EECS40 RLC Lab guide

Introduction

Second-Order Circuits

Second order circuits have both inductor and capacitor components, which produce one or more resonant frequencies, ω_0. In general, a differential equation for the circuit can be written in the form:

$$\frac{d^2 v(t)}{dt^2} + 2\alpha \frac{dv(t)}{dt} + \omega_0^2 v(t) = f(t) \qquad \text{(Eq. 1)}$$

Where α and ω_0 depend on values for R, L, and C present in the circuit, and the forcing function *f(t)* depends on these values as well as voltage sources present. This differential equation follows the same form as that for a damped harmonic oscillator (ie. A mass oscillating due to a spring (resonance), but experiencing a damping force (friction), and possibly a driving force (on the mass to counteract the spring)). In the case of our second order circuit, the damping is produced by a resistor burning energy, resonance occurs between the inductive and capacitive components, and driving force would be a voltage source to force the circuit against the damping by the resistor.

For a series RLC circuit, the damping factor $\alpha = \frac{R}{2L}$ and resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

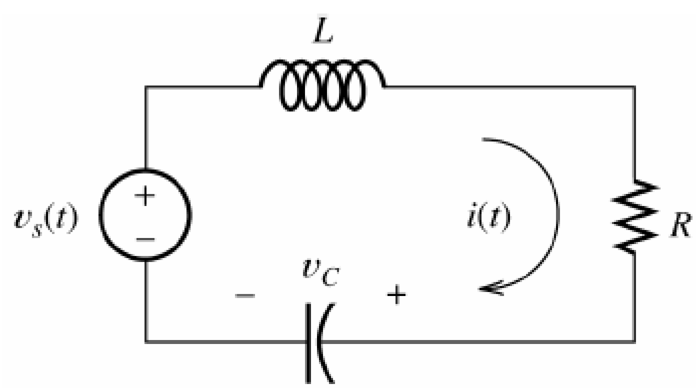


Figure 1: Series RLC circuit

The solution to the differential equation (Eq. 1) depends on these parameters α and ω_0, in that if:

$\alpha > \omega_0$ The circuit is **overdamped**, and the solution is:

$$v(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$\alpha = \omega_0$ The circuit is **critically damped**, and the solution is:

$$v(t) = K_1 e^{s_1 t} + K_2 t e^{s_2 t}$$

$\alpha < \omega_0$ The circuit is **underdamped**, and the solution is:

$$v(t) = e^{-\alpha t}(K_1 \cos(\omega_n t) + K_2 \sin(\omega_n t))$$

The roots s_1 and s_2 of the over- and critically damped solutions are given by:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

The natural frequency ω_n of the underdamped case is given by:

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2}$$

In this lab, we will observe the step response of a series RLC circuit. The step response is how the circuit behaves in response to a forcing function that is a step function (f(t) = 0 for t <0, and f(t) = 1 if t≥0) which we apply periodically to our circuit with a square wave. The solution is shown in Figure 2.6, where the values are normalized (amplitude of the forcing function divided out).

