EE40
Lecture 10
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2/13/08

Reading: Chap. 3

Chapter 3

- Outline
  - The capacitor
  - The inductor

The Capacitor

Two conductors (a,b) separated by an insulator:

difference in potential = V_{ab}

=> equal & opposite charge ***Q*** on conductors

$$Q = CV_{ab}$$

(stored charge on each plate in terms of voltage)

where ***C*** is the **capacitance** of the structure,

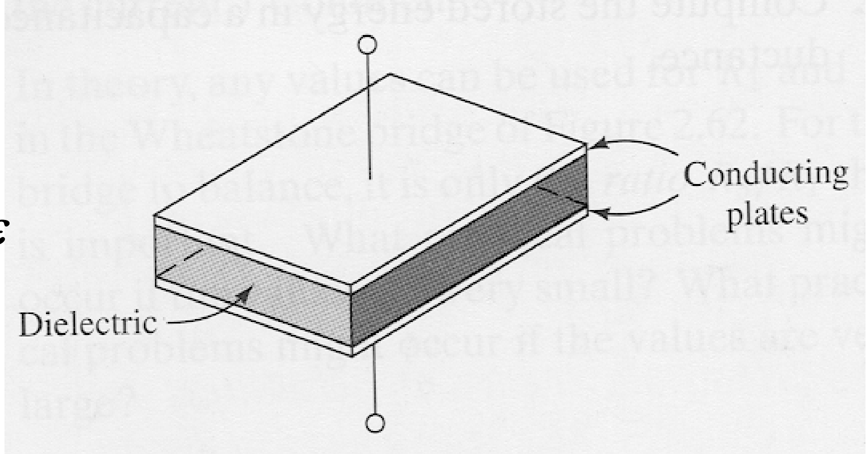
- positive (+) charge is on the conductor at higher potential.
- the net charge is zero.

Parallel-plate capacitor:

- area of the plates = A ***(m²)***
- separation between plates = d ***(m)***
- ***dielectric permittivity*** of insulator = ε ***(F/m)***

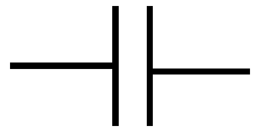
=> capacitance

$$C = \frac{A\varepsilon}{d} \quad (F)$$



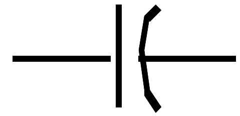
Capacitor

Symbol:

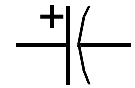


C

or



C



C

Electrolytic (polarized) capacitor

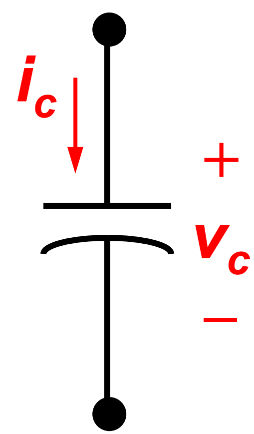
These have high capacitance and cannot support voltage drops of the wrong polarity

Units: **Farads (Coulombs/Volt)**

(typical range of values: 1 pF to 1 μF; for "supercapa-citors" up to a few F!)

Current-Voltage relationship:

$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt}$$



To write this it is important to have use a passive convention, otherwise you need a minus sign.

Note: v_c **must be a continuous function of time since the charge stored on each plate cannot change suddenly**

Voltage in Terms of Current

$$Q(t) = \int_0^t i_c(t)dt + Q(0)$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t)dt + \frac{Q(0)}{C} = \frac{1}{C}\int_0^t i_c(t)dt + v_c(0)$$

Uses: Capacitors are used to store energy for camera flashbulbs, in filters that separate various frequency signals, and they appear as undesired "parasitic" elements in circuits where they usually degrade circuit performance

At higher frequencies capacitors become increasingly like short circuits

Stored Energy

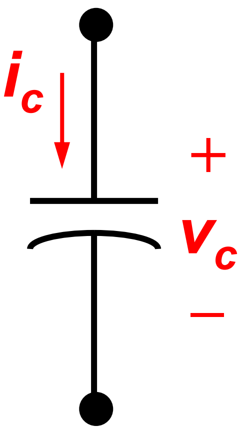
CAPACITORS STORE ELECTRIC ENERGY

During charging, the average voltage across the capacitor was only half the final value of ***V*** for a linear capacitor.

