EE40
Lecture 5
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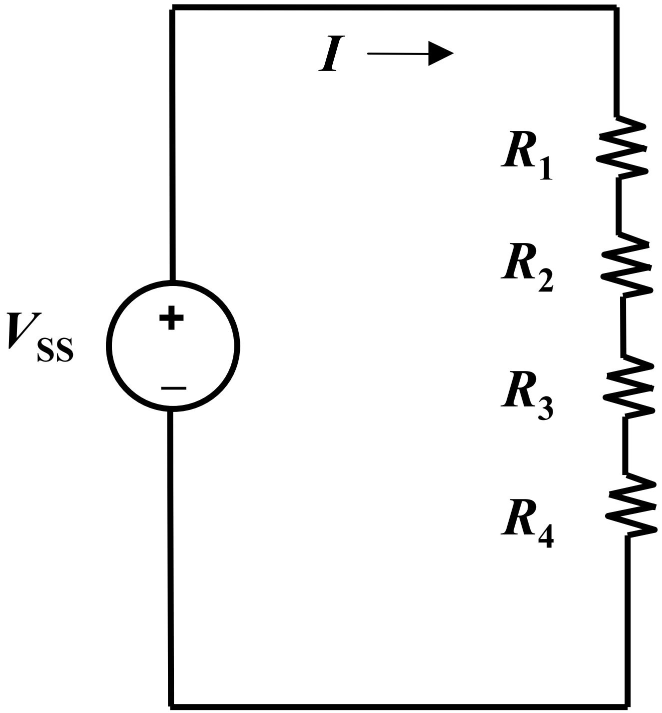
2/1/08

Reading: Chap. 2

Voltage and Current Dividers. Nodal analysis.

Resistors in Series

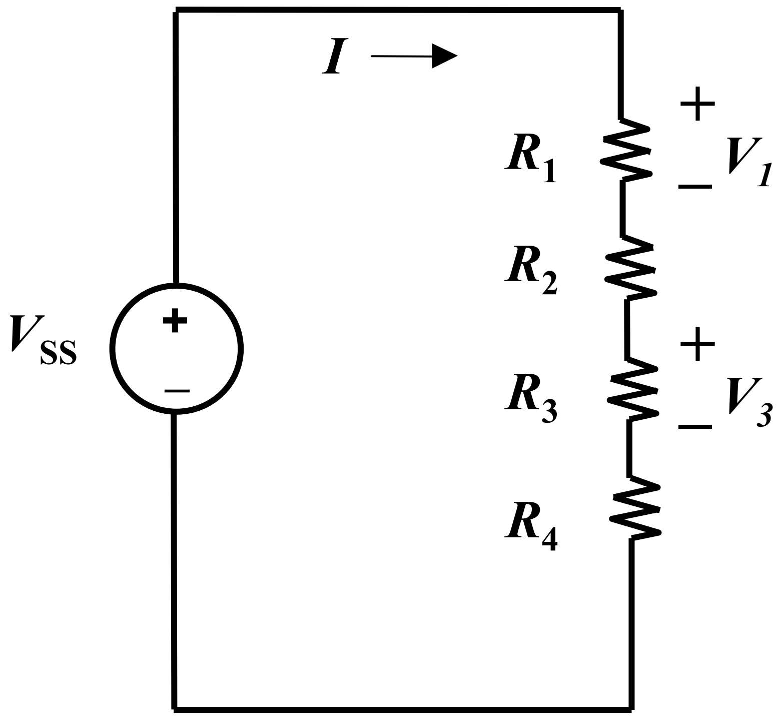
Consider a circuit with multiple resistors connected in series. Find their "equivalent resistance".



