (b)
The AC equivalent (small signal) circuit is: Since $R_L = 1\ k\Omega$ and $v_{gs}(t) =$

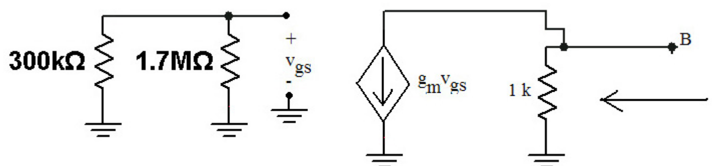




$v_{in}(t)$, the voltage gain is:

$$A_v = -g_m \frac{1}{2} = -0.75$$

The input resistance is found by looking into the circuit across the AC voltage source and ground when the load is replaced by an open circuit. Here this is the parallel combination of 1.7 $M\Omega$ and 300 $k\Omega$, which is 255 $k\Omega$. The output resistance is found by looking into the circuit across the load terminals as shown below (when the input voltage source is replaced by a short circuit). Here



$v_{gs} = 0$, so the controlled current source may be replaced by an open circuit. The output resistance is thus seen equal to 1 $k\Omega$.

Problem 12.55

From formula (12.42) for the voltage gain of the source follower, we see that it can never exceed 1 in absolute value whereas from formula (12.35) for the voltage gain of the common-source amplifier we see that it could, in principle, exceed 1. Thus we would need to choose a common-source amplifier if we desire a voltage gain greater than 1 in absolute value.

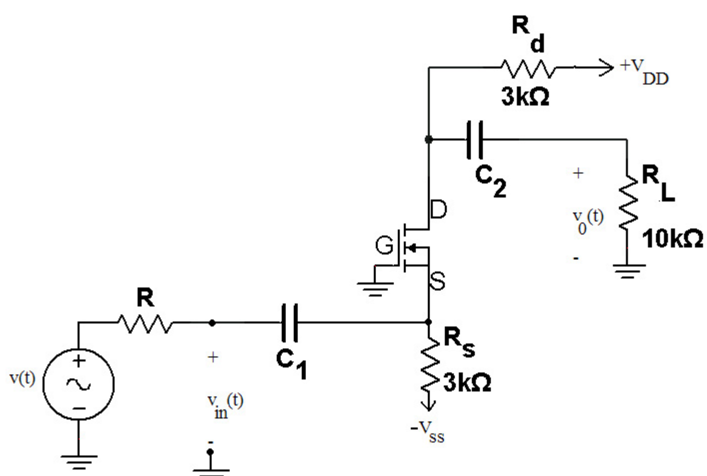
Consider formula (12.37) for the output resistance of the common-source amplifier. Note that r_d is typically very large (and often modelled by ∞).

Next consider formula (12.45) for the output resistance of the source follower. Note that if g_m is large, then this can be quite small.

This suggests that the source follower is a better configuration to choose for an amplifier if we desire a small output resistance.

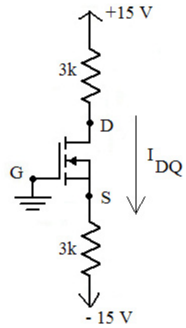
Problem 12.57

The parameters for the circuit are: $k_p = 0.05\ mA/V^2$, $W = 600\ \mu m$, $L =$



$10\ \mu m, v_{to} = 1\ V,\ r_d = \infty,\ V_{DD} = 15\ V,\ V_{SS} = 15\ V$. We first compute $K = (\frac{W}{L})\frac{k_p}{2} = 1.5\ mA/V^2$.

Part (a) To find the Q-point, consider the DC equivalent circuit. If the MOS-



FET in this DC equivalent circuit were in the cutoff region, we would have $I_{DQ} = 0$. This would imply that $V_{GSQ} = 15\ V$. But this exceeds v_{to}, contradicting our assumption. Thus we must have $I_{DQ} > 0$ and the MOSFET is not in cutoff. We have:

$$V_{GSQ} = 15 - 3I_{DQ}$$

where I_{DQ} is measured in mA and V_{GSQ} is measured in volts. Also,

$$V_{DSQ} = 30 - 6I_{DQ}$$

Bot these equations result from KVL. Since $V_{DSQ} = 2V_{GSQ} \geq V_{GSQ} - v_{to}$, where the second equation comes from $V_{GSQ} \geq v_{to}$, we know that the MOSFET is in the saturation region at the Q-point. This gives the equation:

$$\begin{aligned} I_{DQ} &= K(V_{GSQ} - v_{to})^2 \\ &= 1.5((15 - 3I_{DQ}) - 1)^2 \\ &= 1.5(14 - 3I_{DQ})^2. \end{aligned}$$

Solving: $I_{DQ} = 4.114\ mA,\ V_{DSQ} = 5.316\ V,\ g_m = 4.968\ mS,\ R_L = 2.308\ k\Omega,\ A_v = 11.465\ R_{in} = 188.6\ \Omega$