EE 40: Introduction to Microelectronic Circuits
Spring 2008: HW 6
Solution

Venkat Anantharam

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Referenced problems from Hambley, 4th edition.

1. P5.56

   Note that we have not been asked to find the voltage at the node between the 5Ω resistor and the $15j\Omega$ inductor. We may therefore replace the series connection by a single impedance of the value $5 + 15j\Omega$ before starting.

   KCL at node 1 gives

$$\frac{1}{10}\mathbf{V_1} + \frac{1}{5+j15}(\mathbf{V}_1 - \mathbf{V}_2) = 1$$

   KCL at node 2 gives

$$-\frac{1}{j10}\mathbf{V}_2 + \frac{1}{5+j15}(\mathbf{V}_2 - \mathbf{V}_1) = 1e^{j\pi/6}$$

   Rewriting, we see that

$$(\frac{1}{10} + \frac{1}{5+15j})\mathbf{V}_1 - (\frac{1}{5+15j})\mathbf{V}_2 = 1 \quad (1)$$

$$(-\frac{1}{5+15j})\mathbf{V}_1 + (\frac{1}{10} + \frac{1}{5+15j})\mathbf{V}_2 = e^{\pi j/6} \quad (2)$$

   We choose to manipulate these two equations so that when we add them together the $\mathbf{V}_1$ will cancel, giving a single equation in $\mathbf{V}_1$.

   Equation 1 can be multiplied by $2*(5+j15)$ to eliminate the denominators, yielding

$$3(1+j)\mathbf{V}_1 - 2\mathbf{V}_2 = 10 + j30.$$

   Multiplying this by $1 - j$ (to get a real coefficient for $\mathbf{V}_1$), we get

$$6\mathbf{V}_1 - 2(1-j)\mathbf{V}_2 = 40 + 20j \quad (3)$$

.

Working with eq. 2 now, we first multiply through by $2*(5+j15)$ to clear the denominators and obtain

$$-2\mathbf{V}_1 + (j-1)\mathbf{V}_2 = 10(1+j3)(\frac{\sqrt{3}}{3} + \frac{j}{2}) = 5(\sqrt{3} - 3 + (3\sqrt{3}+1)j). \quad (4)$$

We can multiply this by 3, in order to add it to eq. 3. The sum of $3 * eq.\ 4 + eq.\ 3$ is

$$5(j-1)\mathbf{V}_2 = (15\sqrt{3} - 5) + (45\sqrt{3} + 35)j,$$

or, dividing through by 5,

$$(j-1)\mathbf{V}_2 = (3\sqrt{3} - 1) + (9\sqrt{3} + 7)j. \quad (5)$$

We can multiply each side by $j+1$ to obtain

$$2\mathbf{V}_2 = (6\sqrt{3} + 8) - (12\sqrt{3} + 6)j,$$

or

$$\mathbf{V}_2 = (3\sqrt{3} + 4) - (6\sqrt{3} + 3)j$$

.

It remains to find

$$\mathbf{V}_2$$

. We plug eq.5 into eq. 4, getting

$$-2\mathbf{V}_1 + (3\sqrt{3} - 1) + (9\sqrt{3} + 7)j = (5\sqrt{3} - 15) + (15\sqrt{3} + 5)j,$$

which we can solve for

$$\mathbf{V}_1 = (7 - \sqrt{3}) + (1 - 3\sqrt{3})j.$$

Therefore our final answer is

$$\mathbf{V}_2 = (3\sqrt{3} + 4) - (6\sqrt{3} + 3)j = 9.1962 - 13.3923i = 16.2457\angle - 55.5253^\circ \quad (6)$$
$$\mathbf{V}_1 = (7 - \sqrt{3}) + (1 - 3\sqrt{3})j = 5.2679 - 4.1962i = 55.5253\angle - 38.5394 \quad (7)$$

2. P5.59 The KVL equations are

$$10\mathbf{I}_1 + j20(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$j20(\mathbf{I}_2 - \mathbf{I}_1) + 10 + 15(\mathbf{I}_2 - \mathbf{I}_3) = 0$$
$$15(\mathbf{I}_3 - \mathbf{I}_2) + (-j5)\mathbf{I}_3 = 0$$

