# EE40
# Lecture 9
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2/11/08

Reading: Chap. 2

Superposition, Equivalent Circuit, Dependent sources, Wheatstone bridge

# Superposition

## Principle of Superposition:

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the sum of the individual contributions of each independent source acting alone.

# Superposition

<u>Procedure</u>:

1. Determine contribution due to one independent source
   - Set all other sources to 0: Replace independent voltage source by short circuit, independent current source by open circuit
2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

# Open Circuit and Short Circuit

- Open circuit → i=0 ; Cut off the branch
- Short circuit → v=0 ; replace the element by wire
- Turn off an independent voltage source means
  - V=0
  - Replace by wire
  - Short circuit
- Turn off an independent current source means
  - i=0
  - Cut off the branch
  - open circuit

# Comments on Dependent Sources

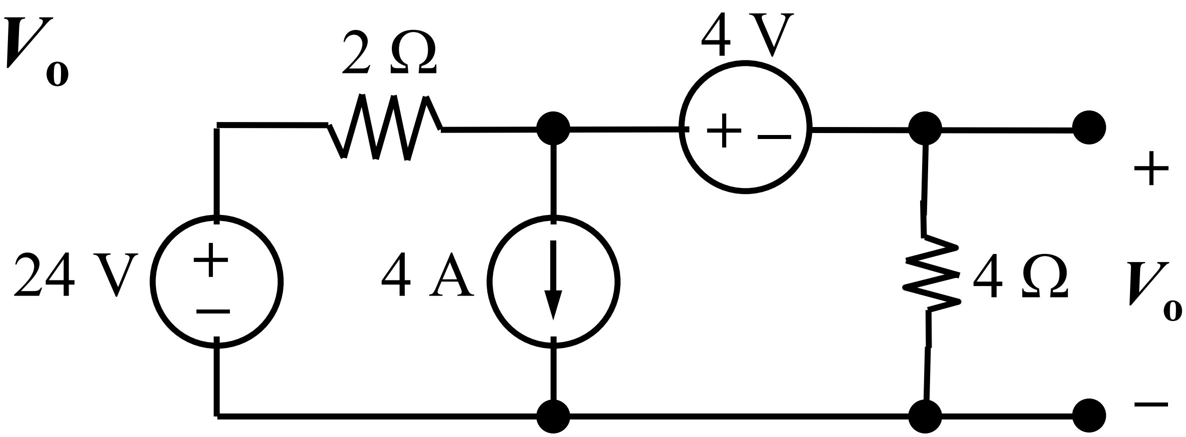
A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.

**The relationship between the dependent source and its reference cannot be broken!**

- Dependent sources cannot be turned off in analysis using superposition.

