EE 40: Introduction to Microelectronic Circuits
Spring 2008: HW 3
Solution

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Referenced problems from Hambley, 4th edition.

1. P2.78
   To find the Thevenin equivalent, we first find Thevenin equivalent voltage V_{TH} as the open circuit voltage across the terminals a and b. By the current divider principle, the current flowing through the series connection of the 5Ω resistors is $1A$ and it flows in a direction such that the voltage across a and b is $-5V$. Hence $V_{TH} = -5V$.
   To find R_{TH}, we zero the independent source. Hence $R_{TH} = 5\Omega||15\Omega = \frac{15}{4}\Omega = 3.75\Omega$.
   The Thevenin equivalent circuit is depicted in Figure 1.

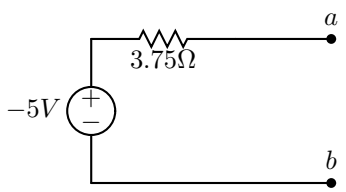

Figure 1: Thevenin Equivalent Circuit

For the Norton equivalent, we have

$$R_N = R_{TH} = 3.75\Omega$$

$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{-5V}{3.75\Omega} = -\frac{4}{3}A$$

The Norton equivalent circuit is depicted in Figure 2.

We could also have found I_N by computing the short circuit current through a short circuit across terminals a and b. By the current divider principle this current would be

$$-2A\frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{10}} = -\frac{4}{3}A$$

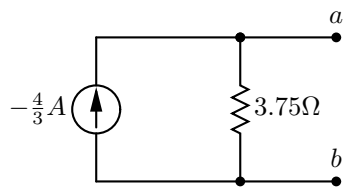

Figure 2: Norton Equivalent Circuit

where the negative sign comes because the current would flow from b to a. This matches the earlier expression.

2. Find the Thevenin and Norton equivalent circuits across terminals a and b for the circuit in Figure 3.

   With an open circuit across terminals a and b the current through the

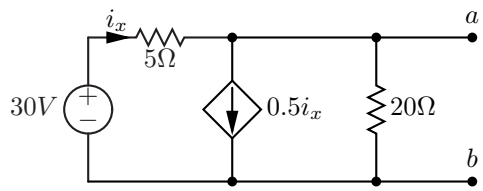

Figure 3: Circuit 1

20Ω resistor must be $0.5\Omega i_x$ and the voltage across it is then $10i_x$. By KVL we also get

$$30V = 5\Omega i_x + 10\Omega i_x$$

So $i_x = 2A$ and we calculate

$$V_{TH} = 10i_x = 20V$$

To find R_{TH} we zero the independent source to get the circuit in Figure 4.

We wish to determine what resistance this circuit is equivalent to across

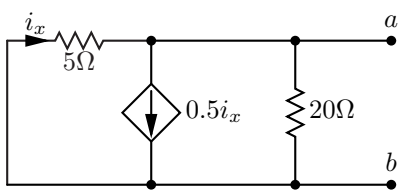

Figure 4: Circuit 1 with zeroed voltage source

the terminals a and b. One way to do this is to imagine connecting an independent voltage source of voltage V_S across the terminals and finding the current drawn from this voltage source. We therefore consider the

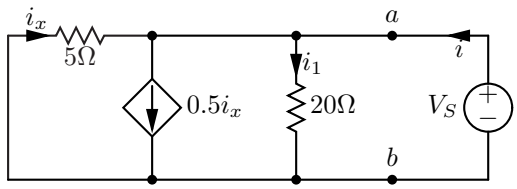

Figure 5: Circuit 1 with test source

circuit in Figure 5.

KVL gives

$$-5\Omega i_x = V_S \rightarrow i_x = -\frac{V_S}{5\Omega}A \tag{1}$$

KVL also gives

$$20\Omega i_1 = V_S \rightarrow i_1 = \frac{V_S}{20\Omega} \tag{2}$$

KCL gives

$$i = i_1 + 0.5i_x - i_x = i_1 - 0.5i_x \tag{3}$$

Plugging (1) and (2) into (3) yields

$$i = \frac{V_S}{20\Omega} + \frac{V_S}{10\Omega} = \frac{3V_S}{20\Omega}$$

Eventually,

$$R_{TH} = \frac{V_S}{i} = \frac{20}{3}\Omega$$

The Thevenin equivalent circuit is depicted in Figure 6.