We simplify each equation individually to write:

$$(1+j2)\mathbf{I}_1 = j2\mathbf{I}_2 \tag{8}$$
$$-j20\mathbf{I}_1 + (j20+15)\mathbf{I}_2 - 15\mathbf{I}_3 = -10 \tag{9}$$
$$10\mathbf{I}_3 = (9+j3)\mathbf{I}_2 \tag{10}$$

Plugging eq. 8 and eq. 10 into eq. 9, we have

$$-4j(2j+4)\mathbf{I}_2 + (j20+15)\mathbf{I}_2 - \frac{3}{2}(9+3j)\mathbf{I}_2 = -10$$

which simplifies to

$$\mathbf{I}_2 = \frac{-10}{9.5-0.5j} = -1.0497 - 0.0552i = -1.0512\angle 3.0102°.$$

Plugging in, we find

$$\mathbf{I}_1 = -0.8177 - j0.4641 = -0.9402\angle 29.5778°$$

$$\mathbf{I}_3 = -0.9282 - j0.3646 = -0.9972\angle 21.4451°$$

3. P5.70

Assuming a passive reference configuration, the current

$$\mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{1500\sqrt{2}\angle 30°}{3 - +j40} = \frac{1500\sqrt{2}\angle 30°}{50\angle 53.13°} = 30\sqrt{2}\angle - 23.13°$$

Since the current lags the voltage we would call the load inductive.

The root mean square values of the voltage and the current are

$$V_{rms} = 1500$$
$$I_{rms} = 30$$

Also, the angle by which the current lags the voltage is

$$\theta = 23.13°$$

The power factor is

$$\cos\theta = \cos(23.13°) = .9196$$

If the underlying frequency is ω the power delivered to the load is a time-varying function given by (see eqn(5.58))

$$p(t) = \frac{V_m I_m}{2}\cos(\theta)\left[1+\cos(2\omega t)\right] + \frac{V_m I_m}{2}\sin(\theta)\sin(2\omega t)$$

where $V_m = 1500\sqrt{2}$ and $I_m = 30\sqrt{2}$ are the peak values of the voltage and the current respectively. Substituting, using $\sin(23.13°) = 0.3928$, we have

$$p(t) = 41382\,[1 + \cos(2\omega t)] + 17676\sin(2\omega t) \text{ watts}$$

The average power delivered to the load is

$$P = \frac{V_m I_m}{2}\cos(\theta) = V_{rms} I_{rms}\cos\theta = 41382 \text{ watts.}$$

The reactive power delivered to the load is

$$Q = V_{rms} I_{rms}\sin\theta = 17676 \text{ VAR}$$

(note that in your book the units are not watts, by convention, to distinguish reactive power from real power, but you can use watts if you like).

Since the power angle is positive (voltage leads the current) the reactive power is positive.

The apparent power delivered to the load is

$$\sqrt{P^2 + Q^2} = (V_{rms} I_{rms}) = 45000 \text{ VA}$$

(here also the units in your book are not watts, but you can use watts if you like).

Note that the peak of the instantaneous power delivered to the load is actually

$$P + \sqrt{P^2 + Q^2} = 86382 \text{ watts.}$$

4. P5.76

We can use KVL to solve for the current from

$$240\sqrt{2}\angle 50° = \mathbf{I}(1 + j2) + 220\sqrt{2}\angle 30°$$

$$218.17 + j260.00 = \mathbf{I}(1 + j2) + 269.44 + j155.56$$

$$\mathbf{I} = \frac{-51.27 + j104.44}{1 + j2} = \frac{116.35\angle 116.15°}{2.236\angle 63.44°} = 52.03\angle 52.71°.$$

The power angle for source A considered in a passive reference is

$$50° - (-127.29°) = 177.29°$$

Since this is not in the range $(-90°, 90°)$, the source is delivering power.

THe power angle for source B considered in a passive reference is

$$30° - (52.71°) = -22.71°$$

Since this is in the range $(-90°, 90°)$, the source is having power delivered to it.

(The above statements apply to real power. Reactive power is also being exchanged between the sources and the inductor).

5. P5.91 We first find the open circuit voltage (phasor) across the terminals a and b.

   KCL tells us that

   $$\mathbf{I}_x = 0.5\mathbf{I}_x$$

   This implies that $\mathbf{I}_x = 0$.