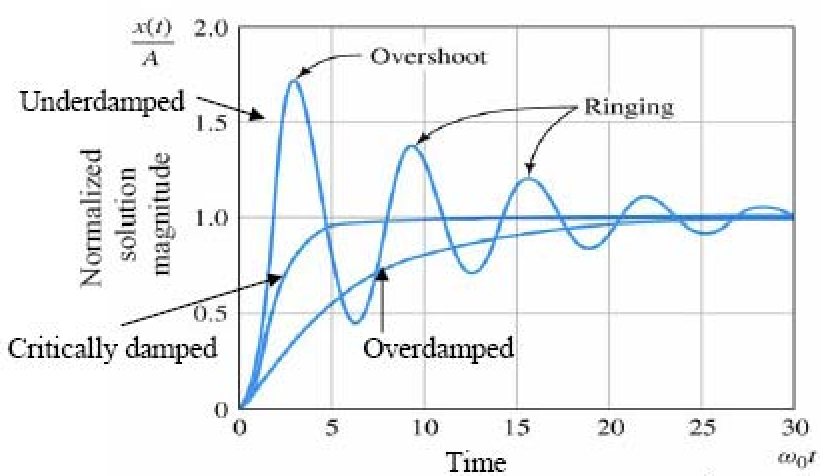


Figure 2: Normalized Step Response of 2nd order circuit

Quality Factor and Bandwidth

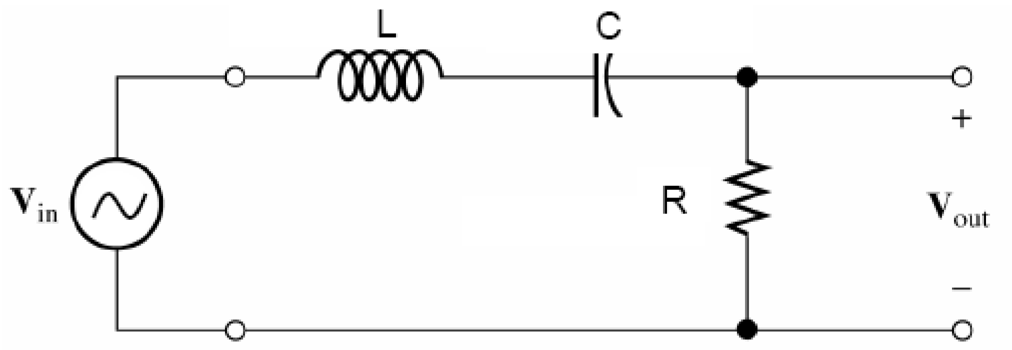


Figure 3: Series RLC Bandpass Filter, with AC source

The circuit shown in Figure 3 has a resonance frequency ω_0, which has the same value as shown on page 1. At resonance, the reactance of the capacitor cancels out the reactance of the inductor so they must be equal in magnitude.

The quality factor Q, of a series RLC filter is defined as the ratio of the inductive reactance to the resistance, at the resonant frequency:

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R}$$

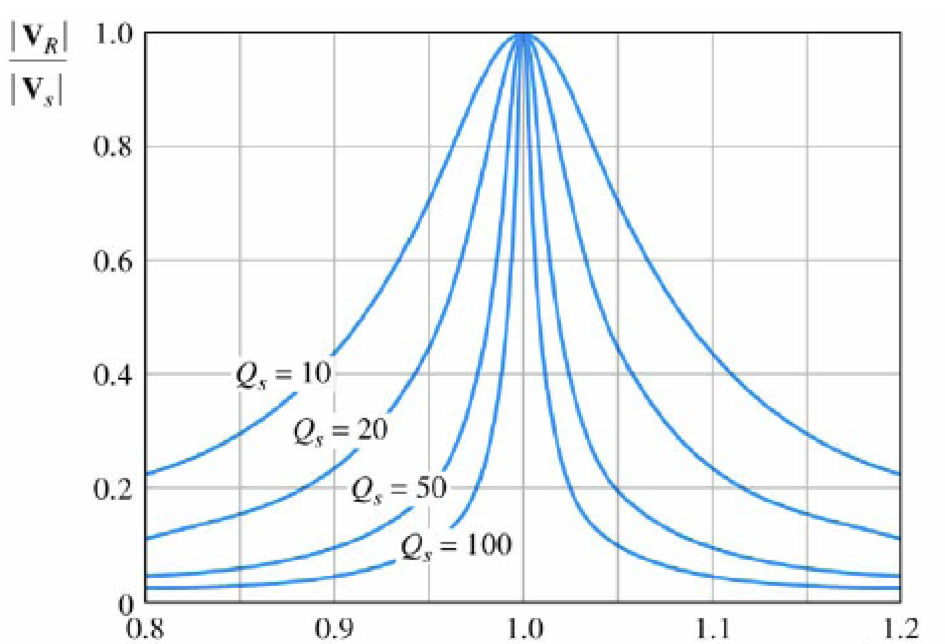


Figure 4: Plots of Transfer Function magnitude, for different values of Qs, the quality factor of a series RLC circuit. Note that this plot is not on a log-log scale (which Bode plots feature).

