EE40
Lecture 19
Venkat Anantharam

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Reading: Chap. 6: Filters, two-terminal elements, Bode plots.

Bel and Decibel (dB)

- A **bel** is a unit of measure of ratios of power levels, i.e. relative power levels.
  - The name was coined in the early 20th century in honor of Alexander Graham Bell, a telecommunications pioneer.
  - The bel is a logarithmic measure. The number of bels for a given ratio of power levels is calculated by taking the logarithm, to the base 10, of the ratio.
  - one bel corresponds to a ratio of 10:1.
- The bel is too large for everyday use, so the **decibel (dB)**, is more commonly used.
  - $10 \log_{10}(P_1/P_2)$ is the power ratio measure in decibels
- dB are used to measure
  - Electric power, Gain or loss of amplifiers, Insertion loss of filters.

Logarithmic Measure for Power

- To express a power in terms of decibels, one starts by choosing a reference power, $P_{reference}$, and writing

  Power P in decibels = $10 \log_{10}(P/P_{reference})$

- Exercise:
  - Express a power of 50 mW in decibels relative to 1 watt.
  - P (dB) $=10 \log_{10} (50 \times 10^{-3}) = -13$ dB
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 mW.
  - P (dB) $=10 \log_{10} (50) = 17$ dB.
- One uses dBm to express absolute values of power relative to a milliwatt.
  - dBm = $10 \log_{10}$ (power in milliwatts / 1 milliwatt)
  - 100 mW = 20 dBm
  - 10 mW = 10 dBm

Logarithmic Measures for Voltage or Current

From the expression for power ratios in decibels, we can readily derive the corresponding expressions for voltage or current ratios.

Suppose that the voltage V (or current I) appears across (or flows in) a resistor whose resistance is R. The corresponding power dissipated, P, is V^2/R (or I^2R). We can similarly relate the reference voltage or current to the reference power, as

$$P_{\text{reference}} = (V_{\text{reference}})^2/R \text{ or } P_{\text{reference}} = (I_{\text{reference}})^2R.$$

Hence,

Voltage, V in decibels = $20\log_{10}(V/V_{\text{reference}})$

Current, I, in decibels = $20\log_{10}(I/I_{\text{reference}})$

Logarithmic Measures for Voltage or Current

Note that the voltage and current expressions are just like the power expression except that they have 20 as the multiplier instead of 10 because power is proportional to the square of the voltage or current.

Exercise: How many decibels larger is the voltage of a 9-volt transistor battery than that of a 1.5-volt AA battery? Let $V_{reference} = 1.5$. The ratio in decibels is

$$20 \log_{10}(9/1.5) = 20 \log_{10}(6) = 16 \text{ dB}.$$

Logarithmic Measures for Voltage or Current

The gain produced by an amplifier or the loss of a filter is often specified in decibels.

The input voltage (current, or power) is taken as the reference value of voltage (current, or power) in the decibel defining expression:

$$\text{Voltage gain in dB} = 20 \log_{10}(V_{output}/V_{input})$$
$$\text{Current gain in dB} = 20\log_{10}(I_{output}/I_{input}$$
$$\text{Power gain in dB} = 10\log_{10}(P_{output}/P_{input})$$

Example: The voltage gain of an amplifier whose input is 0.2 mV and whose output is 0.5 V is

$$20\log_{10}(0.5/0.2\text{x}10^{-3}) = 68 \text{ dB}.$$

Bode Plot

- Plot of magnitude of transfer function vs. frequency
  - Both x and y scale are in log scale
  - Y scale in dB
- Log Frequency Scale
  - Decade → Ratio of higher to lower frequency = 10
  - Octave → Ratio of higher to lower frequency = 2

