EE 40: Introduction to Microelectronic Circuits
Spring 2008: HW 9
(due 4/28, 5 pm)

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Referenced problems from Hambley, 4th edition.

1. P10.83
   In this case we would say (roughly) that

$$V_{DQ} = 5V \qquad I_{DQ} = 3mA$$

(these are the quiescent point voltage and current respectively: note the notational conventions) and

$$v_d(t) = 0.01V\cos(\omega t) \qquad i_d(t) = 0.2mA\cos(\omega t)$$

(these are the small signal ac voltage and current respectively: note the sign conventions) and

$$r_d = \frac{v_d(t)}{i_d(t)} = 50\Omega$$

(this does not depend on t). Note that

$$v_D(t) = V_{DQ} + v_d(t) \qquad i_D(t) = I_{DQ} + i_d(t)$$

2. Design a clipper circuit for a negative limit of $-2.1V$ and a positive limit of $2.2V$. The input voltage is peak limited to $\pm 5V$.
   You are allowed to use two diodes, two DC voltage sources and a resistor.
   The maximum allowable current through each diode is $2.5mA$ and the threshold voltage of each diode is $0.7V$. Use an ideal voltage with threshold, ignoring breakdown.
   Hint: See Figure 10.32 in textbook.
   Following the hint, we consider a circuit like that in Figure 1. We would need to choose

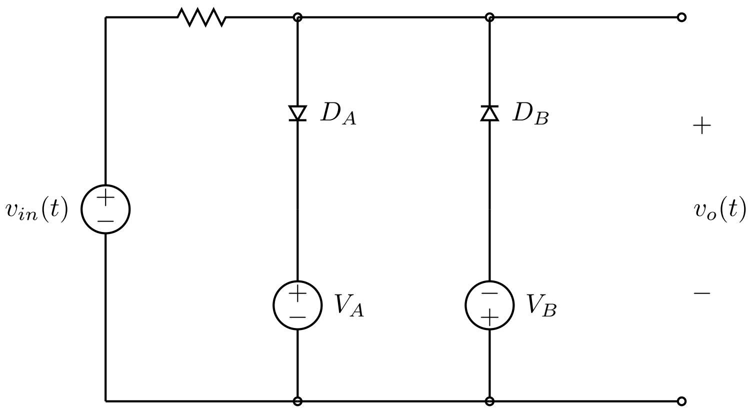

Figure 1: Clipper Circuit

$$V_A = 1.5V \qquad\qquad V_B = 1.4V$$

in oder to get the desired negative and positive limits on the output voltage.
Sine the input voltage is peak-limited to $\pm 5V$, the maximum current through the diode D_A is

$$\frac{5V - V_A - 0.7}{R} = \frac{3.5V}{R}$$

The maximum current through the diode D_B is at most

$$\frac{5V - V_B - 0.7V}{R} = \frac{3.6V}{R}$$

To meet the required limit on the maximum current through the diodes we need

$$\frac{2.9V}{R} \leq 2.5mA \rightarrow R \geq 1.16k\Omega$$

3. A voltage source produces a periodic voltage $v_{in}(t)$ with period $T = 2s$ with waveform as in Figure 2. The positive peaks are of $6V$ and the negative peaks are at $-2V$. Not that

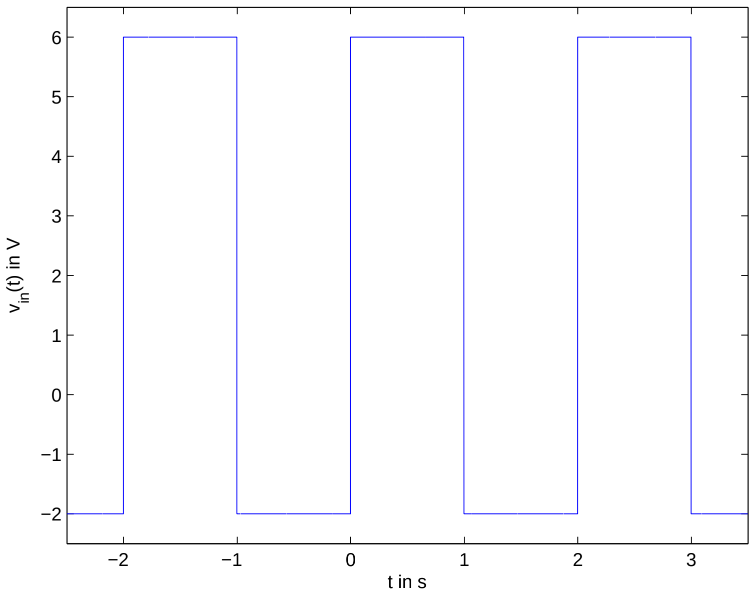

Figure 2: Waveform Input Voltage

$$v_{in}(t) = v_d + v_a(t)$$

where $v_d = 2V$ and $v_a(t)$ is a periodic signal with zero dc component.
It is desired to clamp $v_{in}(t)$ to a positive peak of $10V$. You are allowed to use at most one of each of the following:

(a) an ideal diode.
(b) an ideal Zener diode of arbitrary breakdown voltage.
(c) a resistor of size $1k\Omega$.
(d) a dc voltage source of arbitrary value.
(e) a capacitor of arbitrary value.

