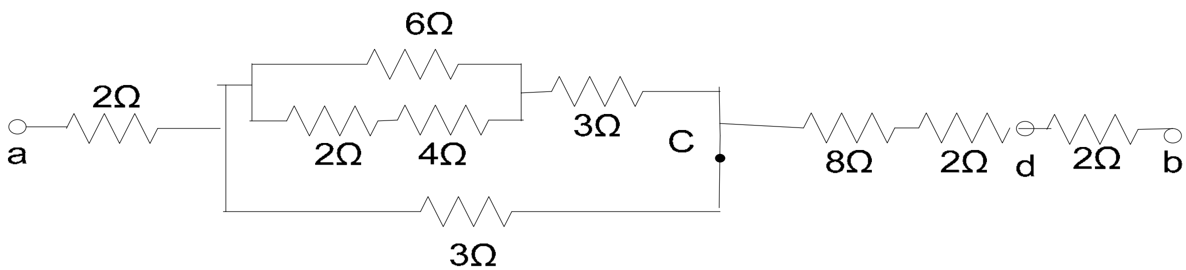
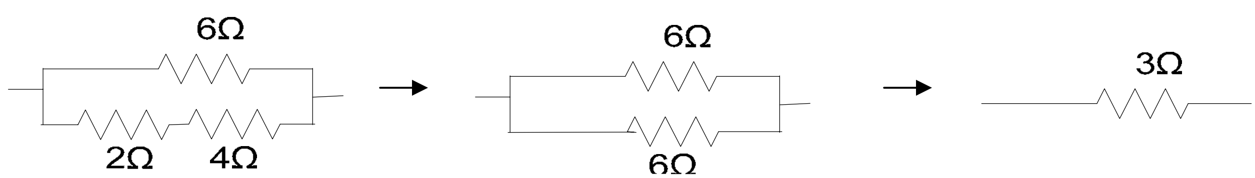
EE40 Spring 2008 Homework 2 Solutions

1) a)

The circuit can be redrawn as



And then the portion:



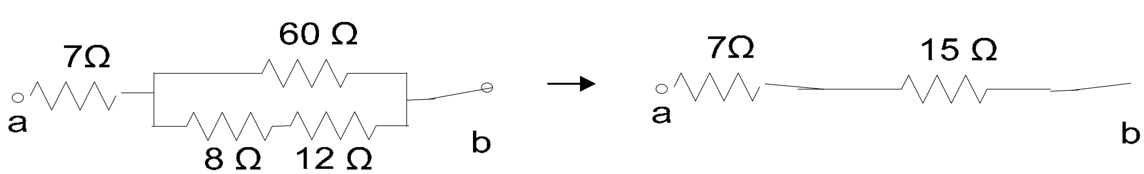
can be replaced by a (2+4)//6 = 6//6 = 3Ω resistor as shown in the drawing. Then we get, by similar reasoning (3+3)//3 = 6//3 = 2Ω for the larger parallel block.
Therefore the total resistance is R_{eq}= 2+2+8+2+2 = 16 Ω.

b)

Shorting c to d means no voltage drop between those two points, so no current on the 8 or 2 Ω resistors, and we can just ignore them. Hence the total resistance is R_{eq}= 2+2+0+2 = 6 Ω.

2) Hambley P 2.17:

Combining the 20Ω resistor with the 30Ω gives 20*30/50 = 60/5=12 Ω,



then we note as in the drawing above that 60 // (8+12) = 60 // 20 = 120/8=15, for a total resistance of 7+15=22 Ω.

3) Hambley P 2.24

Label the blacked nodes in the text by d,a,b,c going counterclockwise from top left.
The total current flowing into and out of the parallel combination of the 6 Ω and 12 Ω resistors on the right is 2A. By the current divider formula, $i_2 = 2*(1/12) /(1/12 + 1/6) = 2/3A$.
The total current flowing into and out of the parallel combination of the 12 Ω and 24 Ω resistors on the left is 2A. By the current divider formula, $i_1 = \frac{1/12}{1/12+1/24} 2 = \frac{4}{3} A$.
Thus

$$v_a - v_b = 12*i_2 = 8V$$
$$v_b - v_c = 8*2 = 16V$$
$$v_c - v_d = 12*i_1 = 16V$$

By KVL, $v = v_a - v_d = 8+16+16 = 40V$

4) Hambley P 2.32

With the switch open, by the voltage division formula $v_2 = 10*R_2 / (R_2 + 6)$ V.
Since $v_2 = 5V$, we have $5(R_2 + 6) = 10R_2$
so $R_2 = 6\Omega$.