The bandwidth of a bandpass filter is the region between which the output is above half the maximum power. This is also the -3dB point, because in decibels, 10 log 0.5 ≈ -3, where 0.5 comes from the power ratio, or $|H(\omega)|^2$. The bandwidth, $\mathbf{B}$, of a series bandpass filter is related to quality factor, Q, by the equation:

$$\mathbf{B} = \frac{f_0}{Q}$$

The voltage measured at the half-power frequency should be ~0.707, or $\sqrt{0.5}$ of the maximum voltage, because power is proportional to the square of voltage.

Phasors and Complex Impedance

Phasors can be used to represent the complex amplitude of a sinusoidal function. For example:

$$A\cos(\omega t + \phi) = A\angle\phi$$

Impedance measures the opposition of a circuit to time varying current. The units of impedance are ohms. A resistor's impedance is simply it's resistance. Capacitors and inductors have complex impedances:

$$\text{Capacitor impedance} = \frac{1}{j\omega C}$$

$$\text{Inductor impedance} = j\omega L$$

An RLC circuit that is driven by a sinusoidal voltage source can be analyzed using KVL, KCL, and Ohm's law. The rules that apply to resistors apply to the complex impedances of the elements of the circuit. For example, V_{out} in Figure 3 can be found easily using the voltage divider equation with the circuit element's impedances. (Doing this is part of a prelab problem)

Hands On
LC Circuit
For this part of the lab, you will need to set your function generator to generate pulses at regular intervals. First, create a square wave with a frequency of 100 Hz, 3 V_{pp}, and a Voltage offset of 1.5V (the offset makes the square wave range between 0V and 6V). Hit the shift key, then the key with “burst” written in small blue letters above it. Use the oscilloscope to make sure the output your function generator looks something like this:

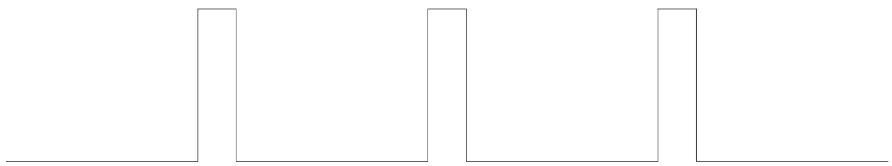


Figure 5: pulses created by the function generator

Now, Attach a 10 μF capacitor, as shown in figure 2. (make sure to check if the capacitor is polarized)

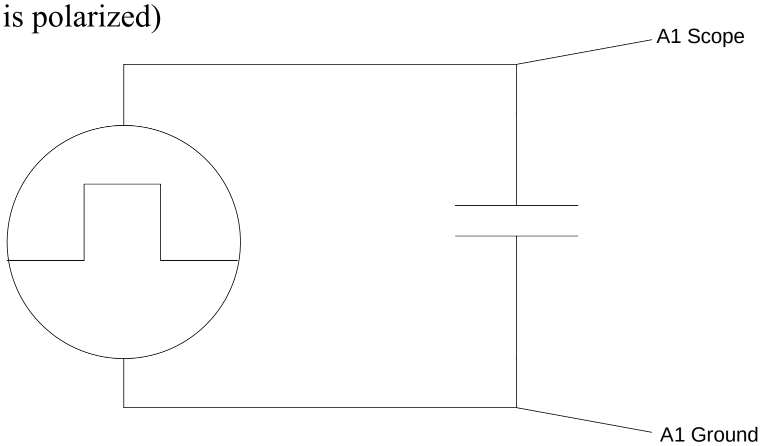


Figure 6: Capacitor attached to a pulse generator

Question 1: Sketch the voltage across the capacitor.

Now, add a 1 mH inductor in parallel with the capacitor, as shown in figure 3.