You may assume implicitly that there is a small resistance in series with the voltage source v_{in} which is so small compared to $1k\Omega$ that it can be neglected.
Design a clamp circuit to perform the desired task, giving some guidelines for the choice of values for the elements (b), (d), and (e).
The circuit in Figure 3 will work. The value of the capacitor C should be such that $C \gg 1mF$ The circuit

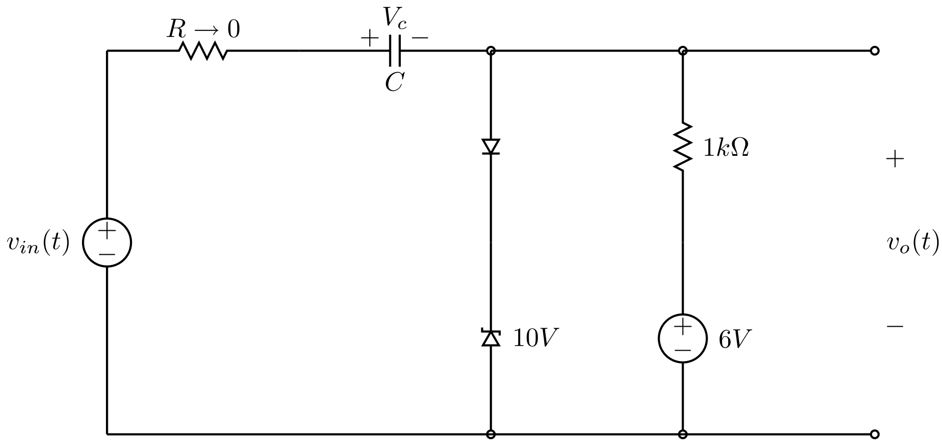

Figure 3: Clamper Circuit

works because in steady state, we have V_C roughly constant at $4V$. We then have $v_o(t) = v_{in}(t) + 4V$ which is thus the level-shifted version of $v_{in}(t)$ clamped to a positive peak of $10V$.

4. In the circuit in Figure 4, the op-amp is assumed to be ideal. The circuit is called a *precision rectifier*. We would need to choose

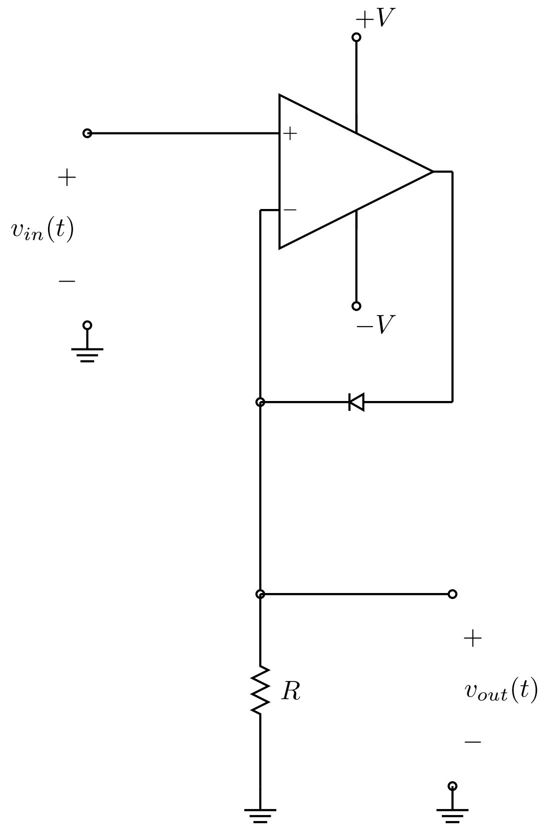

Figure 4: Precision Rectifier

$V_A = 1.5V$ $\quad$ $V_B = 1.4V$

In order to get the desired negative and positive limits on the output voltage.
Since the input voltage is peak-limited to $\pm 5V$ the maximum current through the diode D_A is at most
Here $\pm V$ denotes the supply voltages. Take $V = 12V$. Note that $v_{out(t)}$ is defined at the $-$ input of the op-amp.

(a) Let $v_{in}(t) = V_m \sin(\omega t)$. Let V_m be significantly smaller than V, e.g. $V_m = 6V$. Determine $v_{out}(t)$ under the assumption that the diode is ideal.
When $v_{in}(t) > 0V$, in order to satisfy the summing point constraint the voltage at the $-$ input of the op-amp take on the same value. This is consistent with current flowing to ground from the output of the op-amp, through the ideal diode (which will be in the short-circuit regime) and through the resistors. Thus we have $v_{out}(t) = v_{in}(t)$ when $v_{in} > 0V$. Note that the output of the op-amp is also at $v_{out}(t)$ in this case.
When $v_{in} < 0V$, to satisfy the summing point constraint the voltage at the $-$ input of the op-amp would like to equal $v_{in}(t)$. However, this would entail current flowing through the diode in reverse direction, which is not possible. Thus, the summing point constraint cannot be saitsfied. The output of the op-amp will go to the negative supply voltage level, the diode will be an open circuit in reverse bias, and the $-$ input of the op-amp will be tied to ground.
Thus $v_{out}(t) = 0$ when $v_{in}(t) > 0$. We see that

$$v_{out} = \begin{cases} V_m \sin(\omega t) & \text{if } \sin(\omega t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

i.e. it is a half-wave rectified version of $v_{in}(t)$.