With the switch closed, by the voltage division formula,
$v2 = (R_2//R_L) / (R_2//R_L + 6) *10$ V.
Since $v_2 = 4V$, we have
$4*(R_2//R_L + 6) = 10* R_2//R_L$
so $R_2//R_L = 4\ \Omega$.
We use that $1/ (R_2 // R_L) = 1/R_L + 1/R_2$ to find that $1/R_L = 1/ (R_2 // R_L) – 1/R_2 = ¼ - 1/6 = 1/12\ \Omega$,
so $R_L = 12\ \Omega$.

5) Hambley P 2.49

KCL at node 1 gives

$$(v_1)/5 + (v_1-v_2)/15 + (v_1-v_3)/15 = 0$$

KCL at node 2 gives

$$(v_2-v_1)/15 + (v_2-v_3)/15 = 4$$

KCL at node 3 gives

$$(v_3)/25 + (v_3-v_2)/15 + (v_3-v_1)/15 = 0$$

Multiplying each equation through by 15 and rearranging gives

$$5 v_1 – v_2 – v_3 = 0$$
$$- v_1 +2 v_2 – v_3 = 60$$
$$+ v_1 + v_2 – (13/5) v_3 = 0$$

Adding eq2 and eq3 gives

$3\ v_2 - (18/5)\ v_3 = 60$

Adding 5*eq2+eq1 gives

$9\ v_2 - 6\ v_3 = 300$

Adding -3*eq4+eq5 gives

$(24/5)\ v_3 = 300$

i.e. $v_3 = 25$ V.

Substituting into either eq4 or eq5 gives

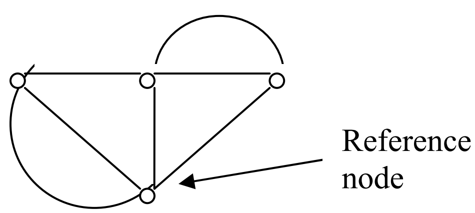
$v_2 = 50$ V.

Substituting into any one of eqs 1,2,or 3 gives

$v_1 = 15$ V.

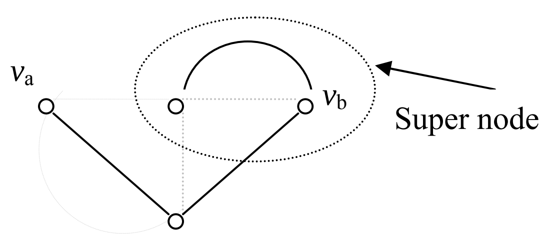
6) Hambley P 2.52 modified:

The graph of the circuit is



with the reference node indicated.

We may choose the tree (this is not unique)



A supernode is indicated.

There are two unknowns, v_a and v_b. The voltage at the other node in the supernode containing node b is v_b+20.