- KCL tells us that the same current (I) flows through every resistor
- KVL tells us the sum of the voltage drops across the resistors equals V_{SS}

We conclude that the equivalent resistance of resistors in series is the sum of the individual resistances

Voltage Divider



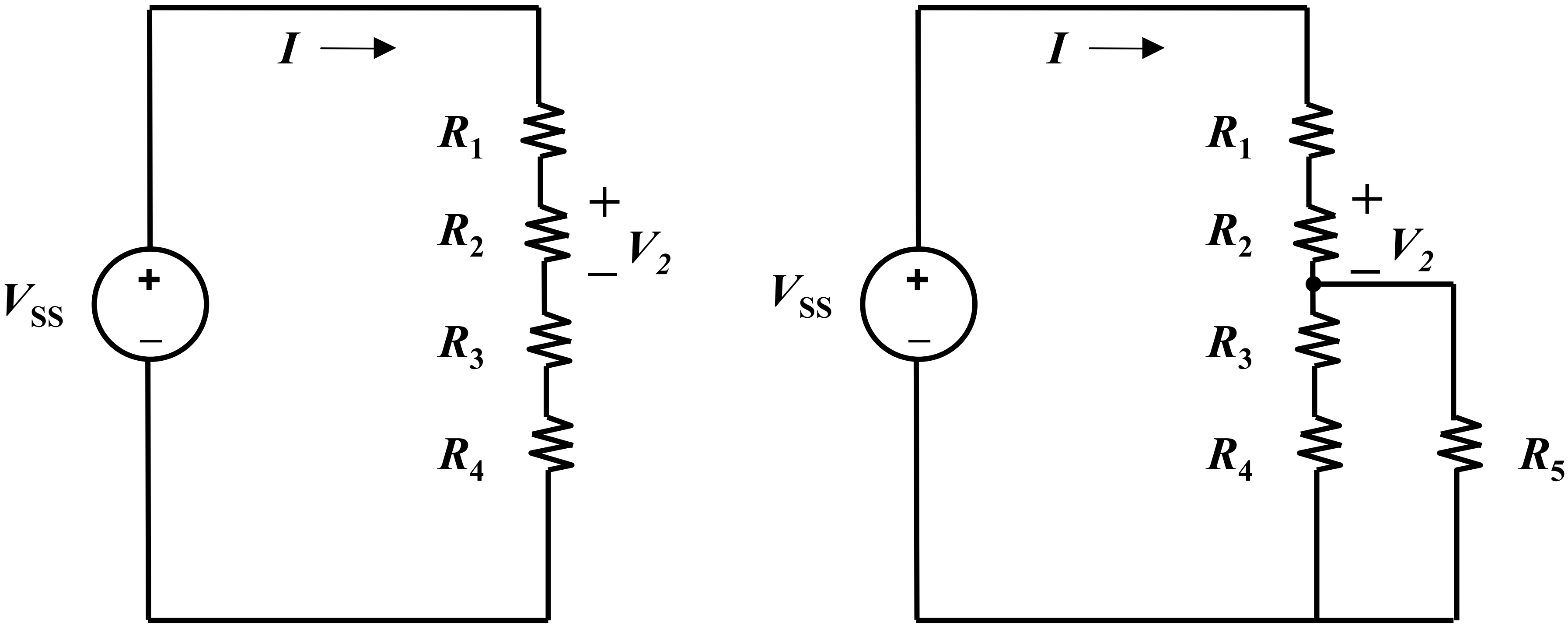
$$I = V_{SS} / (R_1 + R_2 + R_3 + R_4)$$

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Etc.

When can the Voltage Divider Formula be Used?