(b) Now, suppose that the diode is modelled as having a threshold voltage $v_{th} = 0.7V$, being an ideal short circuit at threshold voltage and an ideal open circuit below the threshold voltage. Again, determine $v_{out}(t)$.
When $v_{in}(t) > 0$, the same reasoning as in case (a) can be used to conclude that the summing point constraint will be satisfied, giving $v_{out}(t) = v_{in}(t)$. The difference her is that the output of the op-amp will be at $v_{out}(t) = v_{in}(t) + 0.7V$.
When $v_{in}(t) < 0$ the same reasoning as in case (a) can be used to conclude that the output of the op-amp goes to the negative supply voltage and we have $v_{out}(t) = 0$.
Thus, we again have that $v_{out}(t)$ is a half-wave rectified version of $v_{in}(t)$:

$$v_{out} = \begin{cases} V_m \sin(\omega t) & \text{if } \sin(\omega t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) Why is the circuit called a precision rectifier? Note that even when the diode has a non-zero threshold voltage, the signal $v_{out}(t)$ is a (nearly) perfectly rectified version of $v_{in}(t)$. There is no dead zone due to the threshold voltage, unlike the rectifiers we considered earlier. This is why the circuit is called a precision rectifier.

5. Consider the circuit in Figure 5. This circuit is known as an *inverting precision rectifier.*
Assume that the op-amp has supply voltages at $\pm 12V$ and that $v_{in}(t) = 6V \sin(\omega t)$. Further assume that the diode has a threshold voltage of $0.7V$ but is otherwise ideal.
Determine the voltage $v_{out}(t)$.
When $v_{in}(t) < 0$, the $-$ input of the op-amp gets tied to ground to satisfy the summing point constraint and current flows into the output of the op-amp through the diode, which is in forward bias having come from the input voltage source through the resistance. Thus, we have

$$v_{out}(t) = 0 \qquad \text{when} \quad v_{in}(t) > 0$$

Note that the output of the op-amp is at $-0.7V$ in this case.
When $v_{in}(t) < 0$, the voltage of the $-$ input of the op-amp would like to get tied to ground to satisfy the summing point constraint. However, this would require current flow through the diode in reverse bias, which is not possible. Thus the summing point constraint cannot be satisfied. The output of the op-amp goes to the positive supply voltage, the diode is in reverse bias and we have $v_{out}(t) = v_{in}(t)$

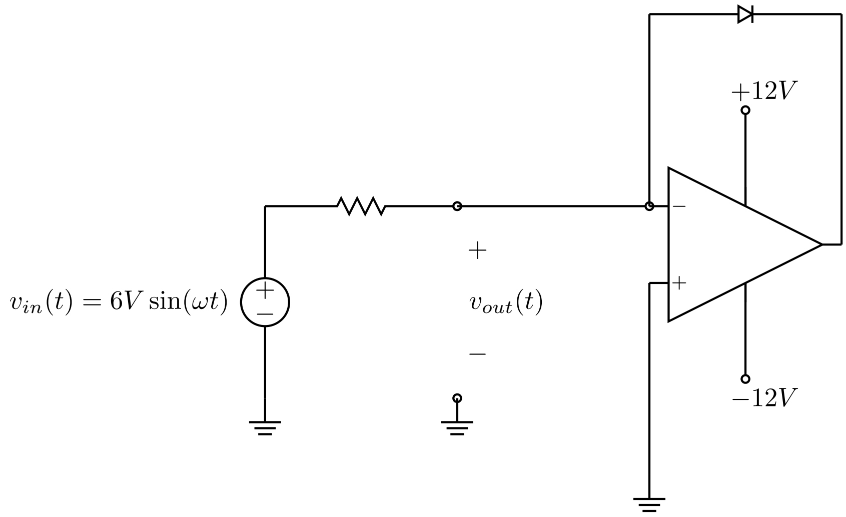

Figure 5: Inverting Precision Rectifier

because no current flows through the resistor.
We have

$$v_{out}(t) = \begin{cases} V_m \sin(\omega t) & \text{if } \sin(\omega t) < 0 \\ 0 & \text{otherwise} \end{cases}$$

i.e. this is a half-wave rectifier that picks the negative excursions.

6. For this problem, note that the intrinsic carrier concentration of both electrons and holes in pure Si at room temperature ($\approx 300K$) can be taken to be $10^{10}cm^{-3}$, while that in pure Ge can be taken to be $2 * 10^{13}cm^{-3}$.
Identify the majority carrier and find the electron and hole concentrations at room temperature in the following semiconductors.
Let n_i denote the intrinsic carrier concentrations of both holes and electrons in each case. We use the mass action law and the approximation that the majority carrier concentration equals the corresponding net dopant concentration.