Writing KCL at node a gives

$2 = v_a/2 + (v_a - (v_b + 20))/5$

Writing KCL at the supernode gives

$1 = (v_b + 20)/10 + (v_b + 20 - v_a)/5.$

Multiplying both eqs by 10 and rearranging,

$7\ v_a - 2\ v_b = 60$

$-2\ v_a + 3\ v_b = -50.$

3*eq1+2*eq2 gives

$17\ v_a = 80$

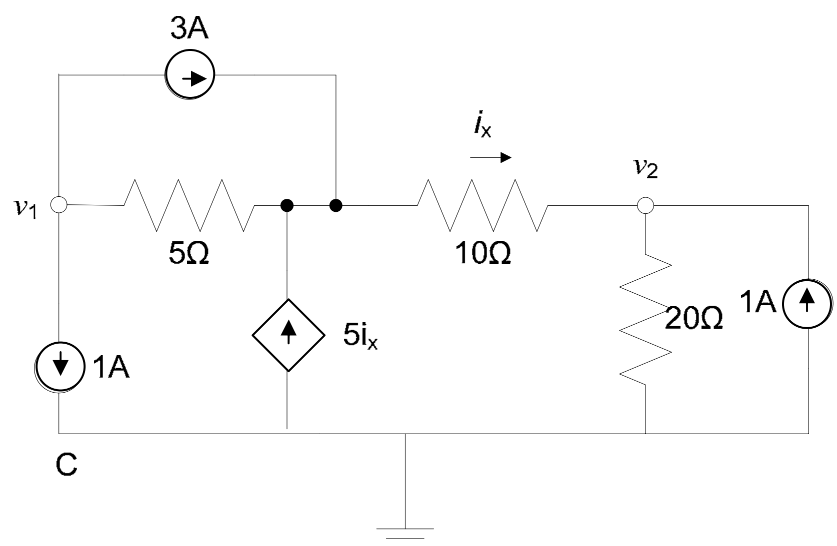
Hence

$v_a = 80/17.$

Substituting into either equation gives $v_b = -230/17$ V.
Finally, $i_1 = (v_b + 20)/10 = 11/17$A.

7)

There was a typo in the problem statement. The dependent current source was intended to be a dependent voltage source. As the problem is currently written, it could be solved by inspection by using KCL at each node without any reference to the voltages. However, for demonstration purposes we will solve it with Node Voltage Analysis. Following the solution to the written problem is an optional section, on how to solve the intended problem.



First we name the current from the controlled current source as if it is a known current source. Later we will substitute for its defining relation. There will be three equations in three variables, followed by the substitution at the end.

Writing KCL at node 1

$$(v_1 - v_3)/5 + 3 + 1 = 0$$

Writing KCL at node 2 gives

$$1 + (v_3 - v_2)/10 = v_2/20.$$

Writing KCL at the node between 1 and 2 (call it v_3) gives

$$5i_x + (v_1 - v_3)/5 + 3 + (v_2 - v_3)/10 = 0$$

And finally, we have the relation that

$$i_x = (v_3 - v_2)/10.$$

Plugging the last equation into the third and multiplying through by 10 gives:

$$v_1 + 2v_2 - v_3 = 15.$$

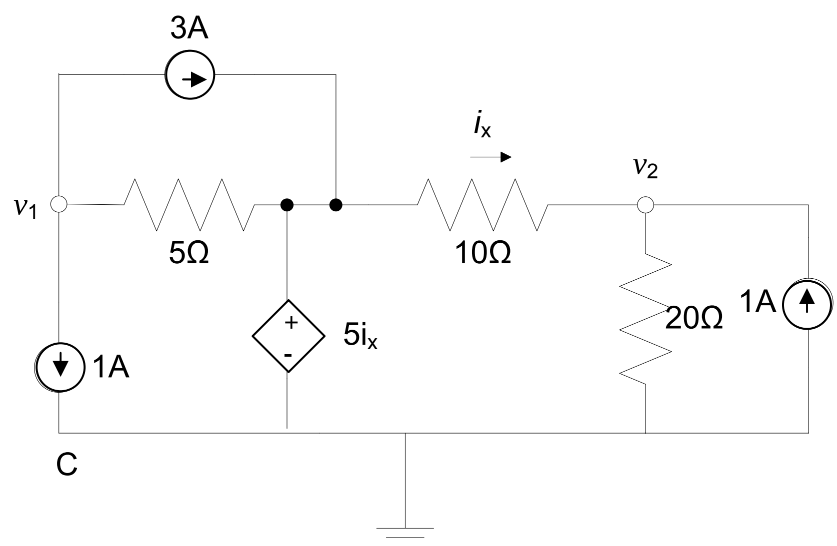
Together with the first two equations, which can be rewritten as

$v_1 + v_3 = 20$
$3\,v_2 - 2\,v_3 = 20,$

We can solve the three equations to find that
$v_1 = 7.5V$, $v_2 = 25V$, and $v_3 = 27.5V$.

Then, referring back to either equation involving i_x, we find that $i_x = ¼$.

OPTIONAL: if you were to do the intended problem, with a dependent voltage source instead of the dependent current source, shown below, then you would have the following solution (go ahead and try it!):



We treat the controlled voltage source as if it were an independent voltage source with unknown voltage V_c. Later we will get an additional equation for V_c by using the defining relation of the controlled voltage source.

