EE40
Lecture 17
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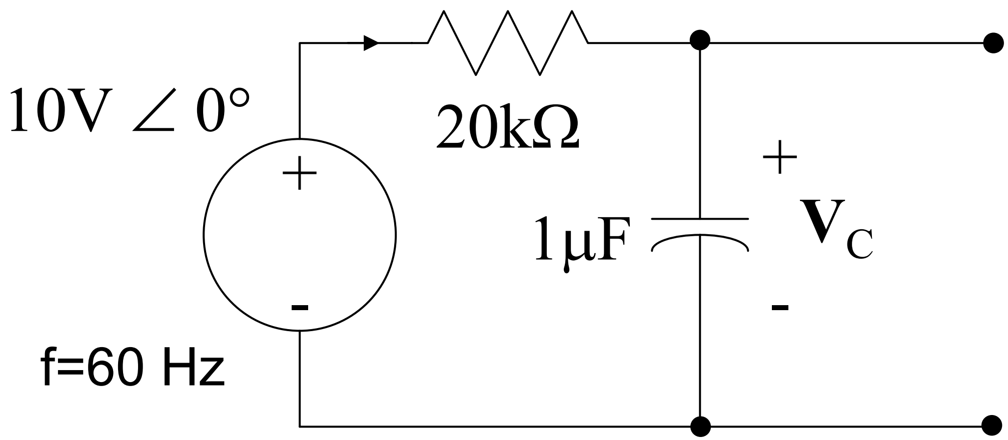
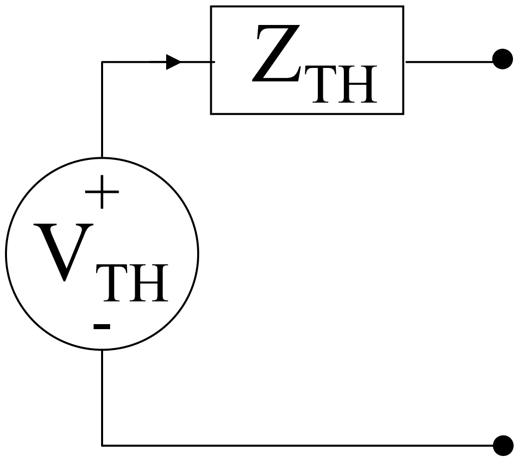
3/05/08

Reading: Chap. 5: phasors.
Chap. 6: Bode plots.

Thevenin Equivalent





$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°

$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

$$\mathbf{V}_{TH} = \mathbf{V}_{OC} = 10\text{V} \angle 0^\circ \left(\frac{2.65\text{k}\Omega \angle -90^\circ}{2.65\text{k}\Omega \angle -90^\circ + 20\text{k}\Omega \angle 0^\circ} \right) = 1.31 \angle -82.4$$

$$\mathbf{Z}_{TH} = \mathbf{Z}_R \,||\, \mathbf{Z}_C = {}^\circ \left(\frac{20\text{k}\Omega \angle 0^\circ \cdot 2.65\text{k}\Omega \angle -90^\circ}{2.65\text{k}\Omega \angle -90^\circ + 20\text{k}\Omega \angle 0^\circ} \right) = 2.62 \angle -82.4$$

Root Mean Square (rms) Values

- rms valued defined as

$$v_{RMS} = \sqrt{\frac{1}{T}\int_{o}^{T} v^2(t)dt} \qquad \text{T} = \text{period}$$

- Assuming a sinusoid gives

$$v_{RMS} = \sqrt{\frac{1}{T}\int_{o}^{T} v_m{}^2 \cos^2(\omega t + \theta)dt}$$

- Using a trigonometric identity gives

$$v_{RMS} = \sqrt{\frac{v_m{}^2}{2T}\int_{o}^{T} [1 + \cos(2\omega t + 2\theta)]dt}$$

- Evaluating at limits gives

$$v_{RMS} = \sqrt{\frac{v_m{}^2}{2T}[T + \frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2w}\sin(2\theta)]} \qquad v_{RMS} = \frac{v_m}{\sqrt{2}}$$

Power: Instantaneous and Time-Average

For a Resistor

- The instantaneous power is

$$p(t) = v(t)i(t) = \frac{v(t)^2}{R}$$

- The time-average power is

$$P_{AVE} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T \frac{v(t)^2}{R}dt = \frac{1}{R}[\frac{1}{T}\int_0^T v(t)^2 dt] = \frac{v_{rms}^2}{R}$$

$v(t)i(t$

For an Impedance

- The instantaneous power is

$$p(t) = v(t)i(t)$$

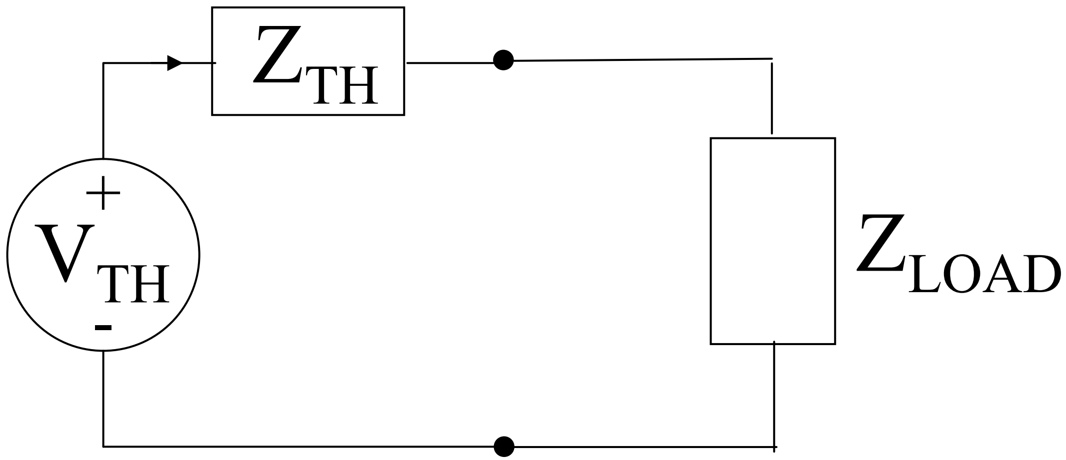
- The time-average power is

$$P_{AVE} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v(t)i(t)dt = \mathrm{Re}\{\mathbf{V}_{rms} \cdot \mathbf{I}_{rms}{}^*\}$$

- The reactive power at 2ω is

$$Q = \mathrm{Im}\{\mathbf{V}_{rms} \cdot \mathbf{I}^*_{rms}\} \qquad P^2_{AVE} + Q^2 = (V_{rms} \cdot I_{rms})^2$$

Maximum Average Power Transfer



- Maximum time average power occurs when

$$\mathbf{Z}_{LOAD} = \mathbf{Z}_{TH}^{*}$$

- This presents a resistive impedance to the source

$$\mathbf{Z}_{total} = \mathbf{Z}_{TH} + \mathbf{Z}_{TH}^{*}$$

- Power transferred is

$$P_{AVE} = \mathrm{Re}\{\mathbf{V}\mathbf{I}^{*}\} = \mathrm{Re}\{\mathbf{V}\frac{\mathbf{V}^{*}}{2R}\} = \tfrac{1}{2}\frac{V_{rms}^{2}}{R}$$