   (a) Silicon doped with phosphorus at a doping concentration of $10^{16}cm^{-3}$.
   $n_i = 10^{10}cm^{-3}$ for silicon. Phosphorus is a donor. The majority carrier concentration is electrons. We may approximate the electron concentration n by the donor concentration $n = N_d = 10^{16}cm^{-3}$.
   By the mass action law, we have

   $$np = n_i^2 = 10^{20}cm^{-6}$$

   where p denotes the concentration of holes.
   We can solve to get $p = 10^4cm^{-3}$

   (b) Silicon simultaneously doped with arsenic at a concentration of $5 * 10^{17}cm^{-3}$ and with boron at a concentration of $5.1 * 10^{17}cm^{-3}$.
   Arsenic is a donor and boron is an acceptor. Since there is a higher concentration of boron than arsenic, the boron compensates for arsenic. The majority carrier is holes. We may approximate the hole concentration p by the excess of the acceptor concentration over donor concentration i.e. by

   $$p = N_a - N_d = 5.1 * 10^{17} - 5 * 10^{17}cm^{-3} = 10^{16}cm^{-3}$$

   By the mass action law

   $$n = \frac{10^{20}cm^{-6}}{p} = 10^4cm^{-3}$$

   (c) Germanium doped with boron at a concentration of $2 * 10^{15}cm^{-3}$.
   For Germanium, we have $n_i = 2 * 10^{13}cm^{-3}$. Boron is an acceptor. We use the approximation $p \approx N_a = 2 * 10^{15}cm^{-3}$ and use the mass action law

   $$np = n_i^2 = 4 * 10^{26}cm^{-6}$$

to get $n = 2 * 10^{11} cm^{-3}$. The majority carrier is holes.

Consider a slat of silicon as shown in Figure 6. We focus on the x direction and assume that there is no

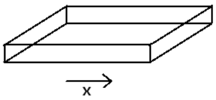

Figure 6: A slat of Silicon

variation in planes transverse to the x-direction.
Recall that Gauss's law tells us that if there is a charge density profile $\rho(x)$ in the slat (measured in $\frac{C}{cm^3}$) then the associate electric field E measured in $\frac{V}{cm}$ and with reference as pointing in the x direction is given by

$$\frac{dE}{dx}(x) = \frac{\rho(x)}{\epsilon}$$

Here, ϵ is the permittivity of the material (here, the material is silicon, for which you can take $\epsilon = 11.7\epsilon_0$, where ϵ_0, the permittivity of vacuum, is $8.85 * 10^{-14} \frac{F}{cm}$).
Further, the associated electrostatic potential $\Phi(x)$, measure in V, satisfies

$$E(x) = -\frac{d\Phi}{dx}(x)$$

which, together with Gauss's law, gives Poisson's equation

$$\frac{d^2\Phi}{dx^2}(x) = -\frac{\rho(x)}{\epsilon}$$

This determines potential differences: the actual value of the potential depends on a choice of reference. This discussion is relevant to the following three problems.

7. Suppose the charge density in a slat of silicon (variations only in the x direction) is given by the graph in Figure 7.

   (a) Verify that the sample as a whole is electrically neutral.
   We integrate the charge density along x:

$$\begin{aligned} \int_{x=-1200nm}^{800nm} \rho(\xi) d\xi &= 80 * 2mC + 60 * 1mC - 10 + 2mC - 50 * 4mC \\ &= 0 \end{aligned}$$

   This shows that the sample as a whole is electrically neutral.

   (b) Use Gauss's law to determine the electric field as a function of x.
   From

$$\frac{dE}{dx}(x) = \frac{\rho(x)}{\epsilon}$$

   we have that

$$E(x) = \frac{1}{\epsilon} \int_{-120nm}^{x} \rho(\xi) d\xi + E(-120nm)$$

   As the overall sample is electrically neutral, we can assume that $E(-120nm) = 0$.
   Performing the integration yields

$$E(x) = \begin{cases} 0 & x \leq -120nm \\ \frac{2(x+120nm)}{\epsilon} \frac{mC}{cm^3} & -120nm < x \leq -40nm \\ \frac{160nm}{\epsilon} \frac{mC}{cm^3} + \frac{x+40nm}{\epsilon} \frac{mC}{cm^3} & -40nm < x \leq 20nm \\ \frac{220nm}{\epsilon} \frac{mC}{cm^3} - \frac{x-20nm}{\epsilon} \frac{mC}{cm^3} & 20nm < x \leq 30nm \\ \frac{200nm}{\epsilon} \frac{mC}{cm^3} - \frac{4(x-30nm)}{\epsilon} \frac{mC}{cm^3} & 30nm < x \leq 80nm \\ 0 & x > 80nm \end{cases}$$

   Plugging in the numerical value of ϵ yields the result plotted in Figure 8.