From this we get the Norton equivalent circuit in Figure 7 where we used

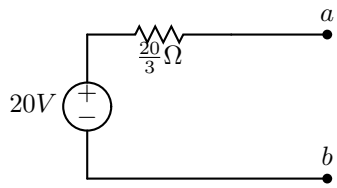

Figure 6: Thevenin Equivalent

$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{20V}{\frac{20}{3}\Omega} = 3A$$

We could have also determined I_N by applying a short circuit across the terminals a and b and computing the current through the short circuit. This involves analyzing the circuit in Figure 8. Here we have by KVL

$$5\Omega i_x = 30V \rightarrow i_x = 6A$$

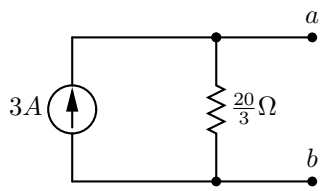

Figure 7: Norton Equivalent

This gives, by KCL

$$i = i_x - 0.5i_x = 0.5i_x = 3A$$

which matches the earlier calculation.

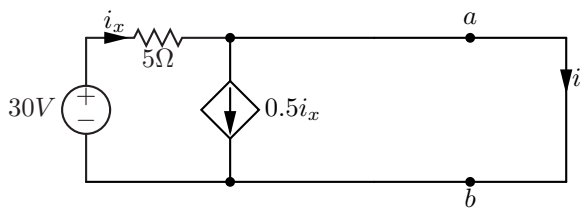

Figure 8: Shortcut circuit

3. P2.100
   As suggested, we first replace the dependent voltage source by an independent voltage source of voltage V_s. Later we will substitute $V_s = 2v_x$ to solve the original circuit.
   To apply superposition, first zero out the voltage source. This requires analyzing the circuit in Figure 9.
   By the current divider principle

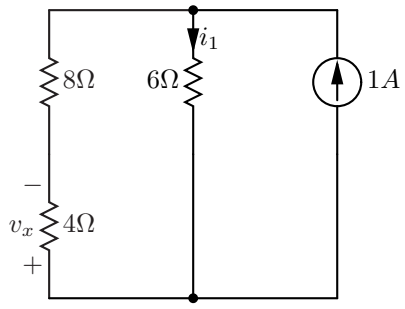

Figure 9: Circuit with zero'd dependent voltage source

$$i_1^1 = \frac{2}{3}A$$

$$v_x^1 = \frac{4}{3}V$$

We next zero out the current source while retaining the voltage source. This involves analyzing the circuit in Figure 10.
We get

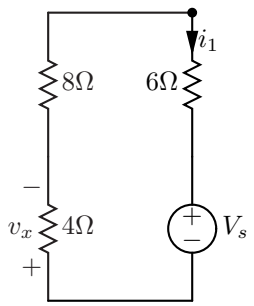

Figure 10: Circuit with zero'd current source

$$i_1^2 = -\frac{V_s}{18\Omega}$$
$$v_x^2 = \frac{2}{9}V_s$$

Superposition tells us that when both source are present, we would have

$$i_1 = i_1^1 + i_1^2 = \frac{2}{3}A - \frac{V_s}{18\Omega}$$
$$v_x = v_x^1 + v_x^2 = \frac{4}{3}V + \frac{2}{9}V_s$$

We now recall that we need to substitute for V_s by $2V_x$. This gives

$$v_x = \frac{4}{3}V + \frac{4}{9}v_x \rightarrow v_x = \frac{12}{5}V$$

Finally, we get

$$i_1 = \frac{2}{3}A - \frac{v_x}{9} = \frac{2}{3}A - \frac{4}{15}A = \frac{2}{5}A$$