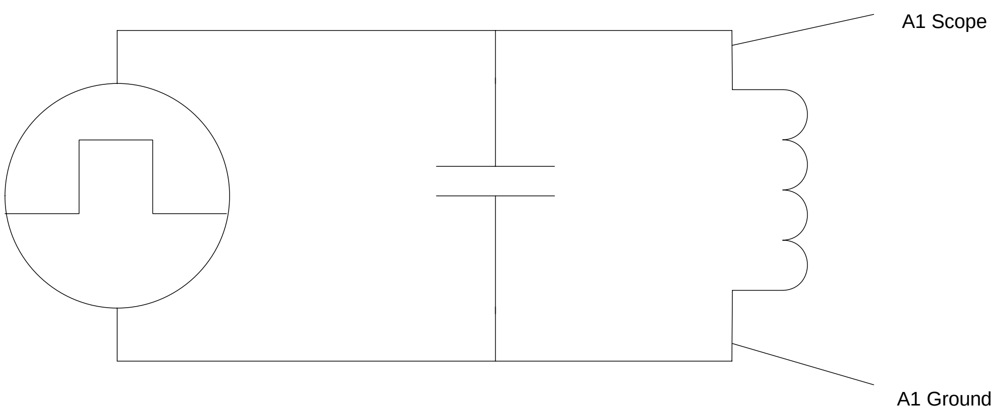


Figure 7: inductor and capacitor attached to a pulse generator

You may have to move the trigger level up or down to get a still picture. If that doesn't work, you can always hit the stop button to freeze the image. DON'T rescale the image, you will need to graph the new output on the same graph as question 1.

Question 2: Sketch the voltage across the capacitor and inductor in parallel, and explain what you see. (Use the same graph as question 1)

Increase the frequency of the function generator to 2 KHz.

Question 3: Sketch what you see when the inductor is removed (like Figure 6). Sketch what you see when the inductor is present (like Figure 7).

Use the two time cursers (hit the curser key, then use the time softkey) to estimate the period of the oscillation you see on your oscilloscope. (You may have to extrapolate the period from a trough to peak (1/4th of a period) to get the most accurate answer)

Question 4: Use the information from the time cursors to estimate the frequency of the oscillation.

Remove the 1 mH inductor and replace it with a 10 mH inductor.

Question 5: Sketch the new waveform.

Series RLC Circuit

Construct the circuit below with a 1 nF Capacitor, 100 mH inductor, and a 2.2 KΩ resistor. Use a sinusoidal input with 6 V_{pp} and 0V offset. Determine the resonance frequency, f_0, and set the function generator to that frequency.

NOTE: make sure you turn off "burst" before continuing.

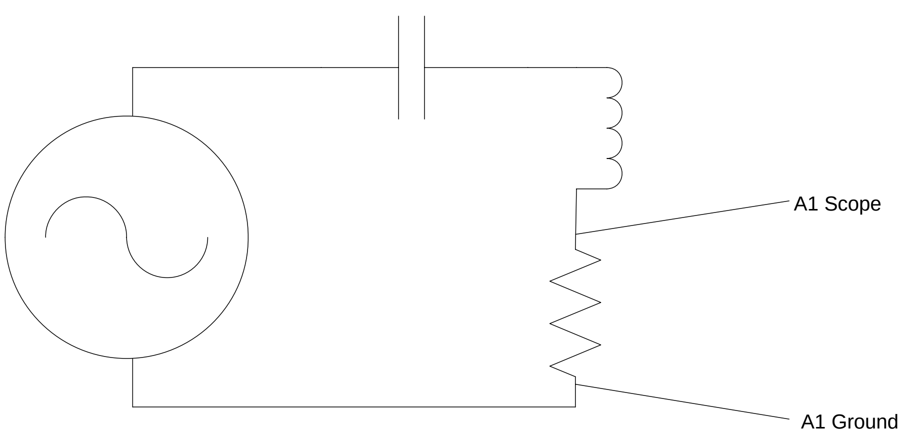


Figure 8: Series RLC circuit

Have the oscilloscope display the resistor's peak to peak voltage, V_{pp}, by hitting the measurement key, then the V_{pp} softkey. Question 6 of the lab report has a table of values to fill on for various frequencies.

Question 7: Once you have filled in the first table of values, you will have to find the true f_0 and -3dB frequencies. To find f_0, sweep the function generator's frequency slowly above and below the calculated f_0 until you find the highest V_{pp}. Multiply this number by .7 to find the -3dB value, and find the frequencies above and below f_0 that that value.