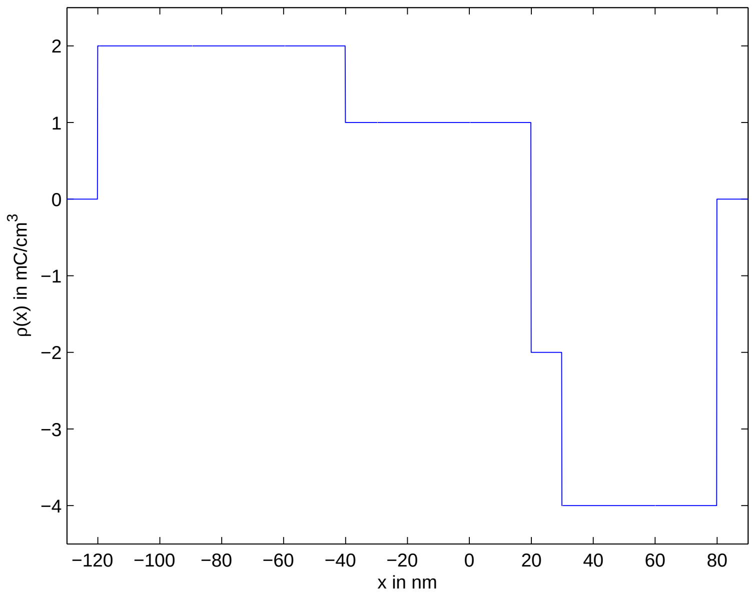

Figure 7: Charge Density Profile

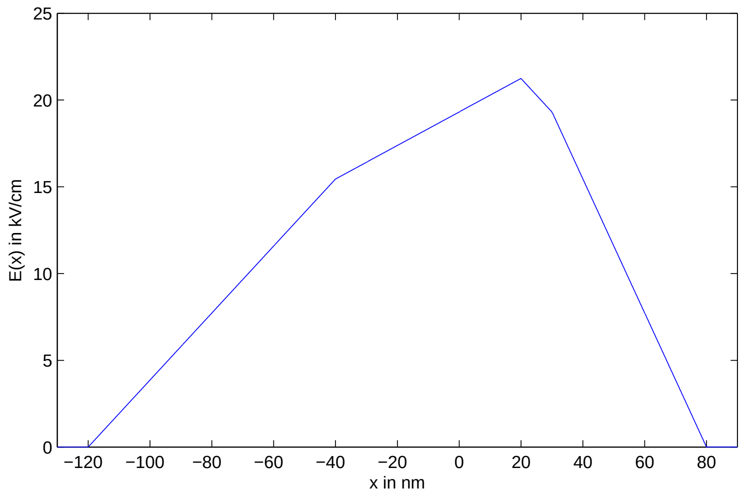

Figure 8: Profile of Electric Field

(c) Explain why the direction of the electric field (which is determined by its sign) does not change throughout the range $-120nm < x < 80nm$.
At any $-120nm < x < 80nm$, the net charge to the left is more positive than the net charge to the right of x.

(d) Determine the potential $\Phi(x)$ as a function of x, assuming as a reference that $\Phi(0) = 0$, thus solving Poisson's equation.
From

$$E(x) = -\frac{d\Phi}{dx}(x)$$

we have that

$$\Phi(x) = -\int_{-120nm}^{\xi} E(x) + \Phi(-120nm)$$

From the assignment, we know that $\Phi(0) = 0$. This can be used to find the unknown constant $\Phi(-120nm)$. We integrate the electric field piecewise to get

$$\Phi(x) = \begin{cases} 0 & x \leq -120nm \\ 10.3mV - 1.867mV\frac{x}{nm} - 9.66\mu V\frac{x^2}{nm^2} & -120nm < x \leq -40nm \\ -1.932mV\frac{x}{nm} - 4.83\mu V\frac{x^2}{nm^2} & -40nm < x \leq 20nm \\ 13.51mV - 2.511mV\frac{x}{nm} - 9.65\mu V\frac{x^2}{nm^2} & 20nm < x \leq 30nm \\ 4.82mV - 3.09mV\frac{x}{nm} + 19.3\mu V\frac{x^2}{nm^2} & 30nm < x \leq 80nm \\ -118.86mV & x > 80nm \end{cases}$$

The potential $\Phi(x)$ is plotted in Figure 9.