$$V_2 = \frac{R_2}{R_1+R_2+R_3+R_4} \cdot V_{SS}$$

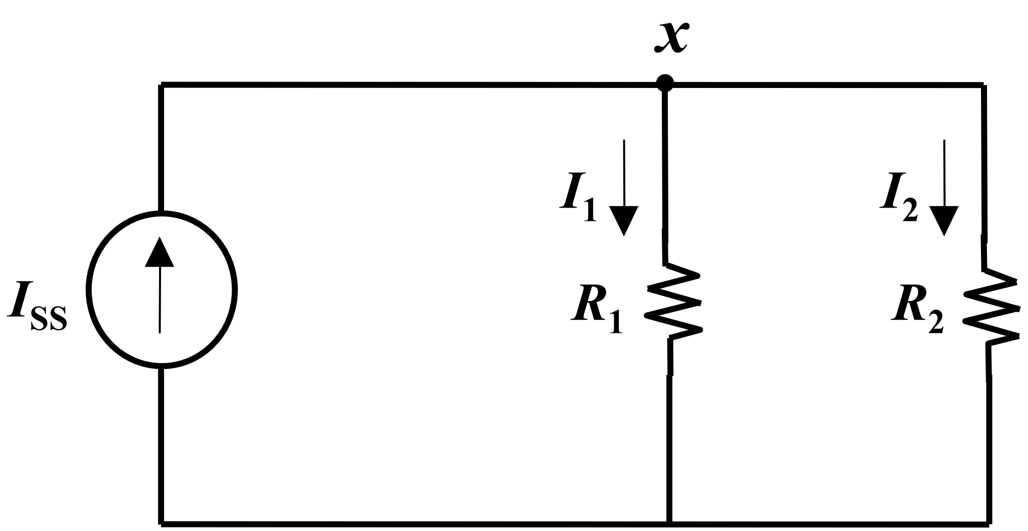
Correct, if nothing else is connected to nodes

$$V_2 \neq \frac{R_2}{R_1+R_2+R_3+R_4} \cdot V_{SS}$$

Why? What is V_2?

Resistors in Parallel

Consider a circuit with two resistors connected in parallel. Find their "equivalent resistance".