4. P2.102

   (a) Balance oocurs when

   $$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

   Substituting the voltage given, this requires

   $$R_3 = \frac{R_1 R_x}{R_2} = 5932\Omega$$

(b) Now we have $R_1 = 10^4\Omega$, $R_2 = 10^4\Omega$, $R_3 = 5933\Omega$ and $R_x = 5932\Omega$. We wish to find the Thevenin equivalent circuit across the terminals a and b.
By the voltage divider principle, the open circuit voltage across terminals a and b is

$$V_{TH} = 10V(\frac{5933}{15933} - \frac{5932}{15932}) \approx 393.94\mu V$$

Zeroing out the voltage source, the resistance across terminals a and b, which equals R_{TH}, is

$$R_{TH} = 10^4||5933\Omega + 10^4||5932\Omega = 7.45k\Omega$$

The current through the detector can be determined by examing the circuit in Figure 11. We find

$$i = \frac{V_{TH}}{R_{TH} + 5k\Omega} = 31.65nA$$

The current is small. Hence the detector must be sensitive.

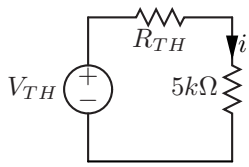

Figure 11: Current Detector connected to Thevenin Equivalent Circuit

5. P2.96

- Zeroing out the $1A$ current source (i.e. replacing it by an open circuit) yields a $2A$ current through element A and for the voltage

$$v_1 = 2i^3 \frac{\Omega}{A^2} = 16V$$

- Zeroing out the $2A$ current source yields a $1A$ current through element A and for the voltage

$$v_2 = 2i^3 \frac{\Omega}{A^2} = 2V$$

- Considering both sources at once yields a $3A$ current through element A (by KCL) and yields a voltage

$$v = 2i^3 \frac{\Omega}{A^2} = 54V$$

Obviously, $v \neq v_1 + v_2$, superposition does not apply. However, there is no reason to expect this since the I/V characteristic of element A is nonlinear.

6. P3.10
   We calculate the following functions for the quantities to be plotted:

$$
\begin{aligned}
v(t) &= \begin{cases} 0V & \text{if } t < 0 \\ 50\frac{V}{s}t & \text{if } 0 \leq t \leq 2 \\ 100V & \text{otherwise} \end{cases} \\
i(t) &= \begin{cases} 5mA & \text{if } 0 \leq t \leq 2 \\ 0mA & \text{otherwise} \end{cases} \\
p(t) &= \begin{cases} 0mW & \text{if } t < 0 \\ 250\frac{mW}{s}t & \text{if } 0 \leq t \leq 2 \\ 500mW & \text{otherwise} \end{cases} \\
w(t) &= \begin{cases} 0mJ & \text{if } t < 0 \\ 500\frac{mJ}{s^2}(\frac{t}{2})^2 & \text{if } 0 \leq t \leq 2 \\ 500mJ & \text{otherwise} \end{cases}
\end{aligned}
$$

   The plots are depicted in Figure 12

7. P3.13
   Since

$$i(t) = C\frac{dv(t)}{dt}$$

   we have

$$
\begin{aligned}
v(t) &= v(0) + \int_0^t \frac{i(\tau)}{C} d\tau \\
&= 0 + \int_0^t \frac{I_m \cos(\omega\tau)}{C} d\tau \\
&= \frac{I_m}{\omega C} \sin(\omega\tau)|_0^\tau \\
&= \frac{I_m}{\omega C} \sin(\omega t)
\end{aligned}
$$

   When the frequency ω is very large the voltage is very small (of course, it is time varying, but its peak is very small). The larger the frequency the more the capacitor approximates a short circuit.

8. P3.25

   (a) We get the equivalent circuit in stages. First we calculate the equivalent capacitor for the series of the $10\mu F$ and $15\mu F$ capacitor