Since the voltage source is connected to the reference node we may treat the non-reference node of the voltage source as part of a reference super node. Thus (besides V_c) there are two variables, v_1 and v_2.

Writing KCL at node 1 gives
$4 = (V_c - v_1)/5.$
Writing KCL at node 2 gives
$1+ \ (V_c - v_2)/10 = v_2/20.$
Writing KCL at node 3 gives

Multiplying eq1 by 5 and eq2 by 20 and rearranging
$v_1 = V_c - 20$
$3\,v_2 = 2\,V_c + 20.$

Now, if V_c were known, this actually solves for v_1 and v_2. However, because V_C is coming from a controlled source we are not done yet. We have to use its defining relation to write

$$V_c = 5*(V_C - v_2)/10,$$

i.e.

$$V_c = -v_2$$

Substituting this in the second eq gives

$$v_2 = 4V$$

and so

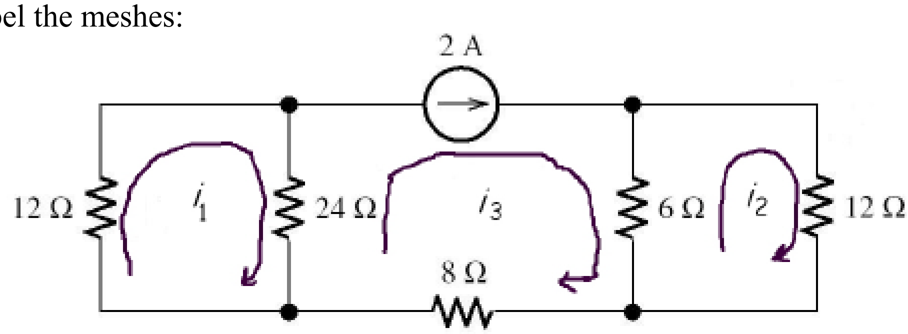
$$V_c = -4V$$

and so

$$v_1 = -24V.$$

8) **Hambley 2.64**

Label the meshes:



Since i_3=2 it is not a variable.

Writing KVL around the first mesh:

$$24*(i_2-i_3) + 12*i_1 = 0$$

Writing KVL around the second mesh:

$$12*i_2 + 6*(i_2 - i_3) = 0$$

Substituting for i_3 and rearranging

$$36*i_1 = 48$$

$$18*i_2 = 12$$

Namely

$$i_1 = 4/3 \text{ A}$$

$$i_2 = 2/3\text{A}$$

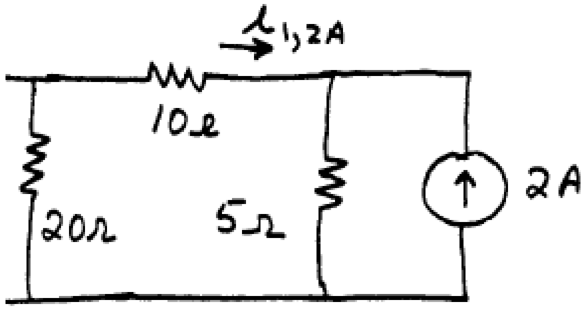
This is what we found in homework problem 3.

9) **Hambley 2.72** modified:

To be turned in with next week's problem set.

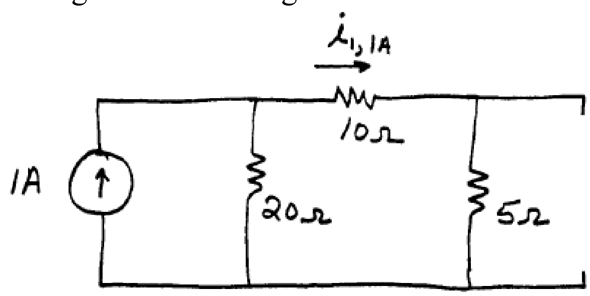
10) Hambley 2.90

Zeroing the 1A source gives



By the current division principle, the contribution of the 2A source to i_1 is 2*(-1/30) / (1/30+1/5) = -2/7A.

Zeroing the 2A source gives



By the current division principle, the contribution of the 1A source to i_1 is 1*(1/15)/(1/15+1/20) = 4/7A.

Hence i_1 = 4/7-2/7 = 2/7A.