EE 40: Introduction to Microelectronic Circuits
Spring 2008: HW 3
Solution

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1. P2.78
   To find the Thevenin equivalent, we first find Thevenin equivalent voltage $V_{TH}$ as the open circuit voltage across the terminals $a$ and $b$. By the current divider principle, the current flowing through the series connection of the 5Ω resistors is 1A and it flows in a direction such that the voltage across $a$ and $b$ is $-5\text{V}$. Hence $V_{TH} = -5\text{V}$.
   To find $R_{TH}$, we zero the independent source. Hence $R_{TH} = 5\Omega || 15\Omega = \frac{15\Omega \cdot 20\Omega}{20\Omega + 15\Omega} = 3.75\Omega$.
   The Thevenin equivalent circuit is depicted in Figure 1.

   ![Figure 1: Thevenin Equivalent Circuit](image)

   Figure 1: Thevenin Equivalent Circuit

For the Norton equivalent, we have

$$R_N = R_{TH} = 3.75\Omega$$
$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{-5\text{V}}{3.75\Omega} = -\frac{4}{3}\text{A}$$

The Norton equivalent circuit is depicted in Figure 2.

We could also have found $I_N$ by computing the short circuit current through a short circuit across terminals $a$ and $b$. By the current divider principle this current would be

$$-2A \cdot \frac{\frac{1}{5} + \frac{1}{10}}{\frac{1}{5} + \frac{1}{10}} = -\frac{4}{3}\text{A}$$
where the negative sign comes because the current would flow from $b$ to $a$. This matches the earlier expression.

2. Find the Thevenin and Norton equivalent circuits across terminals $a$ and $b$ for the circuit in Figure 3.

   With an open circuit across terminals $a$ and $b$ the current through the $20\Omega$ resistor must be $0.5\Omega i_x$ and the voltage across it is then $10i_x$. By KVL we also get
   \[30V = 5\Omega i_x + 10\Omega i_x\]
   So $i_x = 2A$ and we calculate
   \[V_{TH} = 10i_x = 20V\]
   To find $R_{TH}$ we zero the independent source to get the circuit in Figure 4.

   We wish to determine what resistance this circuit is equivalent to across the terminals $a$ and $b$. One way to do this is to imagine connecting an independent voltage source of voltage $V_S$ across the terminals and finding the current drawn from this voltage source. We therefore consider the
circuit in Figure 5.

KVL gives
\[-5\Omega i_x = V_S \rightarrow i_x = -\frac{V_S}{5\Omega} A\]  (1)

KVL also gives
\[20\Omega i_1 = V_S \rightarrow i_1 = \frac{V_S}{20\Omega}\]  (2)

KCL gives
\[i = i_1 + 0.5i_x - i_x = i_1 - 0.5i_x\]  (3)

Plugging (1) and (2) into (3) yields
\[i = \frac{V_S}{20\Omega} + \frac{V_S}{10\Omega} = \frac{3V_S}{20\Omega}\]

Eventually,
\[R_{TH} = \frac{V_S}{i} = \frac{20}{3} \Omega\]

The Thevenin equivalent circuit is depicted in Figure 6. From this we get the Norton equivalent circuit in Figure 7 where we used

\[I_N = \frac{V_{TH}}{R_{TH}} = \frac{20V}{\frac{20}{3} \Omega} = 3A\]

We could have also determined \(I_N\) by applying a short circuit across the terminals \(a\) and \(b\) and computing the current through the short circuit. This involves analyzing the circuit in Figure 8. Here we have by KVL
\[5\Omega i_x = 30V \rightarrow i_x = 6A\]
This gives, by KCL
\[ i = i_x - 0.5i_x = 0.5i_x = 3 \text{A} \]
which matches the earlier calculation.

3. P2.100
As suggested, we first replace the dependent voltage source by an independent voltage source of voltage \(V_s\). Later we will substitute \(V_s = 2v_x\) to solve the original circuit.
To apply superposition, first zero out the voltage source. This requires analyzing the circuit in Figure 9.
By the current divider principle

\[ i_1 = \frac{2}{3} \text{A} \]
\[ v_x^1 = \frac{4}{3}V \]

We next zero out the current source while retaining the voltage source. This involves analyzing the circuit in Figure 10.

