EE40
Lecture 15
Venkat Anantharam

2/29/08

Reading: Chap. 5: phasors

Impedance

- AC steady-state analysis using phasors allows us to express the relationship between current and voltage using a formula that looks likes Ohm’s law:

$$\mathbf{V} = \mathbf{I}\,\mathbf{Z}$$

- **Z** is called impedance.

Some Thoughts on Impedance

- Impedance depends on the frequency ω.
- Impedance is (often) a complex number.
- Impedance allows us to use the same solution techniques for AC steady state as we use for DC steady state.

Resistor I-V relationship

$v_R = i_R R$ .... $\mathbf{V}_R = \mathbf{I}_R R$ where R is the resistance,

$\mathbf{V}_R$ = phasor voltage, $\mathbf{I}_R$ = phasor current,

hence $\mathbf{Z}_R = R$

Capacitor I-V relationship

$i_C = C dv_C/dt$ ....Phasor current $\mathbf{I}_C$ = phasor voltage $\mathbf{V}_C$ /

capacitive impedance $\mathbf{Z}_{C:}$ → $\mathbf{I}_C = \mathbf{V}_C/\mathbf{Z}_C$

where $\mathbf{Z}_C = 1/j\omega C$

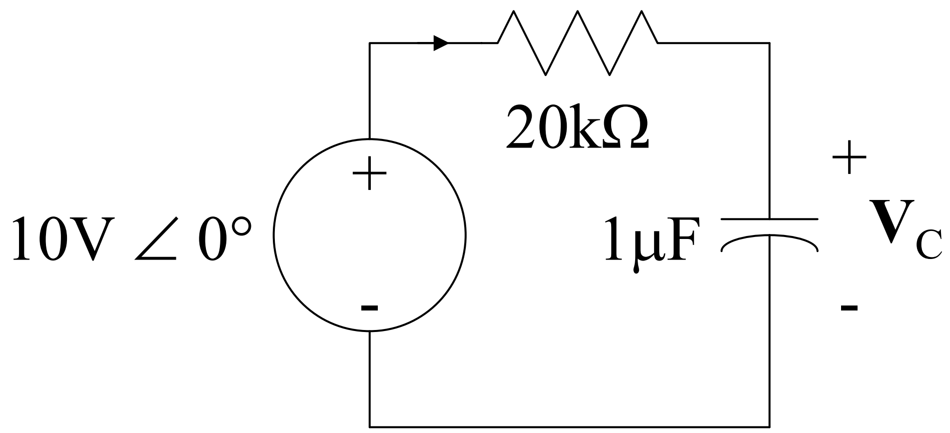
Inductor I-V relationship

$v_L = L di_L/dt$ ....Phasor voltage $\mathbf{V}_L$ = phasor current $\mathbf{I}_L$/

inductive impedance $\mathbf{Z}_L$ → $\mathbf{V}_L = \mathbf{I}_L \mathbf{Z}_L$

where $\mathbf{Z}_L = j\omega L$

Example: Single Loop Circuit



f=60 Hz, $\mathbf{V}_C$=?

