# EE40
# Lecture 20
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3/12/08

Reading: Chap. 6: Bode plots.

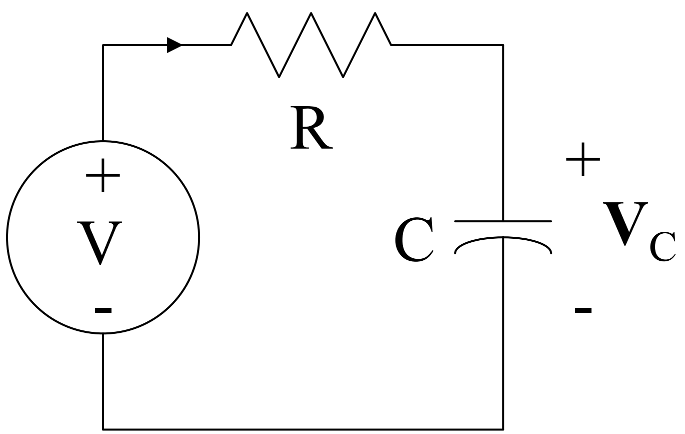
# First-Order Lowpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V_C}}{\mathbf{V}} = \frac{1/(j\omega C)}{1/(j\omega C)+R} = \frac{1}{1+j\omega RC} = \frac{1}{\sqrt{1+(\omega RC)^2}} \angle -\tan^{-1}(\omega RC)$$

$$Let\ \omega_B = \frac{1}{RC} \quad and\ \ f_B = \frac{1}{2\pi RC}$$

$$\mathbf{H(f)} = H(f)\angle\theta$$

$$H(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}} \ ,\theta = -\tan^{-1}\left(\frac{f}{f_B}\right)$$



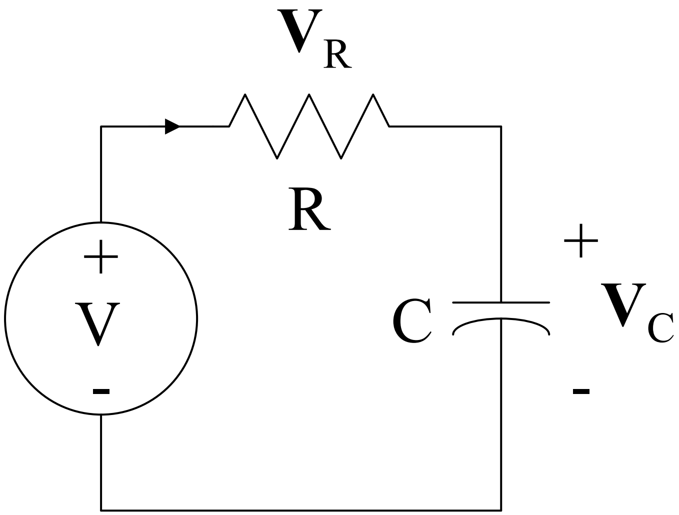

$$H(f_B) = \frac{1}{\sqrt{2}} = 2^{-1/2}$$

$$20\log_{10}\frac{H(f_B)}{H(0)} = 20(-\frac{1}{2})\log_{10} 2 = -3\ \ dB$$

# First-Order Highpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V}_R}{\mathbf{V}} = \frac{R}{1/(j\omega C) + R} = \frac{j\omega RC}{1 + j\omega RC} = \frac{(\omega RC)}{\sqrt{1 + (\omega RC)^2}} \angle\left[\frac{\pi}{2} - \tan^{-1}(\omega RC)\right]$$

$$H(f) = \frac{\left(\frac{f}{f_B}\right)}{\sqrt{1 + \left(\frac{f}{f_B}\right)^2}}, \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{f}{f_B}\right)$$