   From this it follows that the open circuit voltage across a and b, ie $\mathbf{Z}_{th}$, equals $-3\angle 30°$, ie $3\angle -150°$.

   To find $\mathbf{Z}_{th}$, the Thevenin impedance, we zero out the independent source and apply an external voltage (phasor) $\mathbf{V}_{ext}$ across terminals a and b. We then determine $\mathbf{I}_{ext}$. We will have

   $$\mathbf{Z}_{th} = \frac{\mathbf{V}_{ext}}{\mathbf{I}_{ext}}$$

   See the figure (Fig. 1).

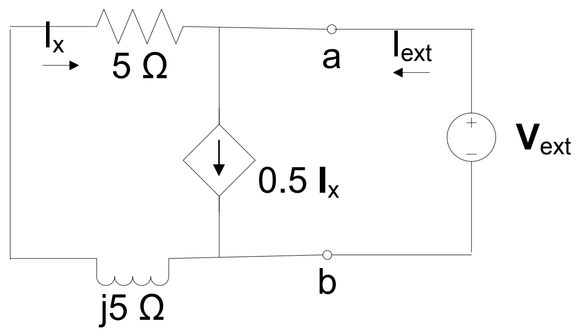

Figure 1:

   KCL tells us that

   $$\mathbf{I}_{ext} + \mathbf{I}_x = 0.5\mathbf{I}_x$$

   Hence

   $$\mathbf{I}_{ext} = -0.5\mathbf{I}_x$$

   KVL tells us that

   $$\mathbf{I}_x(5 + j5) + \mathbf{V}_{ext} = 0$$

   Hence

   $$\mathbf{I}_x = -\frac{\mathbf{V}_{ext}}{5 + j5}$$

$$\mathbf{Z}_{th} = \frac{\mathbf{V}_{ext}}{\mathbf{I}_{ext}} = -2\frac{\mathbf{V}_{ext}}{\mathbf{I}_x} = 10 + j10$$

The Thevenin equivalent circuit is (see Fig.2)

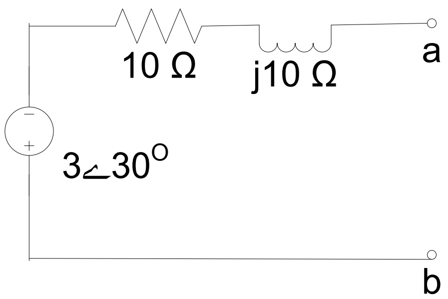

Figure 2:

We can determine the Norton equivalent circuit from this. However, let us instead directly determine $\mathbf{I}_N$ by determining the short circuit current, see the figure (Fig.3):

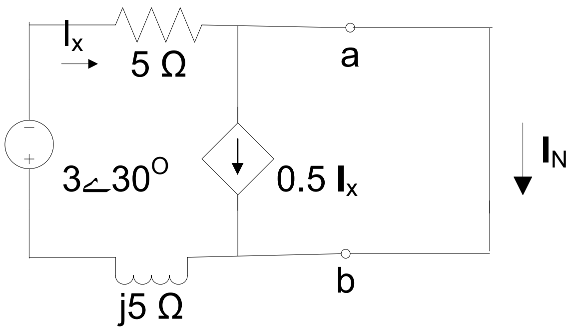

Figure 3:

KCL tells us that $\mathbf{I}_N = 0.5\mathbf{I}_x$.

KVL tells us that $2\angle 30^\circ + (5 + j5)\mathbf{I}_x = 0$.

Hence,

$$\mathbf{I}_N = \frac{-3\angle 30^\circ}{10 + j10}$$

This is consistent with what one would have gotten by direct transformation of the Thevenin equivalent into the Norton equivalent.

6. P6.16

   A (linear) filter whose input is a sinusoid of a given frequency produces an output which is also a sinusoid at the same frequency. From the linearity, the frequencies at which we can determine the value of the filter and the corresponding magnitude and phase may be listed as below:

f	$\lvert H(f)\rvert$	$\angle H(f)$
0	3	$0°$
10^3	2	$30°$
1500	1	$90°$
2000	0	irrelevant

7. P6.25

   We assume that the filter is as in Fig. 6.7. The transfer function is

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

where $f_B = \frac{1}{2\pi RC}$ is the half-power frequency. Here we are given that $f_B = 500$ Hz.