Question 8: If you had any discrepancies, what do you think caused them?

Damping analysis

Construct the circuit from pre-lab (shown below) using L = 10 mH, C = 1nF. For the values of R = 200 Ω, 6.3 kΩ, and 6.9 kΩ, measure your resistors, and fill in the chart on the lab report with calculated values. Apply a V_{in} square wave at 6.0 V_{pp} on the function generator (remember that the actual output will be 12 V_{pp}). For the overdamped and critically damped cases, use a frequency of 3 kHz. For the underdamped case, we need a slower frequency and longer period to observe the damped oscillation, so use a frequency of 1 kHz.

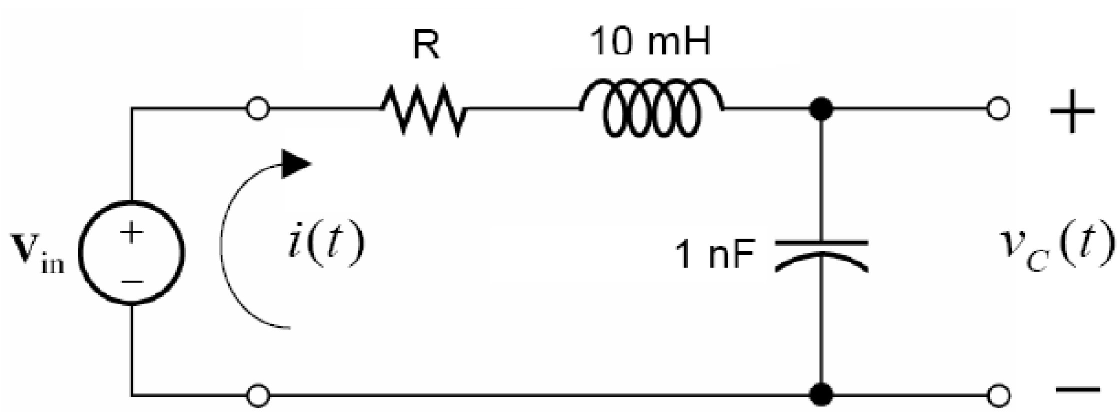
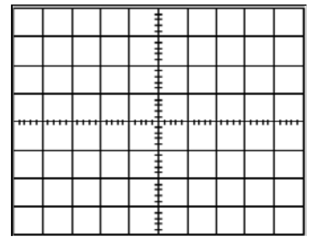


Figure 9: Series RLC Circuit, with DC source

Use the oscilloscope to view the voltage across the capacitor. Make sure you zoom in on the voltage and time scales to observe the damping of the signal. You may need to reposition the plot (adjust the time and/or voltage offsets) to get a more complete view of the waveform. It will help to raise the trigger level to a level that will provide a stable waveform. Also, you can press the Stop button on the oscilloscope to freeze a frame for better viewing.

Question 9: Fill in below with determined values, and plot the observed waveforms.

Values **Waveform**



Overdamped (R, α)
Critically damped (R, α)
Underdamped (R, α, ω_n)

By zooming in on the waveform, and using the cursors (like we did for the RC circuit), we can find the oscillation frequency for the underdamped circuit. By finding the Δt for one period, we can obtain the frequency of the signal (Hz) by $f = 1/\Delta t$. We may also find the angular frequency by the relation $\omega = 2\pi f$

Question 10: For the underdamped case, what is the measured observed oscillation frequency in Hz and rad/s? (Show your calculations) How does this compare to the resonant frequency, ω_0? How does it compare to the natural frequency, ω_n? Is the damping observed in the underdamped case actually occurring at the rate our solution predicts? Use the cursors to find the voltages of 3 consecutive peaks in the waveform. Let the first value be your maximum voltage, say at time $t = 0$. These peaks occur periodically after the maximum voltage, and we know the period of the signal. Thus, we can predict when and how high the subsequent peaks should be, by using our expression for the underdamped solution: they should decay at a rate proportional to $e^{-\alpha t}$.

Question 11: According to the 3 observed peaks, do the consecutive values decay as predicted? List your observed values and predicted values based on the first, reference value. Explain why or why not the predictions are observed.