$$H(f_B) = \frac{1}{\sqrt{2}} = 2^{-1/2}$$

$$20\log_{10}\frac{H(f_B)}{H(0)} = 20(-\frac{1}{2})\log_{10} 2 = -3 \ dB$$

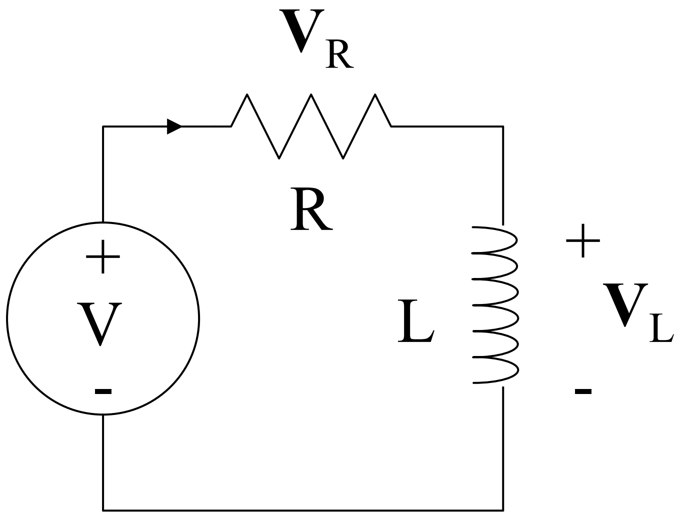
# First-Order Lowpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V}_R}{\mathbf{V}} = \frac{1}{\frac{j\omega L}{R}+1} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$Let\ \omega_B = \frac{R}{L} \quad and\ \ f_B = \frac{R}{2\pi L}$$

$$\mathbf{H(f)} = H(f)\angle\theta$$

$$H(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}}, \theta = -\tan^{-1}\left(\frac{f}{f_B}\right)$$



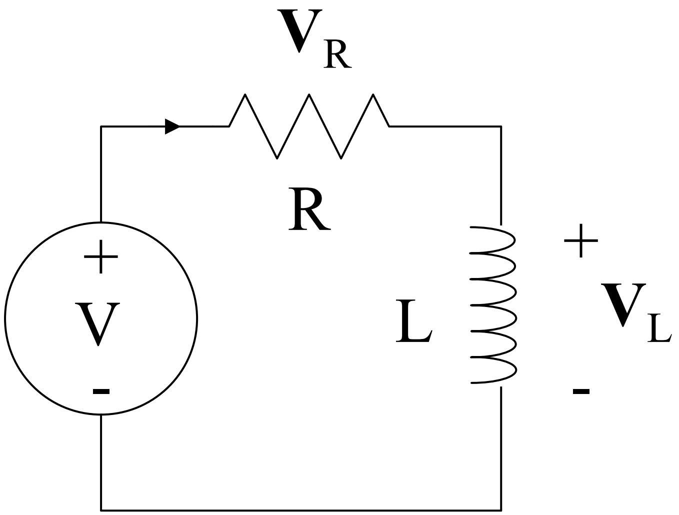
# First-Order Highpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V}_L}{\mathbf{V}} = \frac{\frac{j\omega L}{R}}{\frac{j\omega L}{R}+1} = \frac{\frac{\omega L}{R}}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \angle\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]$$

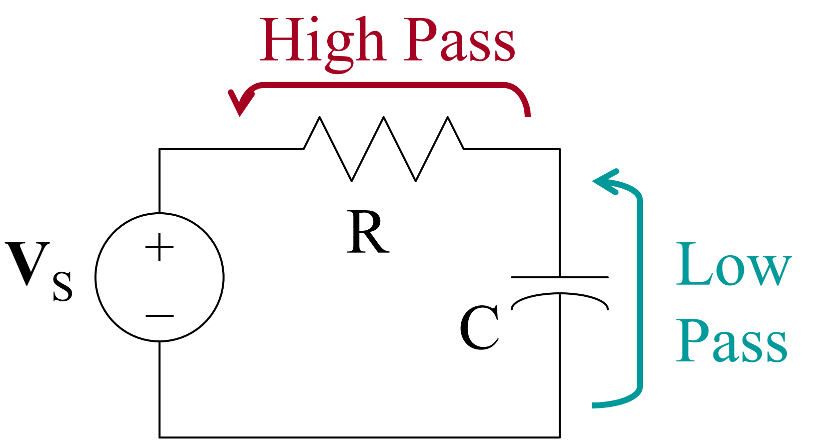
$$Let\ \omega_B = \frac{R}{L} \quad and\ f_B = \frac{R}{2\pi L}$$

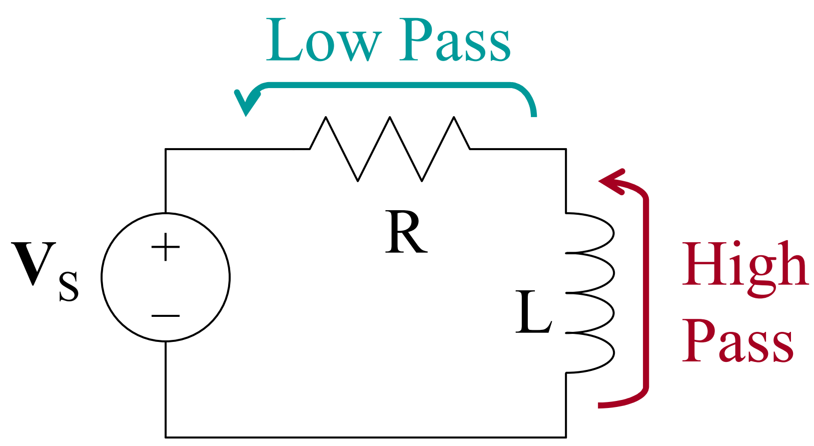
$$\mathbf{H(f)} = H(f)\angle\theta$$

$$H(f) = \frac{\left(\frac{f}{f_B}\right)}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}}, \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{f}{f_B}\right)$$