$$C_s = \frac{10\mu F 15\mu F}{10\mu F + 15\mu F} = 6\mu F$$

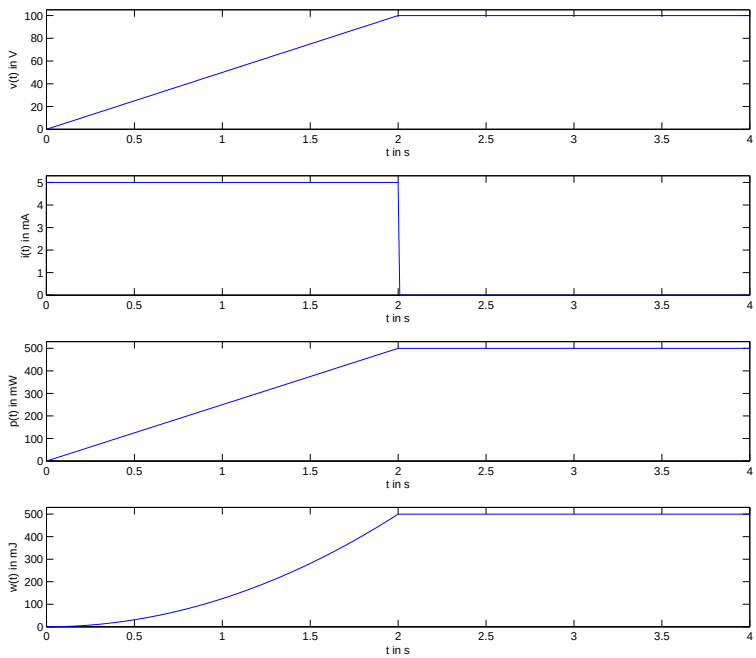

Figure 12: Plot of Voltage, Current, Power, Work

Secondly, we calculate the equivalent capacitor for the parallel combination of the $1\mu F$ and $5\mu F$ capacitor.

$$C_p = 1\mu F + 5\mu F = 6\mu F$$

This yields the circuit in Figure 13. In the next stage, we combine

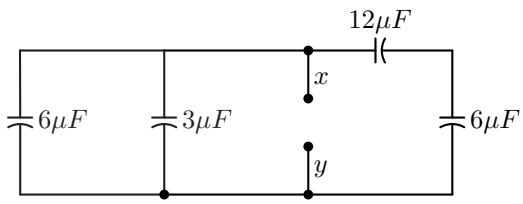

Figure 13: Stage 1

the just calculated $6\mu F$ capacitor with the parallel $3\mu F$ capacitor and the series of the $12\mu F$ and $6\mu F$ capacitors. Analog calculations yield the values in the circuit depicted in Figure 14.

Finally, the parallel $9\mu F$ and $4\mu F$ capacitors are combined, yielding

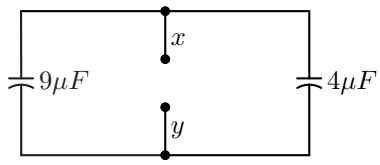

Figure 14: Stage 2

the result that the equivalent capacitance between terminals x and y equals $13\mu F$ (see Figure 15).

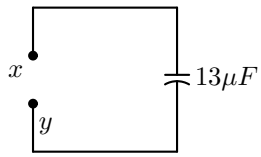

Figure 15: Stage 3

(b) Combining the two parallel combinations on the top and on the bottom leads to the equivalent circuit in Figure 16. Combining the resulting series combination gives the final result (as in Figure 17), the equivalent capacitance between the terminals is $6\mu F$.

9. P3.48

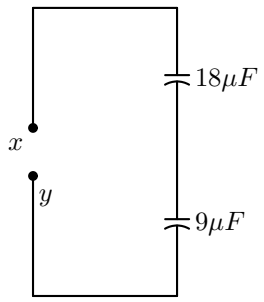

Figure 16: Stage 1

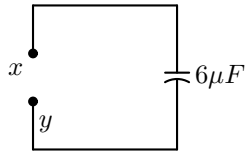

Figure 17: Stage 2

We calculate the following functions for the quantities to be plotted:

$$v(t) = \begin{cases} 10V & \text{if } 0 \leq t \leq 3 \\ -10V & \text{if } 3 < t \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

$$i(t) = \begin{cases} 5\frac{A}{s}t & \text{if } 0 \leq t \leq 3 \\ 30A - 5\frac{A}{s}t & \text{if } 3 < t \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

$$p(t) = \begin{cases} 50\frac{W}{s}t & \text{if } 0 \leq t \leq 3 \\ -300W + 50\frac{W}{s}t & \text{if } 3 < t \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

$$w(t) = \begin{cases} 225\frac{J}{s^2}(\frac{t}{3})^2 & \text{if } 0 \leq t \leq 3 \\ 225\frac{J}{s^2}(\frac{6-t}{2})^2 & \text{if } 3 < t \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

The plots are depicted in Figure 18.