- KVL tells us that the same voltage is dropped across each resistor

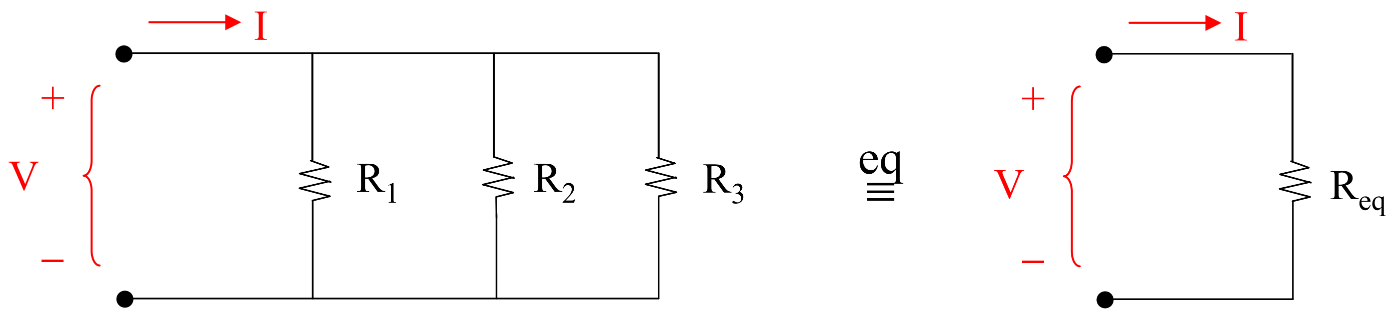
$$V_x = I_1 R_1 = I_2 R_2$$

- KCL tells us that

$$I_{SS} = I_1 + I_2$$

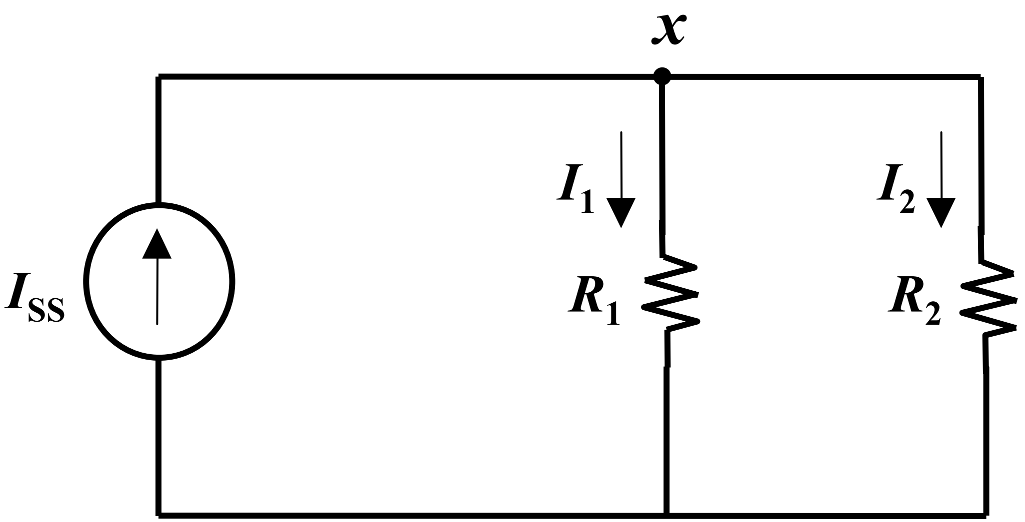
General Formula for Parallel Resistors

What single resistance R_{eq} is equivalent to three resistors in parallel?



Equivalent conductance of resistors in parallel is the sum of their individual conductances

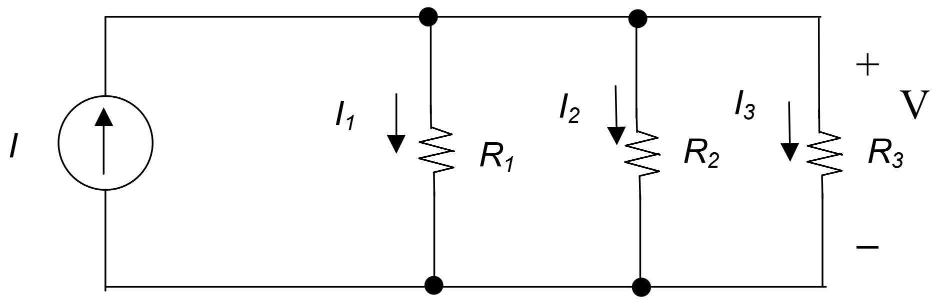
Current Divider



$$V_x = I_1 R_1 = I_{SS} R_{eq}$$

Generalized Current Divider Formula

Consider a current divider circuit with >2 resistors in parallel:



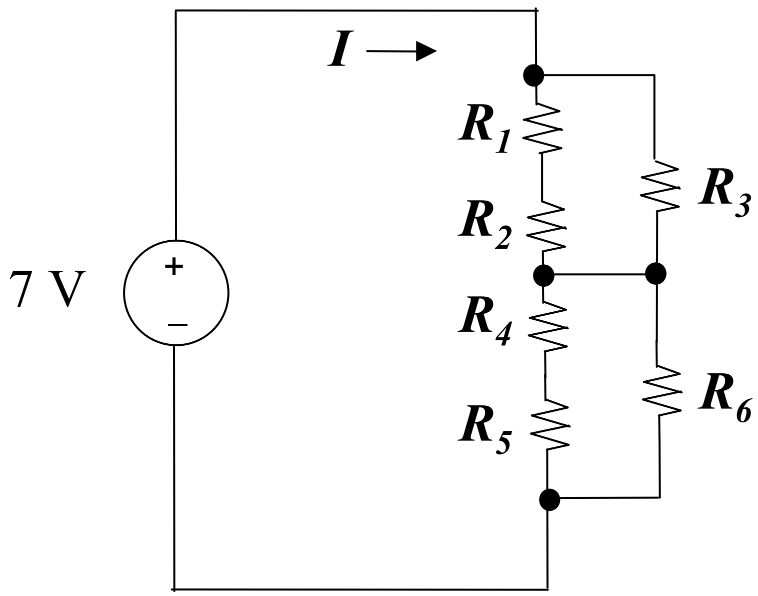
$$V = \frac{I}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)}$$

$$I_3 = \frac{V}{R_3} = I\left[\frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3}\right]$$

Using Equivalent Resistances

Simplify a circuit before applying KCL and/or KVL:

Example: Find I

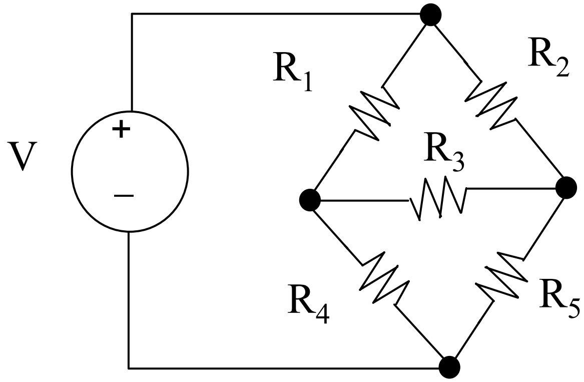


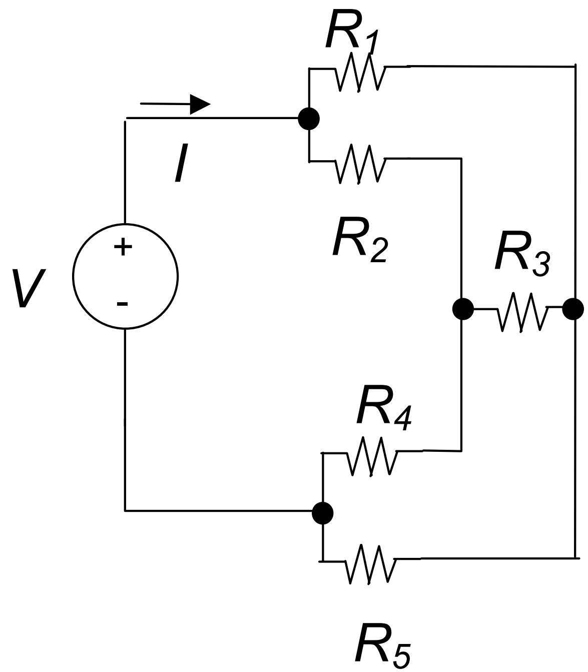
$R_1 = R_2 = 3\text{ k}\Omega$
$R_3 = 6\text{ k}\Omega$

$R_4 = R_5 = 5\text{ k}\Omega$
$R_6 = 10\text{ k}\Omega$

Identifying Series and Parallel Combinations

Some circuits *must* be analyzed (not amenable to simple inspection)





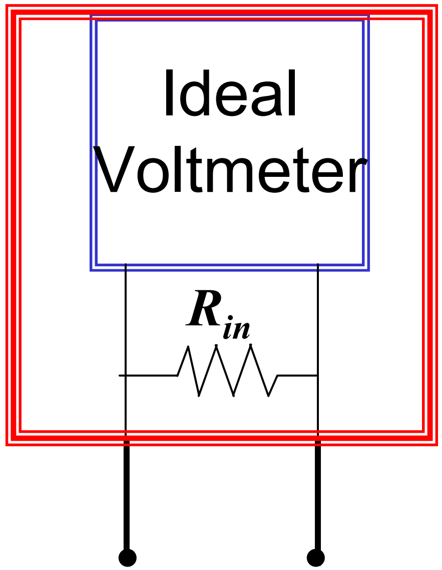
Special cases:

$R_3 = 0$ **OR** $R_3 = \infty$

Measuring Voltage

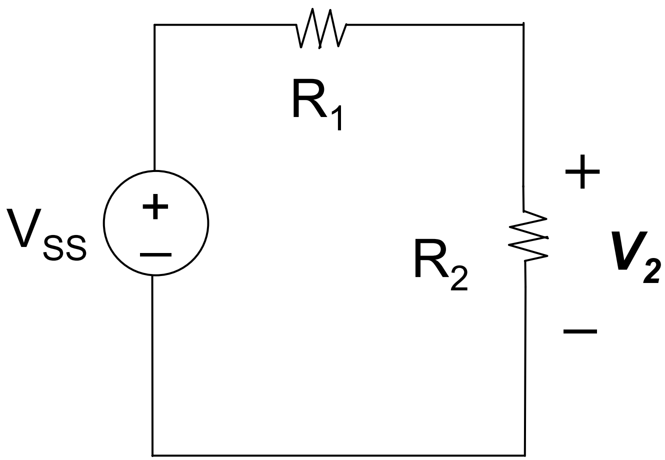
To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.