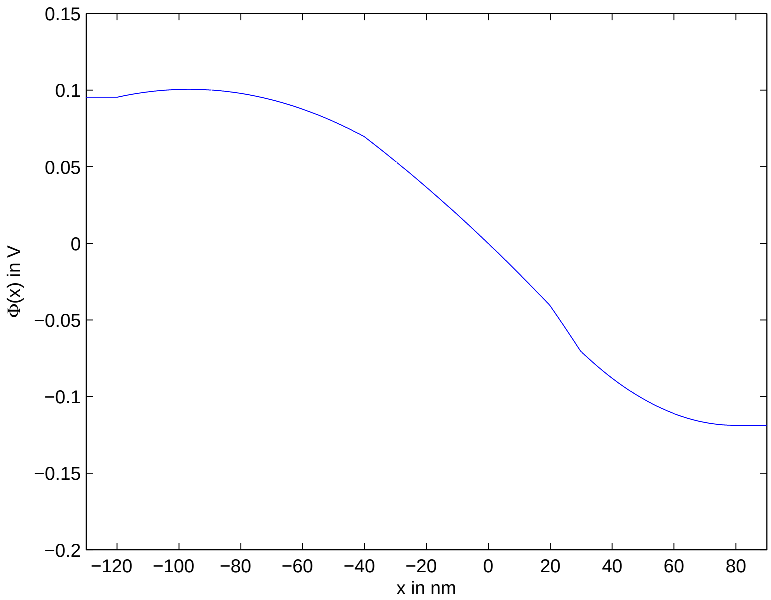

Figure 9: Potential

8. The electric field in a slat of silicon (variations only in the x direction) is given by the graph in Figure 10.

(a) Determine the associated charge density $\rho(x)$. We know that

$$\rho(x) = \epsilon\frac{dE}{dx}(x)$$

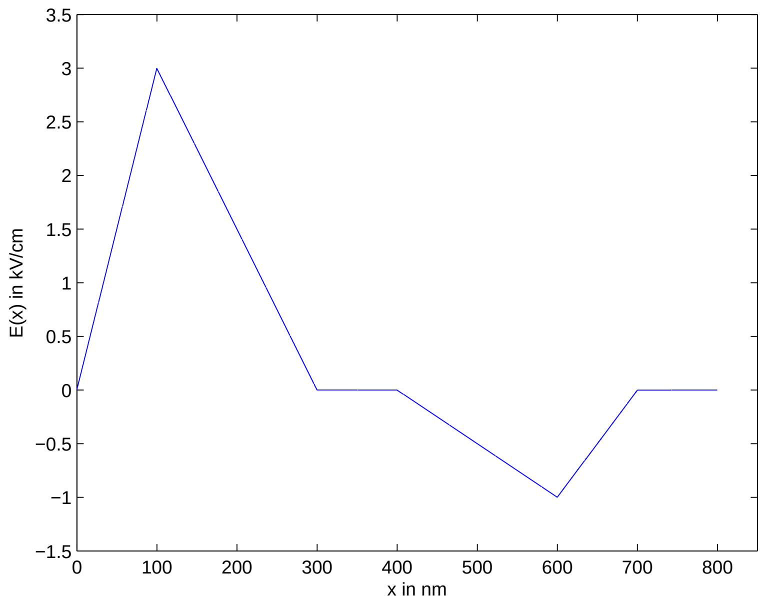

Figure 10: Profile of Electric Field

Hence we can calculate the charge density by differentiating the given function of the electric field and multiplying by ϵ.
We get

$$\rho(x) = \begin{cases} 0 & x < 0nm \\ 31.08\frac{mC}{cm^3} & 0 \leq x < 100nm \\ -15.54\frac{mC}{cm^3} & 100nm \leq x < 300nm \\ 0 & 300nm \leq x < 400nm \\ -5.18\frac{mC}{cm^3} & 300nm \leq x < 600nm \\ 10.36\frac{mC}{cm^3} & 600nm \leq x < 700nm \\ 0 & 700nm \leq x \end{cases}$$

We get the result as plotted in Figure 11. Note that we assumed for the given plot that the material was silicon, even though this was not given in the problem statement.

(b) Assuming as a reference that $\Phi(0) = 0$, solve for the corresponding potential $\Phi(x)$.
As in the previous problem, we need to integrate the electric field function to get

$$\Phi(x) = \begin{cases} 0 & x < 0nm \\ -1.5\mu V\frac{x^2}{nm^2} & 0 \leq x < 100nm \\ 22.5mV - 450\mu\frac{x}{nm} + 750nV\frac{x^2}{nm^2} & 100nm \leq x < 300nm \\ -45mV & 300nm \leq x < 400nm \\ -5mV - 200\mu\frac{x}{nm} + 250nV\frac{x^2}{nm^2} & 300nm \leq x < 600nm \\ -275mV + 700\mu\frac{x}{nm} + 500nV\frac{x^2}{nm^2} & 600nm \leq x < 700nm \\ -3mV & 700nm \leq x \end{cases}$$

The function is plotted in Figure 12

(c) Verify that the sample as a whole is electrically neutral.
Since $E(x)$ eventually returns to the same value it started from, the sample as a whole must be electrically neutral. Another way to see this is to verify that the area enclosed by $\rho(x)$ adds up to zero (considering areas below the x-axis to be zero).

In addition to the discussion of Gauss's law and Poisson's equation, the following two problems refer to the depletion approximation for PN-junctions.