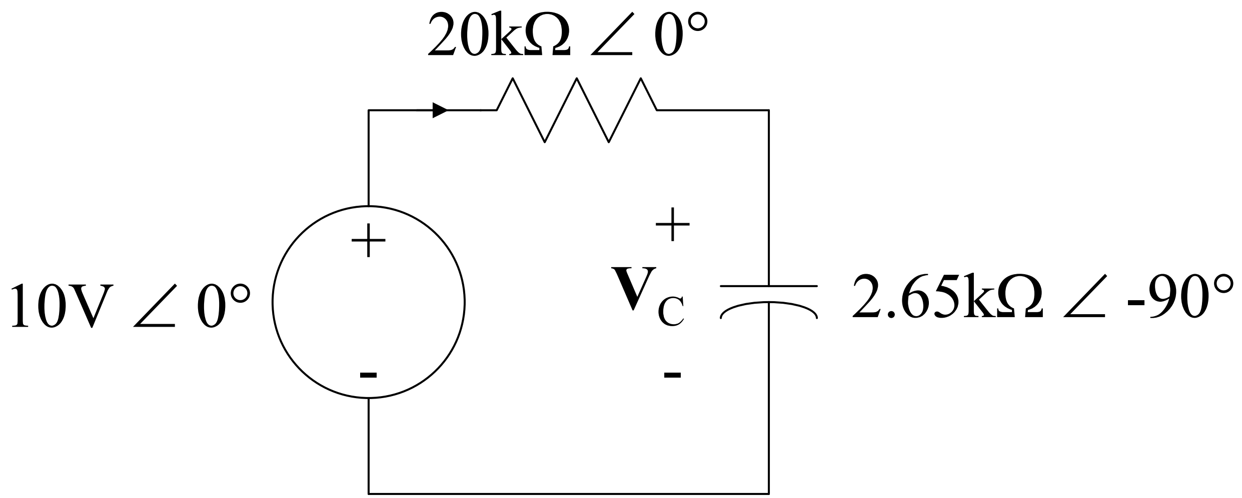
How do we find $\mathbf{V}_C$?

First compute impedances for resistor and capacitor:

$\mathbf{Z}_R = \text{R} = 20\text{k}\Omega = 20\text{k}\Omega \angle 0°$

$\mathbf{Z}_C = 1/j\,(2\pi\text{f x } 1\mu\text{F}) = 2.65\text{k}\Omega \angle -90°$

Impedance Example

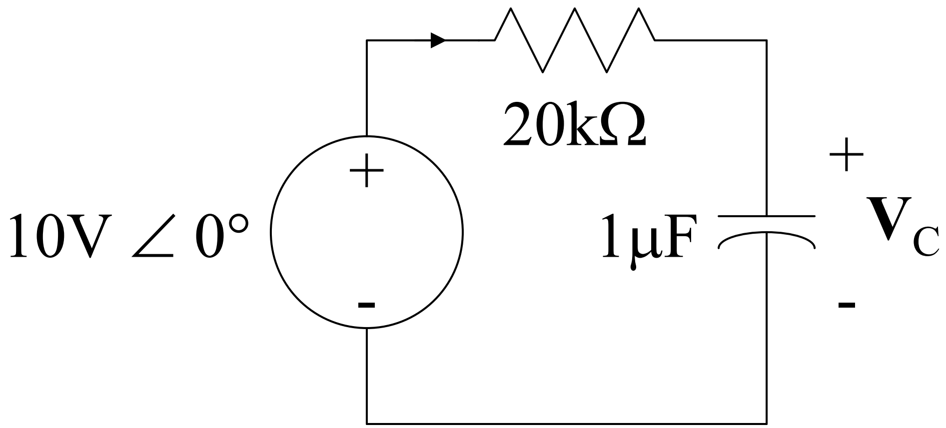


Now use the voltage divider to find $\mathbf{V}_C$:

$$\mathbf{V}_C = 10\text{V} \angle 0° \left(\frac{2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right)$$

$$\mathbf{V}_C = 1.31\text{V} \angle -82.4°$$

What happens when ω changes?



$\omega = 10$

Find $\mathbf{V}_C$

Circuit Analysis Using Complex Impedances

- Suitable for AC steady state.
- KVL

$$v_1(t) + v_2(t) + v_3(t) = 0$$

$$V_1 \cos(\omega t + \theta_1) + V_2 \cos(\omega t + \theta_2) + V_3 \cos(\omega t + \theta_3) = 0$$

$$\mathrm{Re}\left[V_1 e^{j(\omega t + \theta_1)} + V_2 e^{j(\omega t + \theta_2)} + V_3 e^{j(\omega t + \theta_3)} \right] = 0$$