Voltmeters are characterized by their “voltmeter input resistance” (R_{in}). Ideally, this should be very high (typical value 10 MΩ)

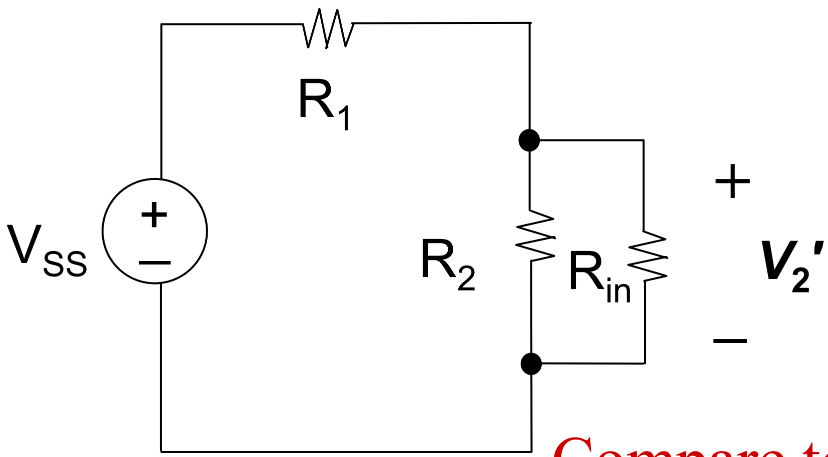


Effect of Voltmeter

undisturbed circuit



circuit with voltmeter inserted



$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right]$$

Compare to R_2

$$V_2' = V_{SS}\left[\frac{R_2 \| R_{in}}{R_2 \| R_{in} + R_1}\right]$$

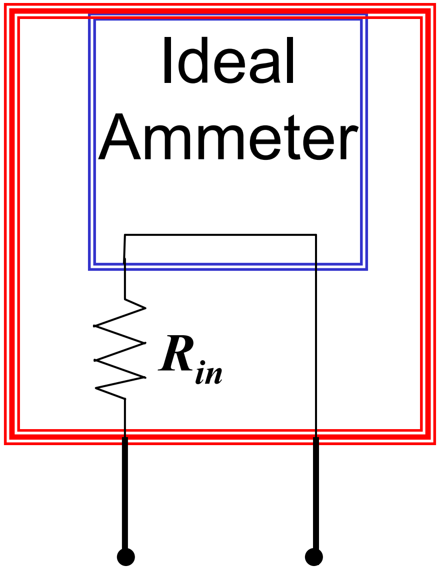
Example: $V_{SS} = 10\,V, R_2 = 100\,K, R_1 = 900\,K \Rightarrow V_2 = 1V$

$R_{in} = 10M, \; V_2' = ?$

Measuring Current

To measure the current flowing through an element in a real circuit, insert an ammeter (digital multimeter in current mode) **in series** with the element.

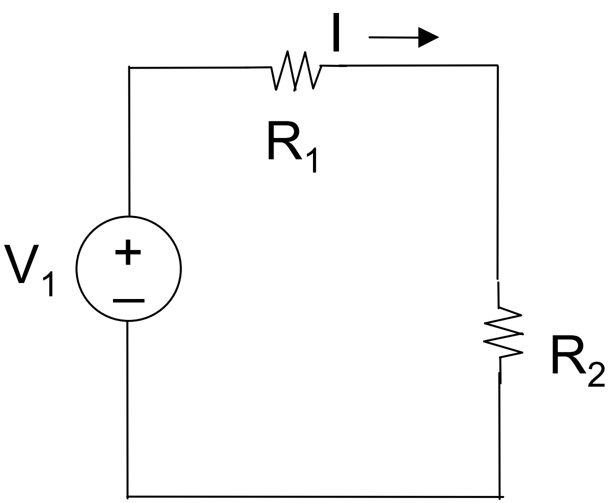
Ammeters are characterized by their "ammeter input resistance" (R_{in}). Ideally, this should be very low (typical value 1Ω).