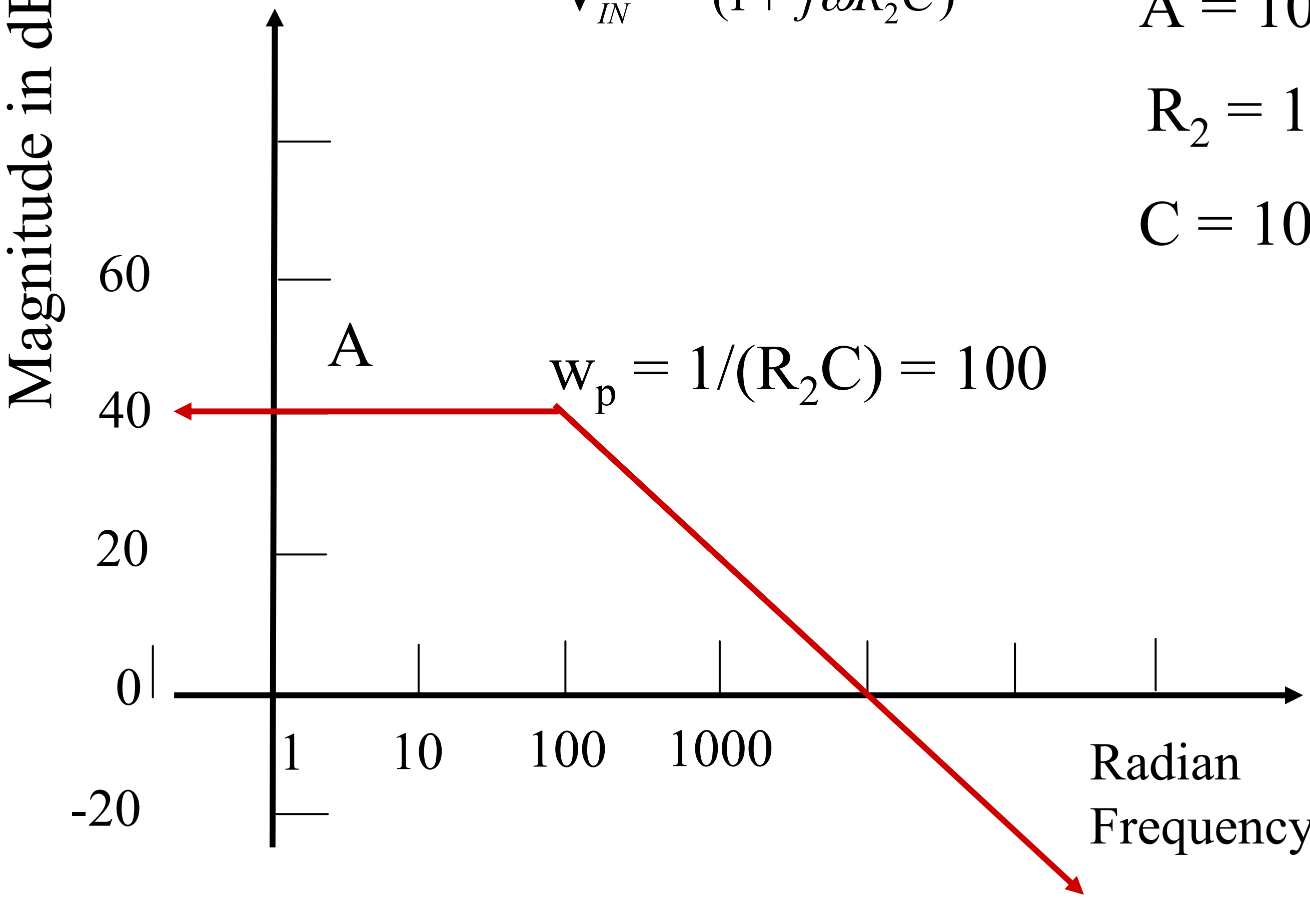
Bode Plot: Label as dB

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A}{(1 + j\omega R_2 C)}$$

A = 100

R_2 = 1000 Ohms

C = 100 uF



Note: Magnitude in dB = $20 \log_{10}(V_{OUT}/V_{IN})$

First-Order Lowpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V_C}}{\mathbf{V}} = \frac{1/(j\omega C)}{1/(j\omega C)+R} = \frac{1}{1+j\omega RC} = \frac{1}{\sqrt{1+(\omega RC)^2}}\angle -\tan^{-1}(\omega RC)$$

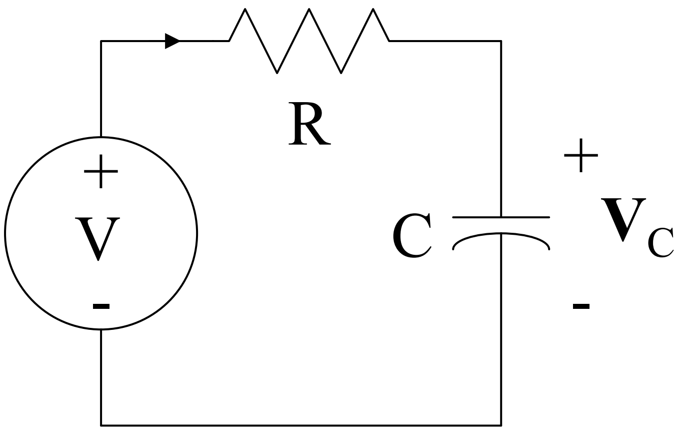
$$Let\ \omega_B = \frac{1}{RC} \quad and\ \ f_B = \frac{1}{2\pi RC}$$

$$\mathbf{H(f)} = H(f)\angle\theta$$

$$H(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}}\ ,\theta = -\tan^{-1}\left(\frac{f}{f_B}\right)$$

$$H(f_B) = \frac{1}{\sqrt{2}} = 2^{-1/2}$$

$$20\log_{10}\frac{H(f_B)}{H(0)} = 20(-\frac{1}{2})\log_{10}2 = -3\ \ dB$$



High-frequency asymptote of Lowpass filter

The high frequency asymptote of magnitude Bode plot assumes -20dB/decade slope

As $f \to \infty$

$$H(f) = \left(\frac{f}{f_B}\right)^{-1}$$

$$20\log_{10}\frac{H(10f_B)}{H(f_B)} = -20dB$$