The input signal has components at DC ($f = 0$), at $f = 500$ and at $f = 15 * 10^3$. The magnitude and phase of the transfer function at these frequencies can be listed as

f	$H(f)$	$\lvert H(f)\rvert$	$\angle H(f)$
0	1	1	$0°$
500	$\frac{1}{1+j}$	$\frac{1}{\sqrt{2}}$	$-45°$
$15 * 10^3$	$\frac{1}{1+j30}$	$\frac{1}{\sqrt{901}}$	$-\arctan(30) = -88.0908°$

Because the filter is linear, the corresponding output voltage is

$$v_{out}(t) = 4 + \sqrt{2}\sin(1000\imath t - 15°) + \frac{5}{\sqrt{901}}\cos(30 * 10^3\pi t - 88.0908°).$$

Note that the component of the input at the high frequency $15 * 10^3$ has been significantly attenuated relative to the other frequencies and incurs a phase shift of approximately $-90°$.

8. P6.29

   We seek a filter with transfer function

$$H(f) = \frac{1}{1 + j(\frac{f}{f_B})}$$

with the property that

$$|H(10^4)| = \frac{1}{100}$$

ie

$$\frac{1}{\sqrt{1+(\frac{f}{f_B})^2}} = \frac{1}{100}$$

Working out the algebra gives

$$f_B = \frac{10^4}{10^4-1} \approx \frac{1}{\sqrt{101}}.$$

Note that 1 kHz is still well above the break frequencies of $\approx$ 100 Hz so signals at this frequency is also substantially attenuated by this filter.

9. (a) P6.54

   We write

   $$H(f) = 10\frac{1}{1-j(f/500)}$$

   The magnitude and phase Bode plots of the constant transfer function 10 can be found by writing, for $H_1(f) = 10$,

   $$|H_1(f)| = 10$$

   $$|H_1(f)|_{dB} = 20log10 = 20$$

   because the logarithm is in base 10 and $\angle H_1(t) = 0°$. The magnitude and phase plots for

   $$H(f) = 10\frac{1}{1-j(f/500)}$$

   can be found by first identifying the break frequency

   $$f_B = 500 \text{ Hz}$$

   and then appealing to the discussion in the text.

   The overall plots are the sums of the respective plots for $H_1(f)$ and $H_2(f)$. We draw these as (see Figs. 4&5)

   Note that the phase plot is the negative of that of Fig. 6.16 because the phase of $H_2(f)$ increase to 90° as $f \to \infty$. The asymptotes are used as an aid for plotting the Bode plots, which are the curved graphs. The value of the magnitude Bode plot at the break frequency is roughly 3 dB below its peak, which in this case is 17 dB.

   (b) P6.55

   We have

   $$H(f) = \frac{1-j(f/100)}{1+j(f/100)}$$

   Since $|H(f)| = 1$ for all f (such a filter is called an all-pass filter), we have

   $$|H(f)|_{dB} = 20\log|H(f)| = 0$$

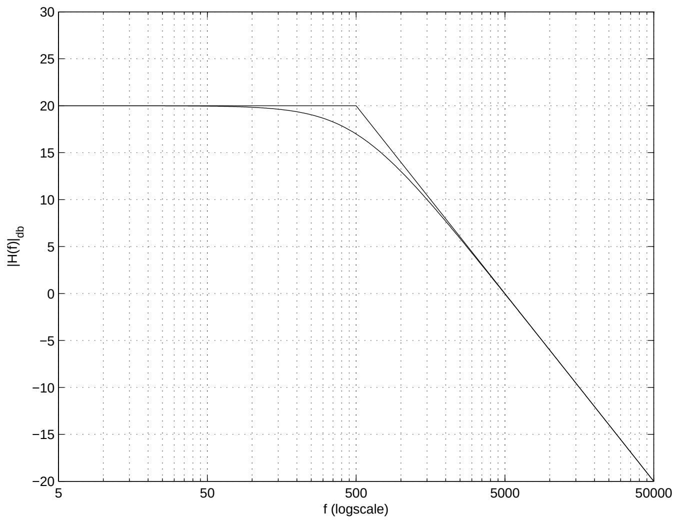

Figure 4: Magnitude plot of H(f) for problem 6.54

Thus the magnitude Bode plot is (see Fig. 6)

For the phase Bode plot we notice that both the numerator $H_1(f) = 1 - j(f/100)$ and the denominator $H_1(f) = 1 + j(f/100)$ have break frequency 100 Hz.