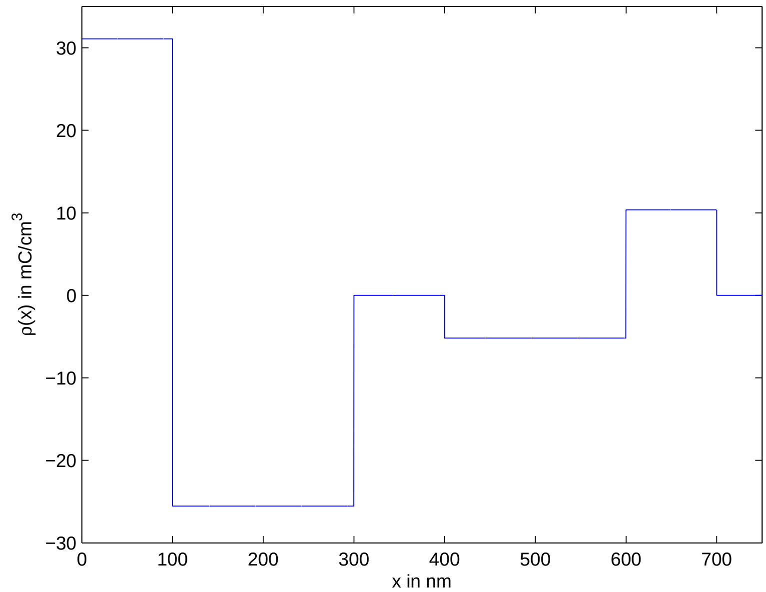

Figure 11: Charge Profile

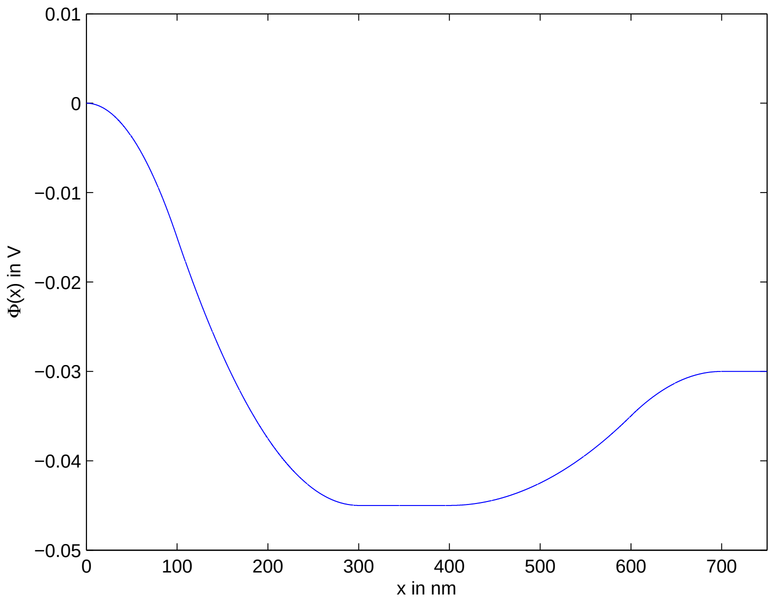

Figure 12: Potential

In addition to the supplementary reader, you should also recall, as discussed in class, that, in the absence of an externally applied bias, the potential in the bulk of the n-type material (away from the depletion region) can be taken to be $V_T \ln\left(\frac{N_d}{n_i}\right)$ and that in the bulk of the p-type material (away from the depletion region) can be taken to be $V_T \ln\left(\frac{n_i}{N_a}\right)$. Here $V_T = \frac{kT}{q}$ denotes the thermal voltage, n_i the intrinsic carrier concentration of holes and of electrons in pure silicon, and N_a and N_d respectively denote the doping densities of acceptors and donors (in the p-type and n-type materials, respectively).
The potentials are thought of as referenced to an "intrinsic" situation - if one solves Poisson's equation exactly with these boundary conditions, the potential will be zero precisely when the electron density and the hole density are equal (each equal to the intrinsic carrier concentration).

9. Consider a Silicon PN-junction in thermal equilibrium at room temperature with no externally applied bias. Assume that $N_a = 10^{19}cm^{-3}$ and $N_d = 10^{17}cm^{-3}$.
   Using the depletion approximation, determine the width of the depletion region in the n-type material, x_{n_0}, the width of the depletion region in the p-type material, x_{p_0}, the electric field $E(x)$ (as a function of the distance x from the junction, assuming the n-type region corresponds to positive x) and the potential $\Phi(x)$ as a function of x.
   Also determine the maximum magnitude of the electric field.
   We have $n_i = 10^{10}cm^{-3}$. This gives the potential for x large and negative (i.e. in the bulk of the p-type material) as $-V_T \ln(10^7)$
   Firstly, we know from the lecture nodes that

$$\Phi(-x_{p_0}) = V_T \ln\left(\frac{n_i}{N_a}\right)$$

   and

$$\Phi(x_{n_0}) = V_T \ln\left(\frac{N_d}{n_i}\right)$$

   Evaluating these formulae with the suitable constants $n_i = 10^{10}cm^{-3}$ and $V_T = 26mV$ as well as the given values for N_A and N_D, we have

$$\Phi(-x_{p_0}) \approx -538.81mV \qquad \Phi(x_{n_0}) \approx 419.07mV$$

   By the depletion approximation, in the interval $[-x_{p_0}, x_{n_0}]$, all free carriers recombine. Using the approximation that the concentration of free majority carriers in the n-type (p-type) material is N_D (N_A), this implies that in the interval $[-x_{p_0}, 0]$ we have a negative charge $-N_A q$ and in the interval $[0, x_{n_0}]$ we have a positive charge $N_D q$. We also have the constant $q = 1.602 * 10^{-19}C$. Hence, we can evaluate

$$\rho_- \approx -1.602\frac{C}{cm^3} \qquad \rho_+ \approx 16.02\frac{mC}{cm^3}$$