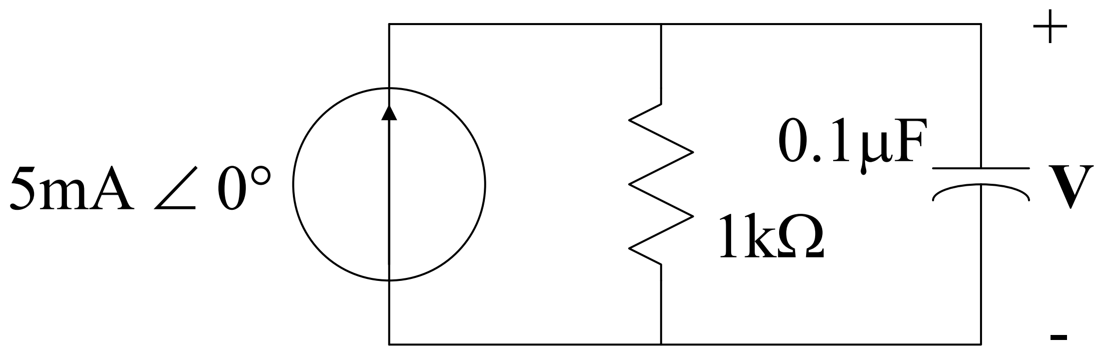
Phasor Form KVL

$$V_1 e^{j(\theta_1)} + V_2 e^{j(\theta_2)} + V_3 e^{j(\theta_3)} = 0$$

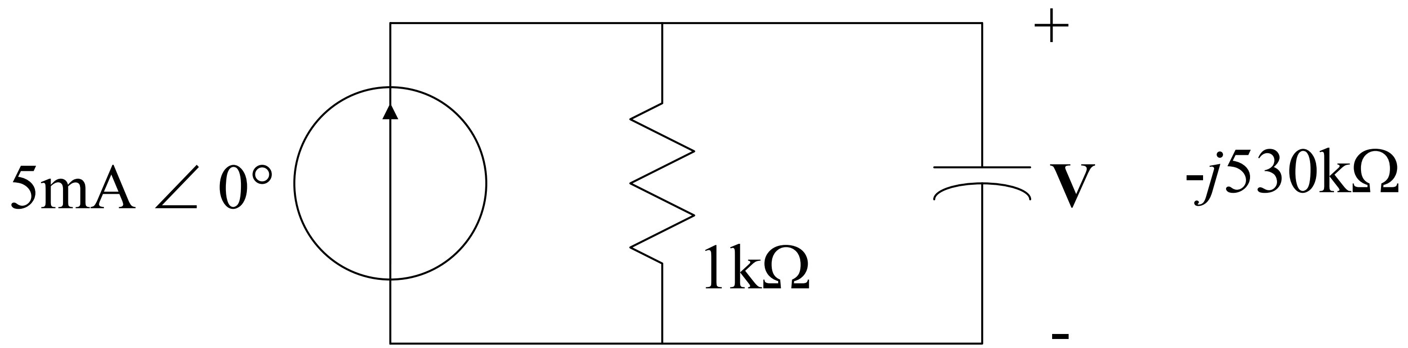
$$\mathbf{V_1} + \mathbf{V_2} + \mathbf{V_3} = 0$$

- Phasor Form KCL $\mathbf{I_1} + \mathbf{I_2} + \mathbf{I_3} = 0$
- Use complex impedances for inductors and capacitors and follow same analysis as in chap 2.