Effect of Ammeter

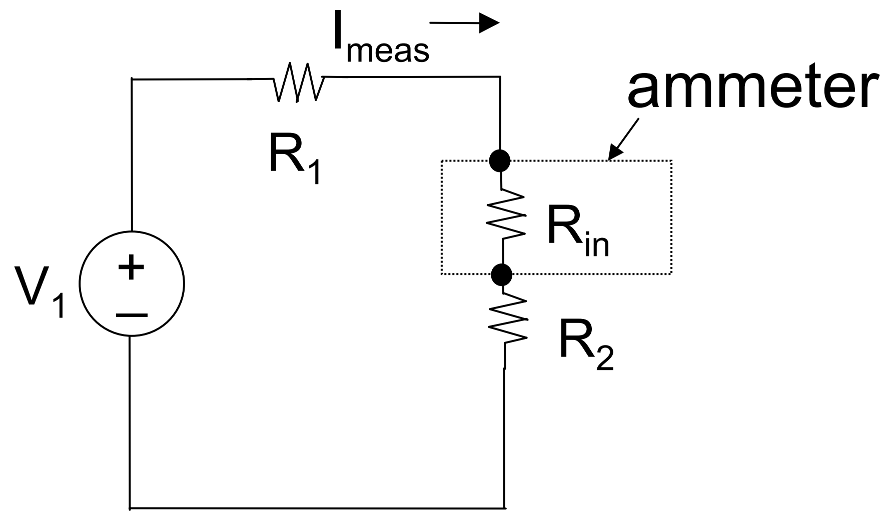
Measurement error due to non-zero input resistance:

undisturbed circuit



$$I = \frac{V_1}{R_1 + R_2}$$

circuit with ammeter inserted



$$I_{meas} = \frac{V_1}{R_1 + R_2 + R_{in}}$$

Example: V_1 = 1 V, R_1= R_2 = 500 Ω, R_{in} = 1Ω

$$I = \frac{1V}{500\Omega + 500\Omega} = 1mA, \quad I_{meas} = ?$$

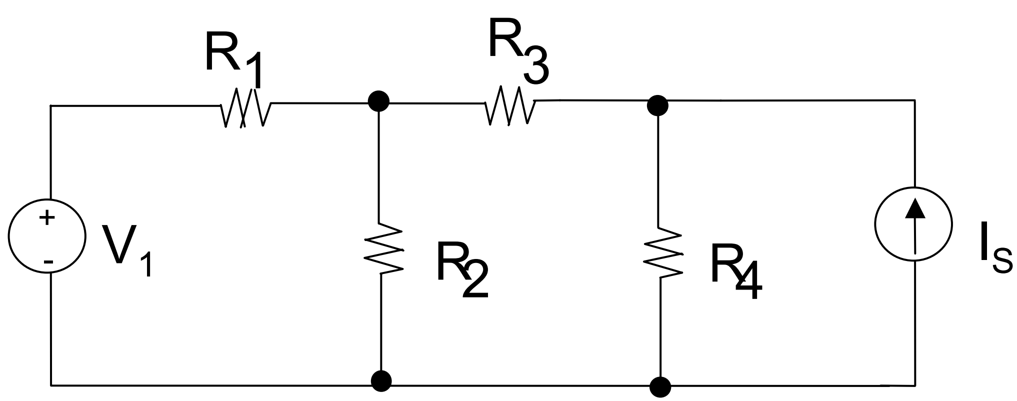
Compare to $R_2 + R_2$

Graph associated to a circuit

- First determine all the nodes in the circuit.
- Draw a line diagram where every pair of nodes that is connected by a circuit element in the circuit is connected by an edge in your diagram.
- This is called the graph of the circuit.
- A tree in this graph is defined as a subdiagram that connects all the nodes and that does not have any loops.
- The total number of edges in the tree is always one less than the total number of nodes.

Example 1

- Here is a circuit
- What are the nodes?
- What is the graph?
- Find some trees in this graph.



Example 2

- Here is a circuit.
- What are the nodes ?
- What is the graph?
- Find some trees in this graph.