   The charge density profile is sketched in Figure 13. Note that the plot is not drawn in scale.
   Even though we do not know the lengths x_{n_0} and x_{p_0} yet, we know that $N_a q x_{p_0} = N_d q x_{n_0}$ as the complete PN-junction is electrically neutral. This allows us to state

$$x_{p_0} = \frac{N_d}{N_a} x_{n_0} \tag{1}$$

   For the same reason, we can assume that $E(-x_{p_0}) = 0$. As in the two previous problems, we use Gauss's law to get the function of the electric field

$$E(x) = \begin{cases} 0 & x < -x_{p_0} \\ \frac{\rho_-}{\epsilon}(x + x_{p_0}) & -x_{p_0} \leq x < 0 \\ \frac{\rho_-}{\epsilon} x_{p_0} + \frac{\rho_+}{\epsilon} x & 0 \leq x < x_{n_0} \\ 0 & x_{n_0} \leq x \end{cases} \tag{2}$$

   From the discussion on page 39 in the supplementary reader, you can infer that the voltage between x_{n_0} and x_{p_0} can be calculated

$$\Phi(x_{n_0}) - \Phi(-x_{p_0}) = \frac{qN_d}{2\epsilon} x_{n_0}^2 + \frac{qN_a}{2\epsilon} x_{p_0}^2 \tag{3}$$

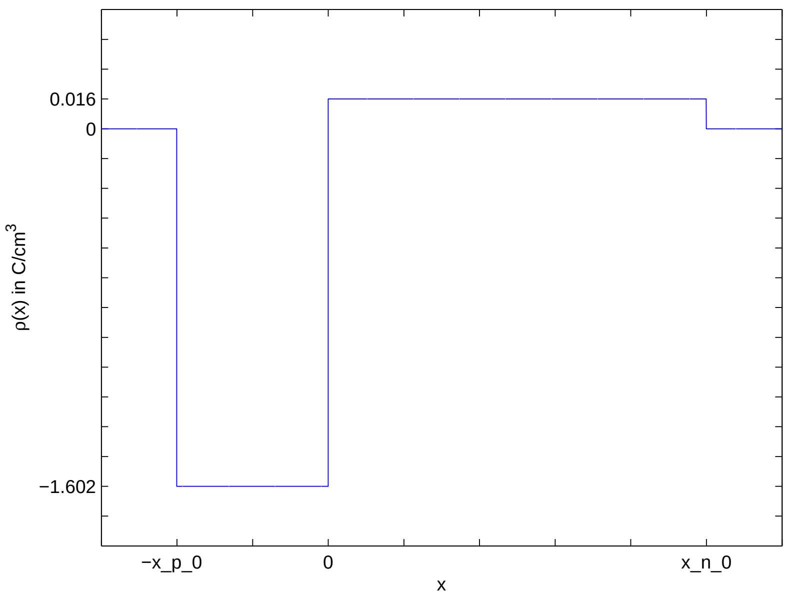

Figure 13: Charge Density Profile

From the discussion above, we also know that

$$\begin{aligned} \Phi_B &= \Phi(x_{n_0}) - \Phi(-x_{p_0}) \\ &= V_T \ln\left(\frac{N_d}{n_i}\right) - V_T \ln\left(\frac{n_i}{N_a}\right) \\ &= V_T \ln\left(\frac{N_d N_a}{n_i^2}\right) \end{aligned} \tag{4}$$

Equations (1), (3), and (4) can be combined to get the solution

$$x_{n_0} = \sqrt{\frac{2\epsilon\Phi_B}{qN_d}\frac{N_a}{N_a + N_d}}$$

and

$$x_{p_0} = \sqrt{\frac{2\epsilon\Phi_B}{qN_a}\frac{N_d}{N_a + N_d}}$$

plugging in the values, we get

$$x_{n_0} \approx 11.07\mu m \qquad x_{p_0} \approx 0.111\mu m$$

Lastly, we can use (2) to determine the maximum of the electric field which is at the junction ($x = 0$).

$$E_{\max} = E(0) = \frac{\rho_-}{\epsilon} x_{p_0} \approx -172\frac{kV}{cm}$$

10. Repeat the preceeding problem for a Ge PN-junction in thermal equilibrium at room temperature with no externally applied bias, assuming that $N_a = 10^{17} cm^{-3}$ and $N_d = 10^{16} cm^{-3}$ using the depletion approximation.
We can use exactly the same formulae as in the previous problem. However, note that the relevant constants of silicon and germanium are different. For germanium, we have

$$n_i = 2 * 10^{13} cm^{-3} \qquad \epsilon_r = 16$$

We can calulate

$$\rho_- \approx -16.02\frac{mC}{cm^3} \qquad \rho_+ \approx 1.602\frac{mC}{cm^3}$$

Refer to the solution of the previous problem for the function of the electric field.

$$\Phi_B = 383.03mV$$

$$x_{n_0} = 24.82\mu m$$

$$x_{p_0} = 2.482\mu m$$

$$E_{\max} = -28.07\frac{kV}{cm}$$