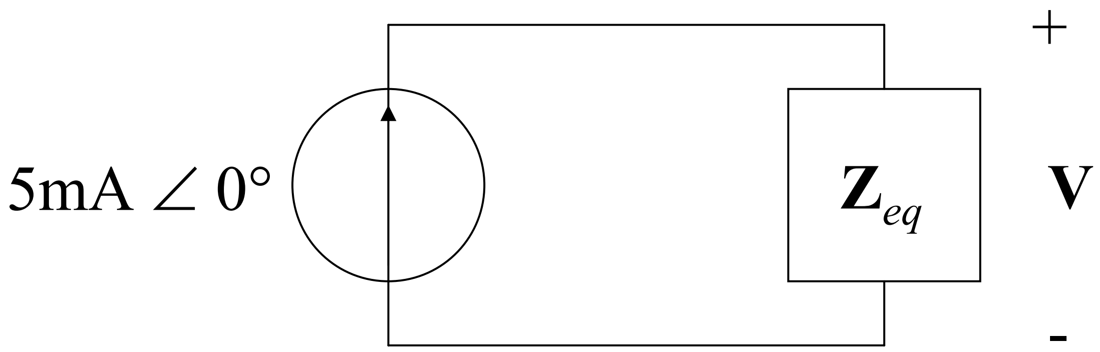
Steady-State AC Analysis



Find $v(t)$ for $\omega = 2\pi\ 3000$



Find the Equivalent Impedance



$$\mathbf{Z}_{eq} = \frac{1000(-j530)}{1000 - j530} = \frac{10^3 \angle 0° \times 530 \angle -90°}{1132 \angle -27.9°}$$

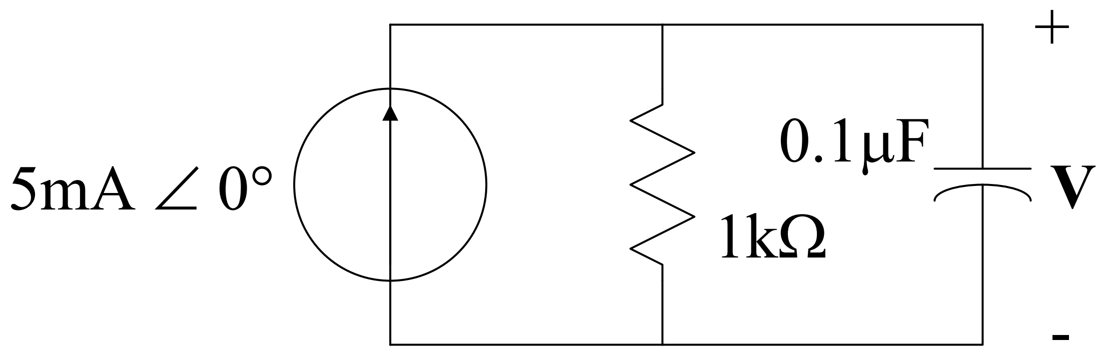
$$\mathbf{Z}_{eq} = 468.2\Omega \angle -62.1°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = 5\text{mA} \angle 0° \times 468.2\Omega \angle -62.1°$$

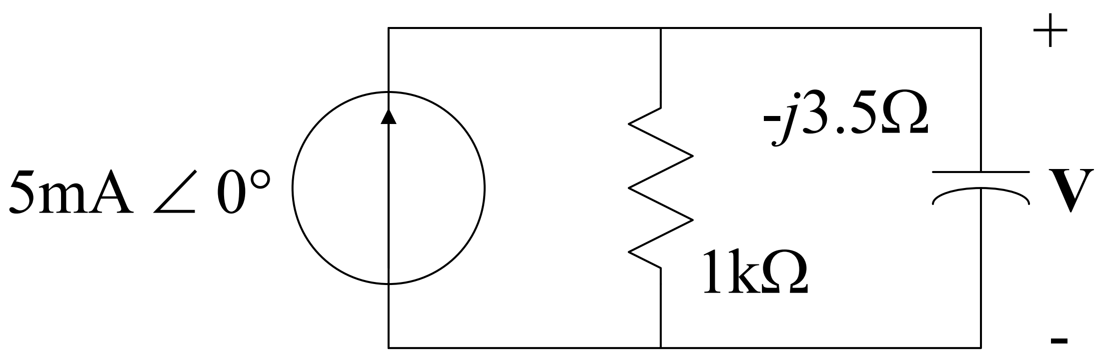
$$\mathbf{V} = 2.34\text{V} \angle -62.1°$$