Thus, the energy needed to build up the charge is $\frac{1}{2}QV = \boxed{\frac{1}{2}CV^2}$.

<u>Example</u>: A 1 pF capacitance charged to 5 Volts has ½$(5V)^2$ (1pF) = 12.5 pJ (A 5F supercapacitor charged to 5 volts stores 63 J; if it discharged at a constant rate in 1 ms energy is discharged at a 63 kW rate!)

A more rigorous derivation



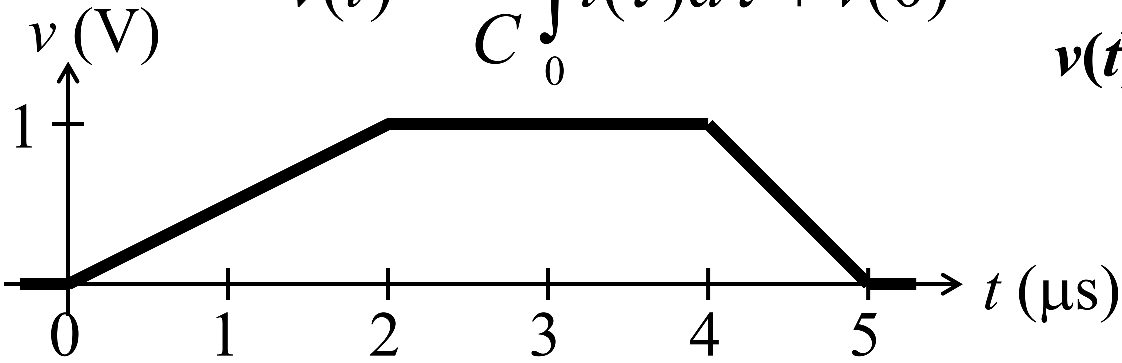
This derivation holds independent of the circuit!

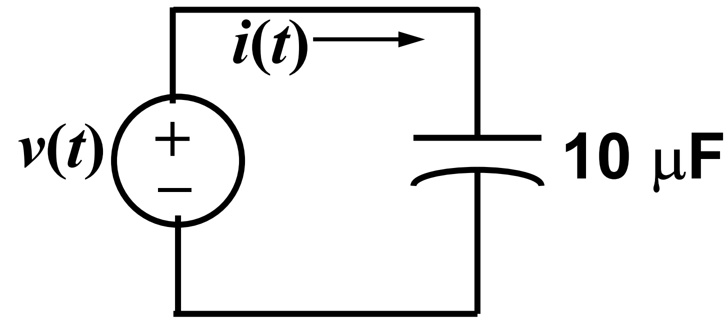
$$w = \int_{t=t_{Initial}}^{t=t_{Final}} v_c \cdot i_c \, dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \frac{dQ}{dt} dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \, dQ$$

$$w = \int_{v=V_{Initial}}^{v=V_{Final}} C v_c \, dv_c = \frac{1}{2} C {V_{Final}}^2 - \frac{1}{2} C {V_{Initial}}^2$$