We get

\begin{align*}
    i_1^2 &= -\frac{V_s}{18\Omega} \\
    v_x^2 &= \frac{2}{9}V_s
\end{align*}

Superposition tells us that when both sources are present, we would have

\begin{align*}
    i_1 &= i_1^1 + i_1^2 = \frac{2}{3}A - \frac{V_s}{18\Omega} \\
    v_x &= v_x^1 + v_x^2 = \frac{4}{3}V + \frac{2}{9}V_s
\end{align*}

We now recall that we need to substitute for \( V_s \) by 2\( V_x \). This gives

\[ v_x = \frac{4}{3}V + \frac{4}{9}v_x \rightarrow v_x = \frac{12}{5}V \]

Finally, we get

\[ i_1 = \frac{2}{3}A - \frac{v_x}{9} = \frac{2}{3}A - \frac{4}{15}A = \frac{2}{5}A \]

4. P2.102

(a) Balance occurs when

\[ \frac{R_3}{R_1} = \frac{R_x}{R_2} \]

Substituting the voltage given, this requires

\[ R_3 = \frac{R_1R_x}{R_2} = 5932\Omega \]
(b) Now we have $R_1 = 10^4 \Omega$, $R_2 = 10^4 \Omega$, $R_3 = 5933 \Omega$ and $R_x = 5932 \Omega$. We wish to find the Thevenin equivalent circuit across the terminals $a$ and $b$.

By the voltage divider principle, the open circuit voltage across terminals $a$ and $b$ is

$$V_{TH} = 10V \left( \frac{5933}{15933} - \frac{5932}{15932} \right) \approx 393.94 \mu V$$

Zeroing out the voltage source, the resistance across terminals $a$ and $b$, which equals $R_{TH}$, is

$$R_{TH} = 10^4 || 5933 \Omega + 10^4 || 5932 \Omega = 7.45 k \Omega$$

The current through the detector can be determined by examining the circuit in Figure 11. We find

$$i = \frac{V_{TH}}{R_{TH} + 5k \Omega} = 31.65 nA$$

The current is small. Hence the detector must be sensitive.

![Figure 11: Current Detector connected to Thevenin Equivalent Circuit](image)

5. P2.96

- Zeroing out the 1A current source (i.e. replacing it by an open circuit) yields a 2A current through element A and for the voltage

$$v_1 = 2i^3 \frac{\Omega}{A^2} = 16V$$

- Zeroing out the 2A current source yields a 1A current through element A and for the voltage

$$v_2 = 2i^3 \frac{\Omega}{A^2} = 2V$$

- Considering both sources at once yields a 3A current through element A (by KCL) and yields a voltage

$$v = 2i^3 \frac{\Omega}{A^2} = 54V$$
Obviously, \( v \neq v_1 + v_2 \), superposition does not apply. However, there is no reason to expect this since the I/V characteristic of element A is nonlinear.

6. **P3.10**

We calculate the following functions for the quantities to be plotted:

\[
\begin{align*}
    v(t) &= \begin{cases} 
        0V & \text{if } t < 0 \\
        50\frac{1}{2}t & \text{if } 0 \leq t \leq 2 \\
        100V & \text{otherwise}
        \end{cases} \\
    i(t) &= \begin{cases} 
        5mA & \text{if } 0 \leq t \leq 2 \\
        0mA & \text{otherwise}
        \end{cases} \\
    p(t) &= \begin{cases} 
        0mW & \text{if } t < 0 \\
        250mA \times t & \text{if } 0 \leq t \leq 2 \\
        500mA & \text{otherwise}
        \end{cases} \\
    w(t) &= \begin{cases} 
        0mJ & \text{if } t < 0 \\
        500mJ \left(\frac{1}{2}\right)^2 & \text{if } 0 \leq t \leq 2 \\
        500mJ & \text{otherwise}
        \end{cases}
\end{align*}
\]

The plots are depicted in Figure 12.