10. P3.63

    (a) Figure 19 depicts a redrawn circuit where the two $2H$ inductances have been composed. Obviously, the $4H$ and $0.5H$ inductances are now in parallel two a short cut (can be considered a $0H$ inductance). Hence the two parallel inductances with the shortcut yields a shortcut and we can redraw the circuit as in Figure 20. We see that the equivalent inductance across the terminals is $1H$.

    (b) Combining the two parallel combinations of inductances ($30H$ and

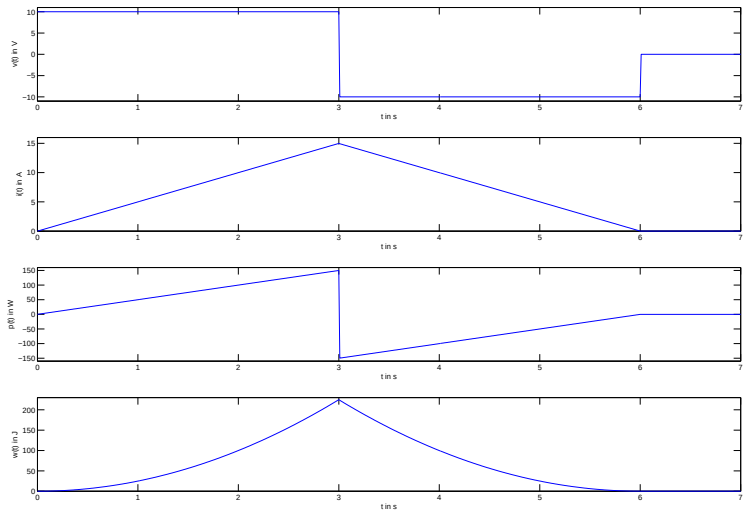

Figure 18: Plot of Voltage, Current, Power, Work

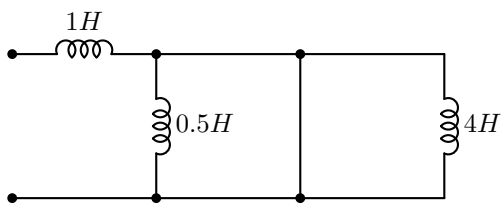

Figure 19: Stage 1

$15H$) and ($6H$ and $3H$) yields two equivalent inductances

$$\frac{30H15H}{30H + 15H} = 10H$$

and

$$\frac{6H3H}{6H + 3H} = 2H$$

respectively. This corresponding circuit is depicted in Figure 21. Now we combine the $2H$ and $10H$ as well as the $2H$ and $2H$ series combinations:

$$2H + 10H = 12H$$

$$2H + 2H = 4H$$

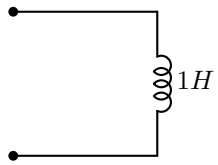

Figure 20: Stage 2

This yields the circuit in Figure 22. Composition of the parallel combination and the resulting series combination yields the final equivalent inductance in Figure 23. We conclude that the equivalent inductance across the terminals equals $4H$.

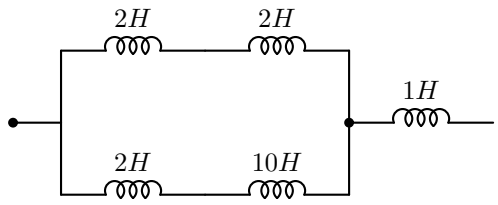

Figure 21: Stage 2

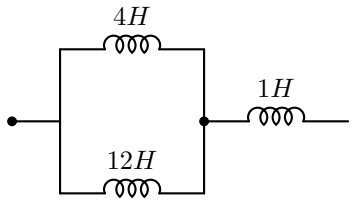

Figure 22: Stage 2

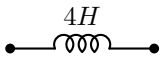

Figure 23: Stage 2