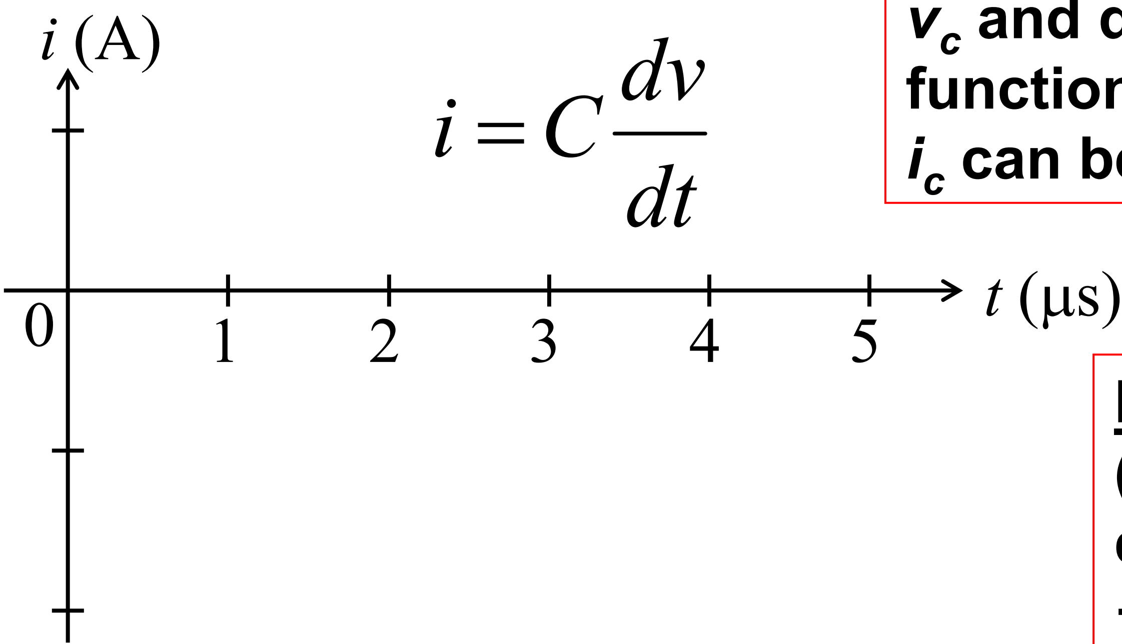
Example: Current, Power & Energy for a Capacitor