7. **P3.13**

Since

\[
i(t) = C \frac{dv(t)}{dt}
\]

we have

\[
v(t) = v(0) + \int_0^t \frac{i(\tau)}{C} d\tau
\]

\[
= 0 + \int_0^t \frac{I_m \cos(\omega \tau)}{C} d\tau
\]

\[
= \frac{I_m}{\omega C} \sin(\omega \tau)|_0^t
\]

\[
= \frac{I_m}{\omega C} \sin(\omega t)
\]

When the frequency \( \omega \) is very large the voltage is very small (of course, it is time varying, but its peak is very small). The larger the frequency the more the capacitor approximates a short circuit.

8. **P3.25**

(a) We get the equivalent circuit in stages. First we calculate the equivalent capacitor for the series of the 10\( \mu \)F and 15\( \mu \)F capacitor

\[
C_s = \frac{10\mu F \times 15\mu F}{10\mu F + 15\mu F} = 6\mu F
\]
Figure 12: Plot of Voltage, Current, Power, Work
Secondly, we calculate the equivalent capacitor for the parallel combination of the 1\( \mu \text{F} \) and 5\( \mu \text{F} \) capacitor.

\[
C_p = 1\mu \text{F} + 5\mu \text{F} = 6\mu \text{F}
\]

This yields the circuit in Figure 13. In the next stage, we combine the just calculated 6\( \mu \text{F} \) capacitor with the parallel 3\( \mu \text{F} \) capacitor and the series of the 12\( \mu \text{F} \) and 6\( \mu \text{F} \) capacitors. Analog calculations yield the values in the circuit depicted in Figure 14.

Finally, the parallel 9\( \mu \text{F} \) and 4\( \mu \text{F} \) capacitors are combined, yielding the result that the equivalent capacitance between terminals \( x \) and \( y \) equals 13\( \mu \text{F} \) (see Figure 15).

(b) Combining the two parallel combinations on the top and on the bottom leads to the equivalent circuit in Figure 16. Combining the resulting series combination gives the final result (as in Figure 17), the equivalent capacitance between the terminals is 6\( \mu \text{F} \).

9. P3.48
We calculate the following functions for the quantities to be plotted:

\[ v(t) = \begin{cases} 
10V & \text{if } 0 \leq t \leq 3 \\
-10V & \text{if } 3 < t \leq 6 \\
0 & \text{otherwise}
\end{cases} \]

\[ i(t) = \begin{cases} 
5A & \text{if } 0 \leq t \leq 3 \\
30A - 5A & \text{if } 3 < t \leq 6 \\
0 & \text{otherwise}
\end{cases} \]

\[ p(t) = \begin{cases} 
50W & \text{if } 0 \leq t \leq 3 \\
-300W + 50W & \text{if } 3 < t \leq 6 \\
0 & \text{otherwise}
\end{cases} \]

\[ w(t) = \begin{cases} 
225\left(\frac{1}{2}\right)^2 & \text{if } 0 \leq t \leq 3 \\
225\left(\frac{6-t}{2}\right)^2 & \text{if } 3 < t \leq 6 \\
0 & \text{otherwise}
\end{cases} \]

The plots are depicted in Figure 18.

10. P3.63

(a) Figure 19 depicts a redrawn circuit where the two 2H inductances have been composed. Obviously, the 4H and 0.5H inductances are now in parallel two a short cut (can be considered a 0H inductance). Hence the two parallel inductances with the shortcut yields a shortcut and we can redraw the circuit as in Figure 20. We see that the equivalent inductance across the terminals is 1H.

(b) Combining the two parallel combinations of inductances (30H and
15H) and (6H and 3H) yields two equivalent inductances

\[
\frac{30H \cdot 15H}{30H + 15H} = 10H
\]

and

\[
\frac{6H \cdot 3H}{6H + 3H} = 2H
\]

respectively. This corresponding circuit is depicted in Figure 21.

Now we combine the 2H and 10H as well as the 2H and 2H series combinations:

\[
2H + 10H = 12H
\]

\[
2H + 2H = 4H
\]
This yields the circuit in Figure 22. Composition of the parallel combination and the resulting series combination yields the final equivalent inductance in Figure 23. We conclude that the equivalent inductance across the terminals equals $4H$. 
Figure 23: Stage 2