# First-Order Filter Circuits




$\mathbf{H_R} = R / (R + 1/j\omega C)$

$\mathbf{H_C} = (1/j\omega C) / (R + 1/j\omega C)$

$\mathbf{H_R} = R / (R + j\omega L)$

$\mathbf{H_L} = j\omega L / (R + j\omega L)$

# Change of Voltage or Current with A Change of Frequency

One may wish to specify the change of a quantity such as the output voltage of a filter when the frequency changes by a factor of 2 (an octave) or 10 (a decade).

For example, a single-stage RC low-pass filter has at frequencies above $\omega = 1/RC$ an output that changes at the rate -20dB per decade.

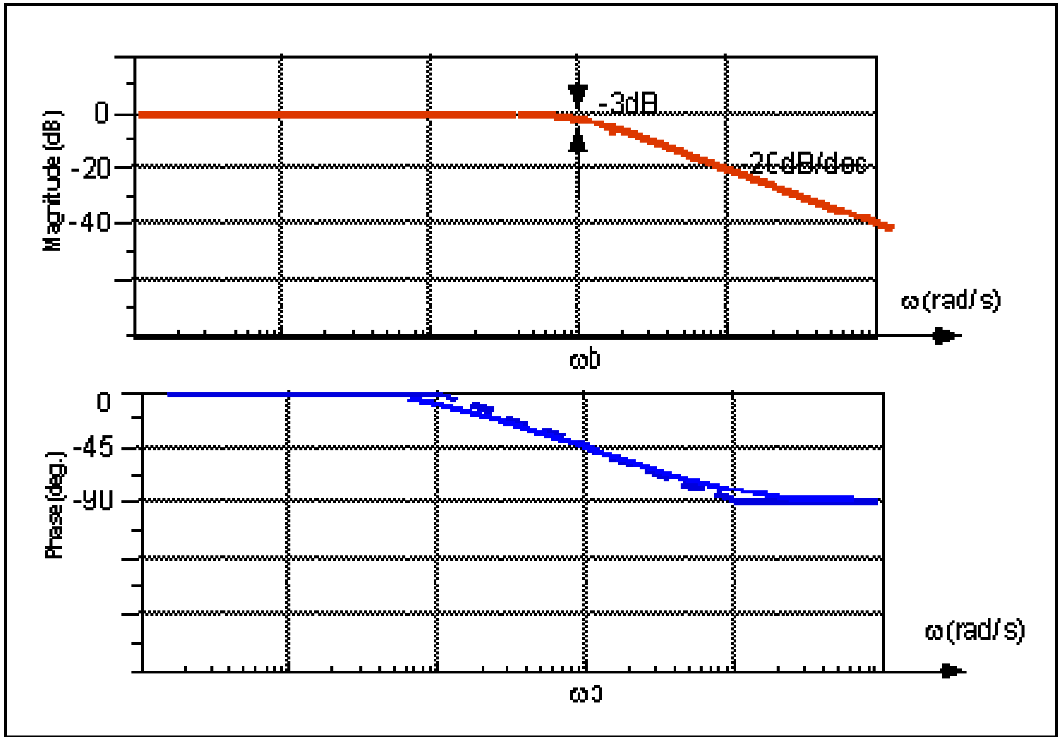
# High-frequency asymptote of Lowpass filter

The high frequency asymptote of magnitude Bode plot assumes -20dB/decade slope

As $f \to \infty$

$$H(f) = \left(\frac{f}{f_B}\right)^{-1}$$

$$20\log_{10}\frac{H(10f_B)}{H(f_B)} = -20dB$$



# Low-frequency asymptote of Highpass filter

$$As\ f \to 0$$

$$H(f) = \frac{\left(\frac{f}{f_B}\right)}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}} \to \left(\frac{f}{f_B}\right)$$

$$f \to \infty$$

$$20\log_{10}\frac{H(f_B)}{H(0.1f_B)} = 20dB$$

The low frequency asymptote of magnitude Bode plot assumes 20dB/decade slope