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0)$$



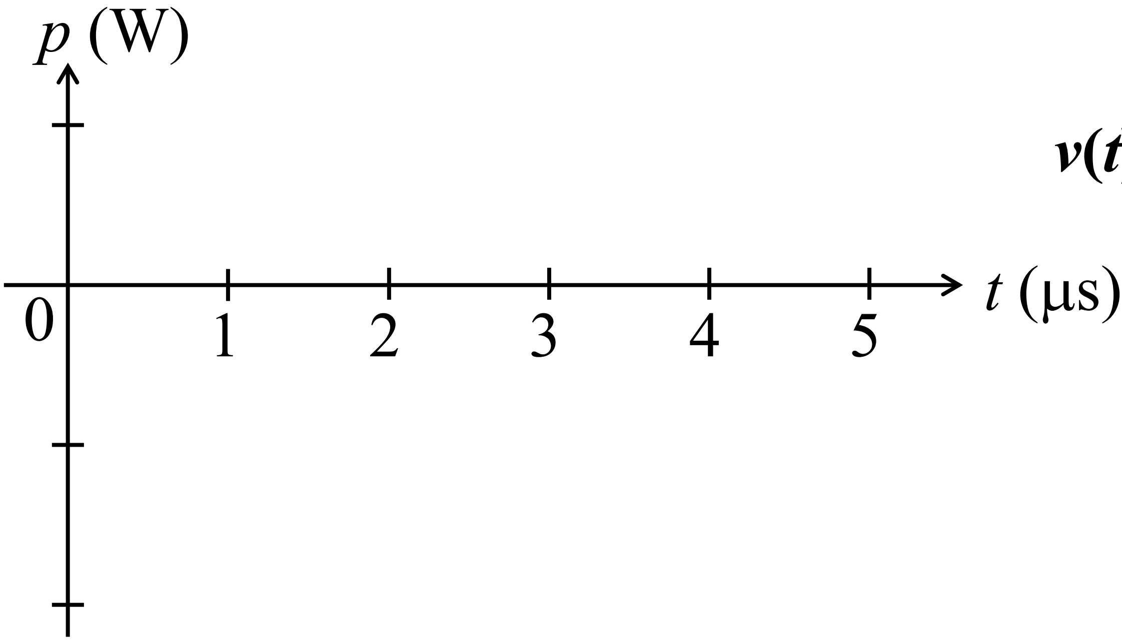


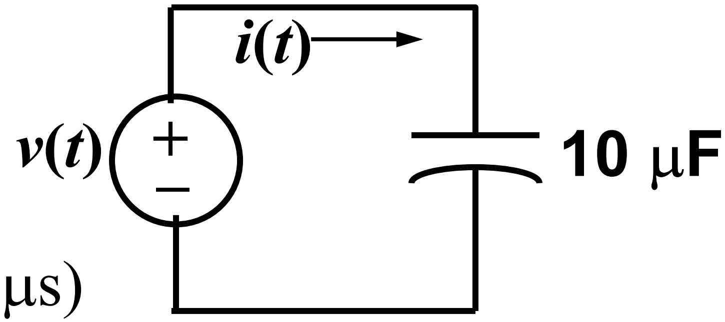
$$i = C\frac{dv}{dt}$$



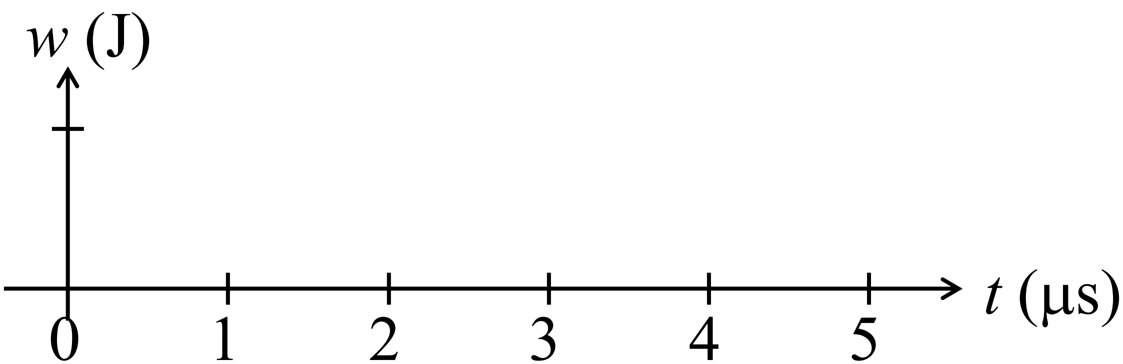
v_c and q must be continuous functions of time; however, i_c can be discontinuous.

<u>Note</u>: In "steady state" (dc operation), time derivatives are zero → *C* is an open circuit



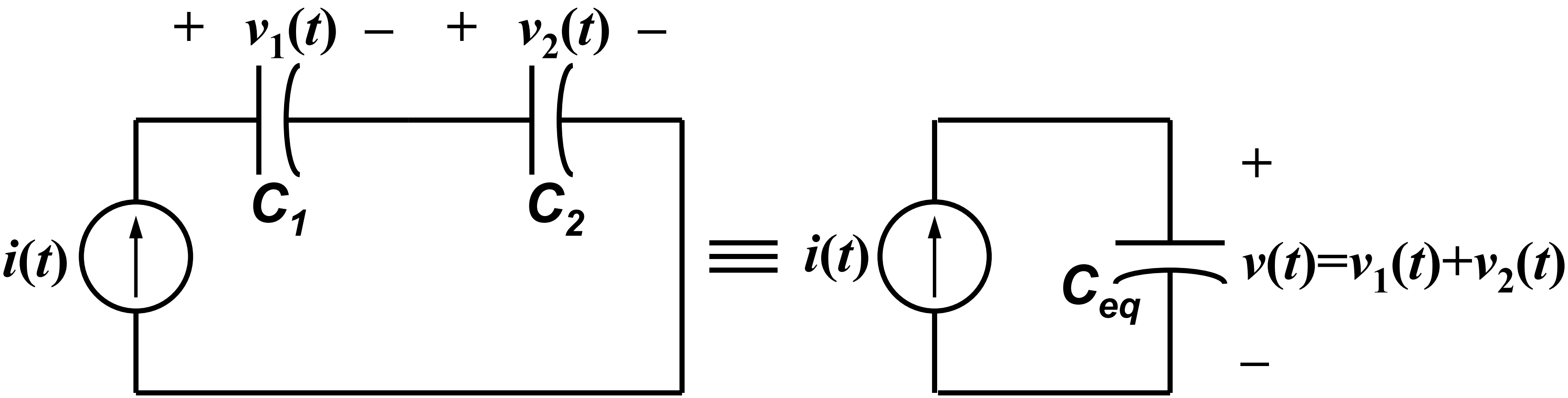


$$p = vi$$



$$w = \int_0^t p\,d\tau = \frac{1}{2}Cv^2$$

Capacitors in series and parallel



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Similarly, for capacitors in parallel, the capacitance adds.