The corresponding phase plots are (see Figs. 7& 8)

We add these to get the Bode phase plot of H(f) (see Fig. 9)

Again, the asymptotes are used only as an aid in sketching the phase Bode plot, which is the curved line.

10. P6.69

(a) By the voltage divider formula,

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_2}{R_2 + \frac{R_1/j2\pi fC}{R_1+1/j2\pi fC}} = \frac{R_2 + R_1 R_2 j2\pi fC}{R_2 + R_1 + j2\pi fCR_1R_2}$$

(Here we are assuming a sinusoidal input voltage at frequency f, in which case the output will also be sinusoidal at frequency f, and we are computing a ratio of phasors).

(b) See Fig. 10.

(c) As $f \to 0$ the transfer function $H(f)$ approaches $\frac{R_2}{R_1+R_2}$. This is consistent with the voltage divider formula when the capacitor is replaced by an open circuit.

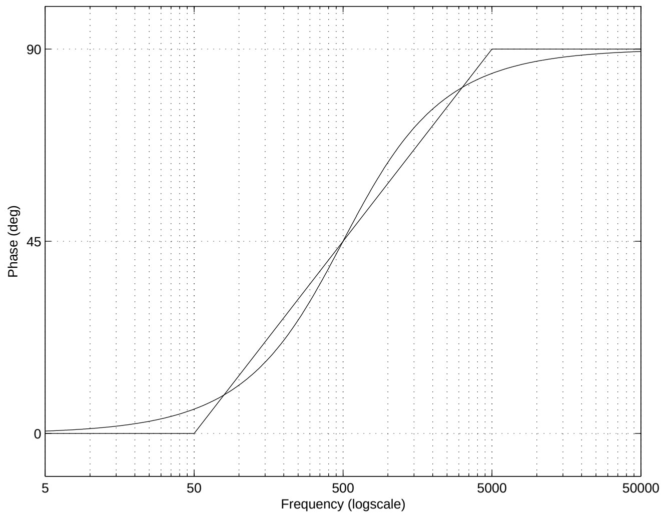

Figure 5: Phase plot of H(f) for problem 6.54

(d) As $f \to \infty$ the transfer function $H(f)$ approaches 1. This is consistent with what the transfer function would be if the capacitor were replaced with a short circuit.