$$v(t) = 2.34\text{V} \cos(2\pi 3000\text{t} - 62.1°)$$

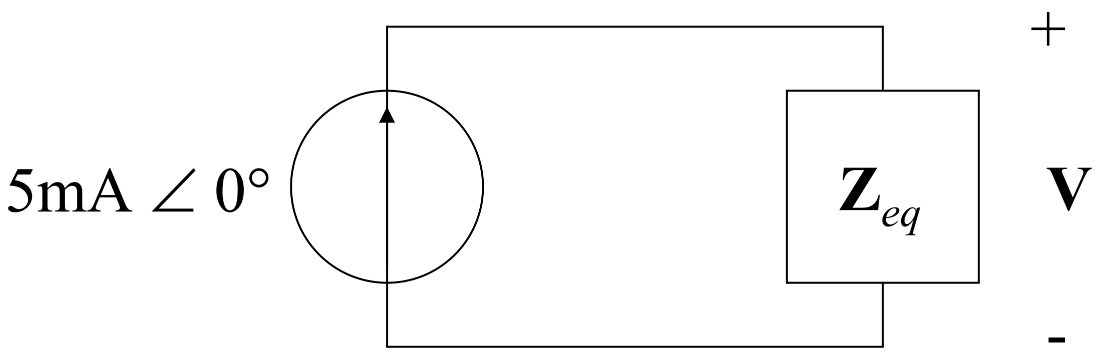
Change the Frequency



Find $v(t)$ for $\omega=2\pi\ 455000$



Find an Equivalent Impedance



$$\mathbf{Z}_{eq} = \frac{1000(-j3.5)}{1000 - j3.5} = \frac{10^3\angle 0° \times 3.5\angle -90°}{1000\angle -0.2°}$$

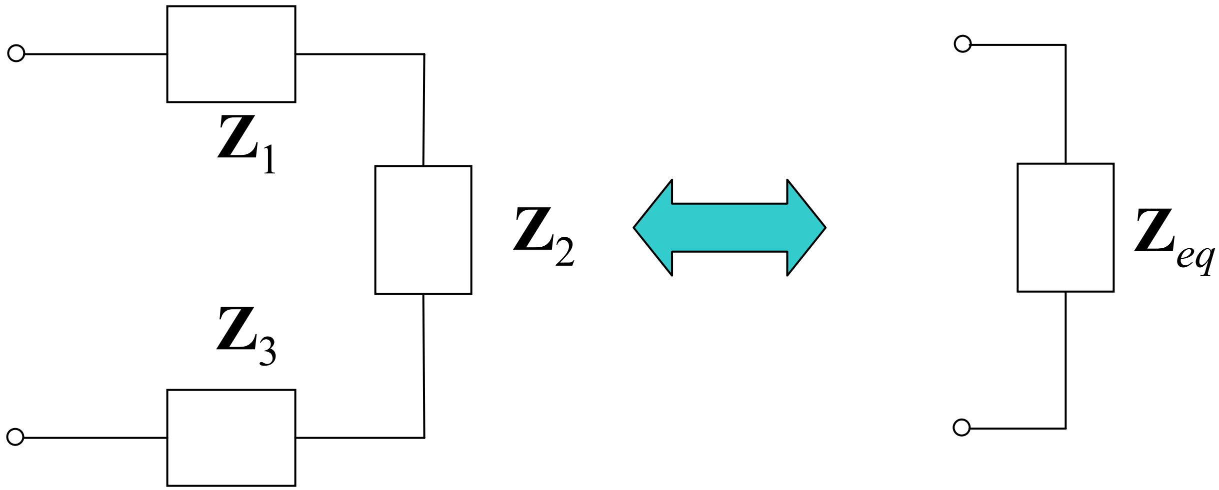
$$\mathbf{Z}_{eq} = 3.5\Omega\angle -89.8°$$