Inductor

Symbol:

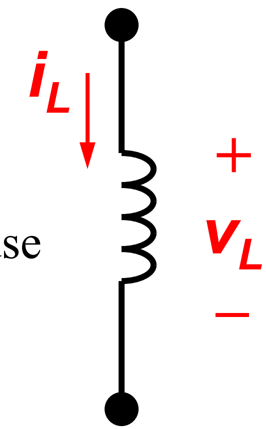
L

Units: **Henrys (Volts • second / Ampere)**

(typical range of values: μH to 10 H)

Current in terms of voltage:

$$di_L = \frac{1}{L} v_L(t)dt$$



To write this it is important to use the passive configuration.

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)d\tau + i(t_0)$$

Note: i_L **must be a continuous function of time because magnetic flux cannot change suddenly**

Stored Energy

INDUCTORS STORE MAGNETIC ENERGY

Consider an inductor having an initial current $i(t_0) = i_0$

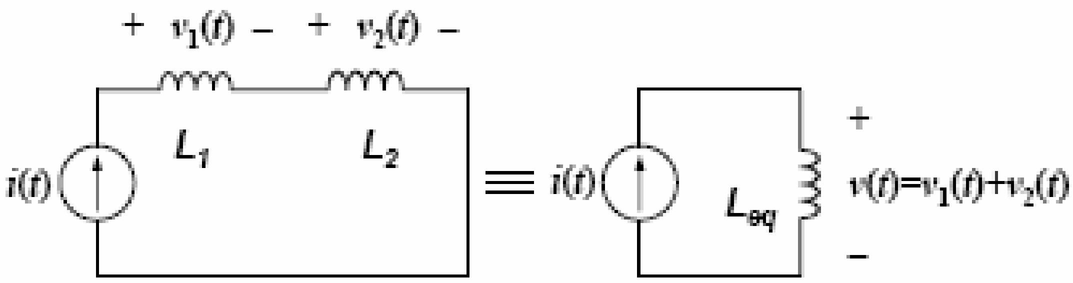
$$p(t) = v(t)i(t) =$$

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau =$$

$$w(t) = \frac{1}{2}Li^2 - \frac{1}{2}Li_0^{\ 2}$$

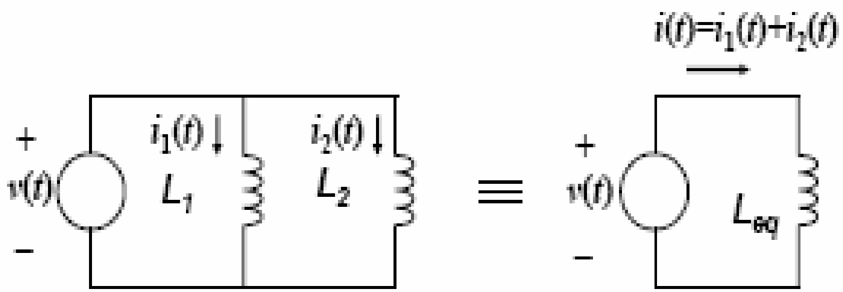
At higher frequencies inductors behave increasingly like open circuits.

Inductors in Series and Parallel



Common Current

$$L_{eq} = L_1 + L_2$$



Common Voltage

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Summary

Capacitor

$$i = C\frac{dv}{dt}; w = \frac{1}{2}Cv^2$$

v cannot change instantaneously

i can change instantaneously

Do not short-circuit a charged capacitor (-> infinite current!)

n cap.'s in series: $$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

n cap.'s in parallel: $$C_{eq} = \sum_{i=1}^{n} C_i$$

In steady state (not time-varying), a capacitor behaves like an open circuit.

Inductor

$$v = L\frac{di}{dt}; w = \frac{1}{2}Li^2$$

i cannot change instantaneously

v can change instantaneously

Do not open-circuit an inductor with current (-> infinite voltage!)

n ind.'s in series: $$L_{eq} = \sum_{i=1}^{n} L_i$$

n ind.'s in parallel: $$\frac{1}{L_{eq}} = \sum_{i=1}^{n} \frac{1}{L_i}$$

In steady state, an inductor behaves like a short circuit.