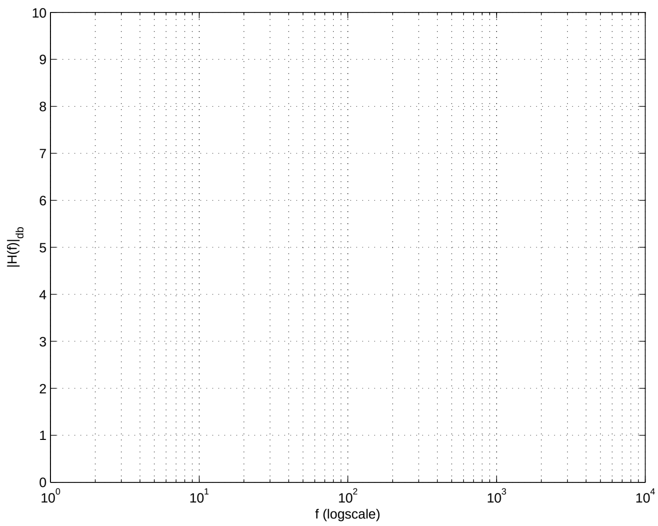

Figure 6: Magnitude plot of H(f) for problem 6.55

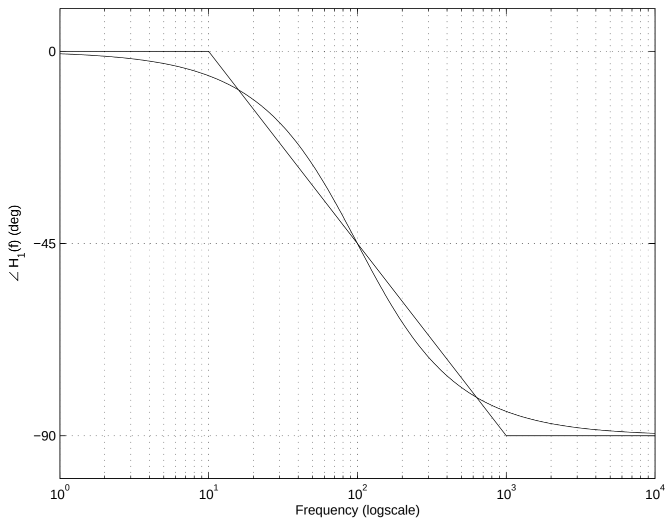

Figure 7: Phase plot of $H_2(f)$ for problem 6.55

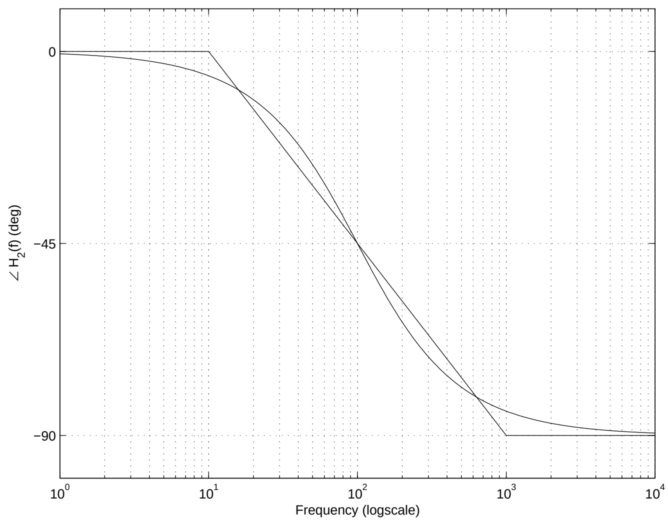

Figure 8: Phase plot of $H_2(f)$ for problem 6.55

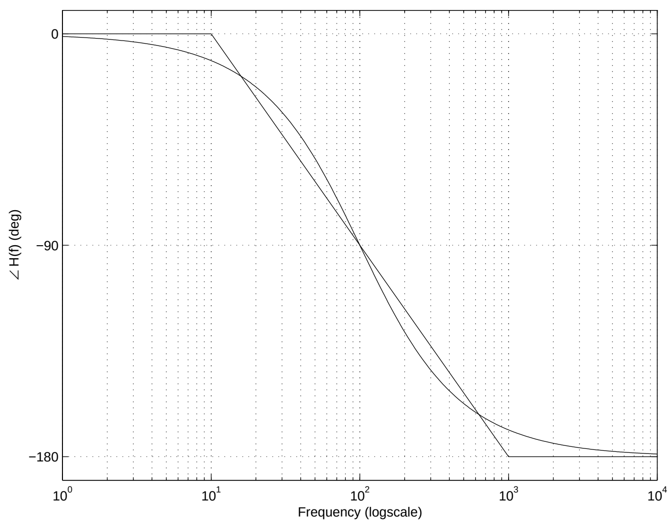

Figure 9: Phase plot of $H(f)$ is the sum of phase plots for $H_1(f)$ and $H_2(f)$ for problem 6.55

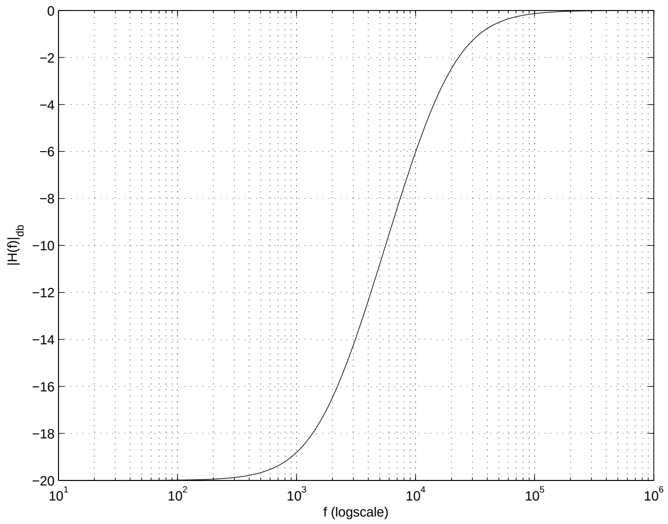

Figure 10: Magnitude plot of H(f) for problem 6.69