$$\mathbf{V} = \mathbf{IZ}_{eq} = 5\text{mA}\angle 0° \times 3.5\Omega\angle -89.8°$$

$$\mathbf{V} = 17.5\text{mV}\angle -89.8°$$

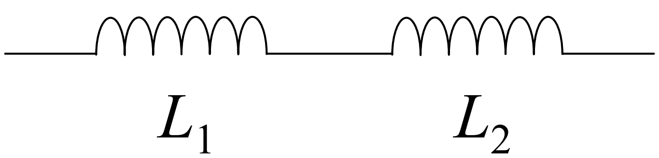
$$v(t) = 17.5\text{mV}\cos(2\pi 455000\text{t} - 89.8°)$$

Series Impedance

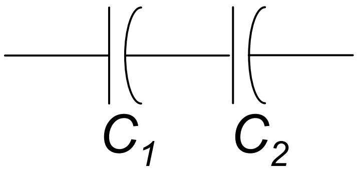


$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$

For example:

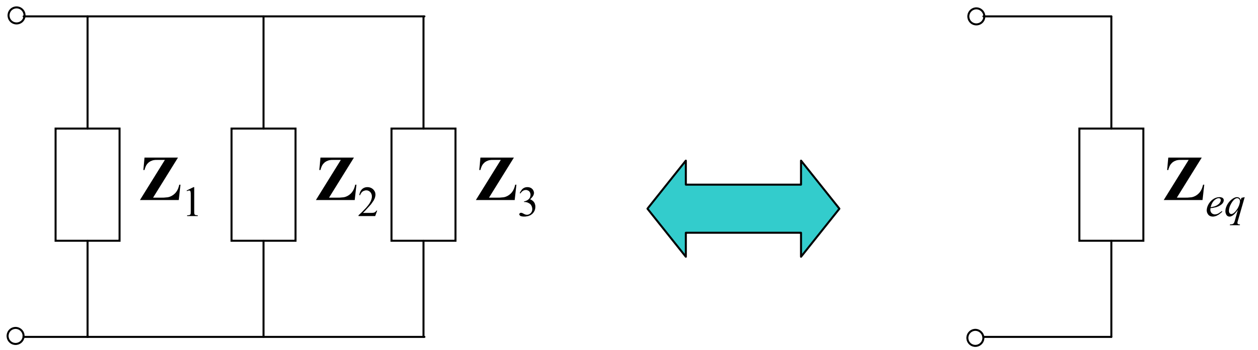


$$Z_{eq} = j\omega(L_1 + L_2)$$



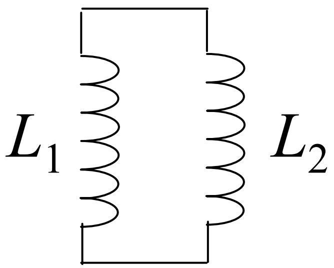
$$Z_{eq} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}$$

Parallel Impedance

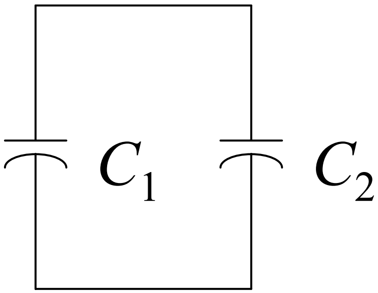


$$1/\mathbf{Z}_{eq} = 1/\mathbf{Z}_1 + 1/\mathbf{Z}_2 + 1/\mathbf{Z}_3$$

For example:



$$\mathbf{Z}_{eq} = j\omega \frac{L_1 L_2}{(L_1 + L_2)}$$



$$\mathbf{Z}_{eq} = \frac{1}{j\omega(C_1 + C_2)}$$