# Superposition: Example 1

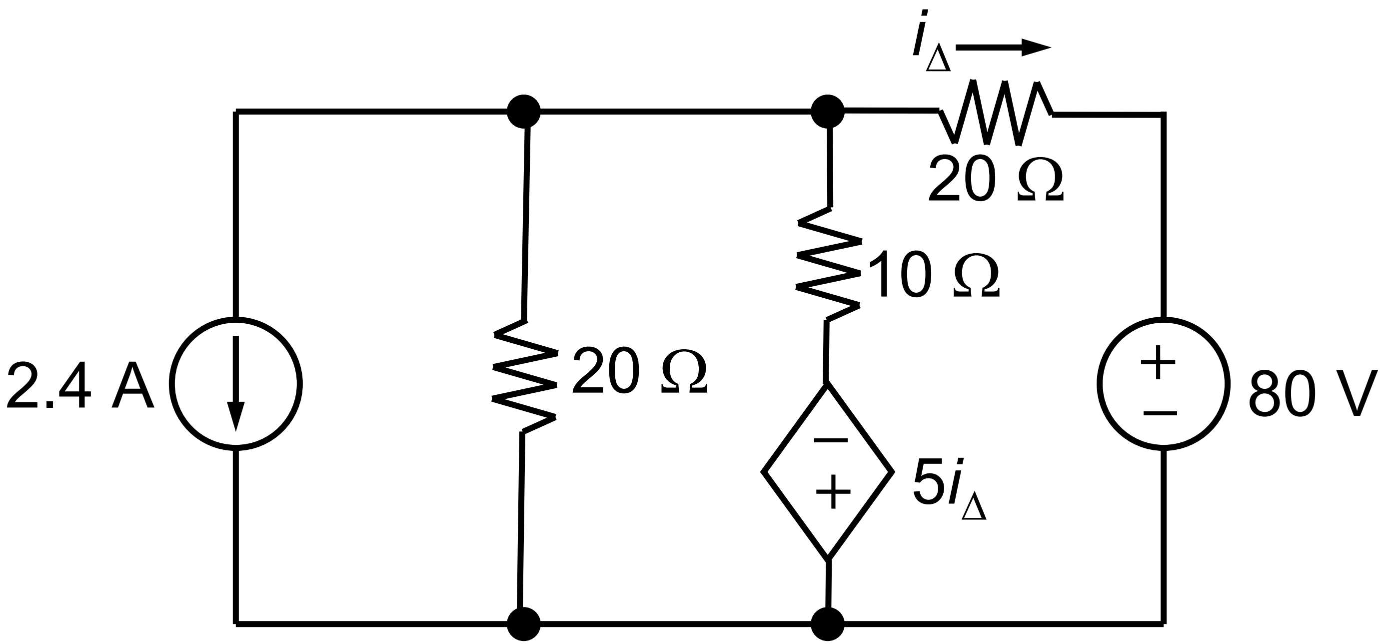
- Find $V_o$




Using superposition there are three separate circuits to analyze. Each of them is trivial to analyze using either the voltage divider formula or the current divider formula.

# Superposition: Example 2

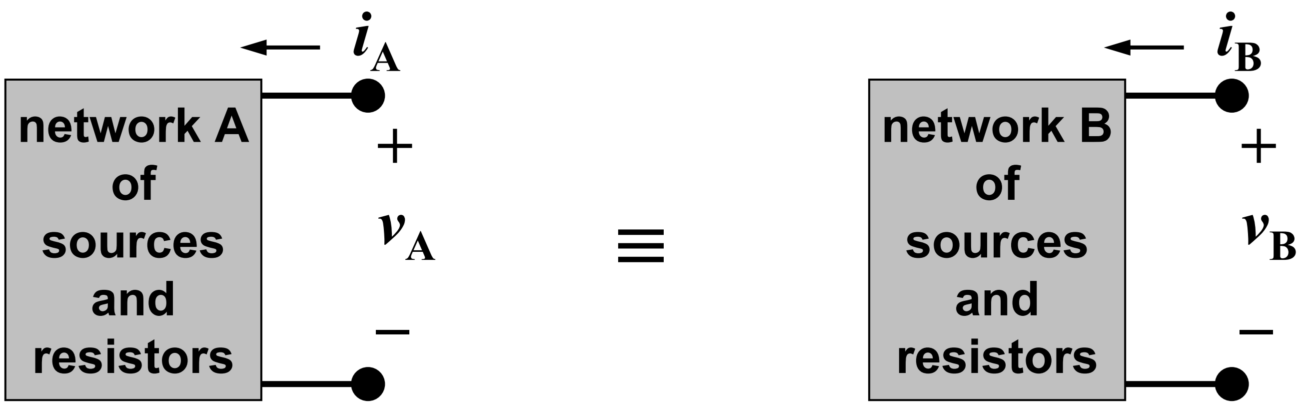
Solve this circuit using superposition:




Using superposition there are two circuits to analyze. In each of these circuits the controlled voltage source appears, tied to its defining current in exactly the same way as in the original circuit.

# Equivalent Circuit Concept

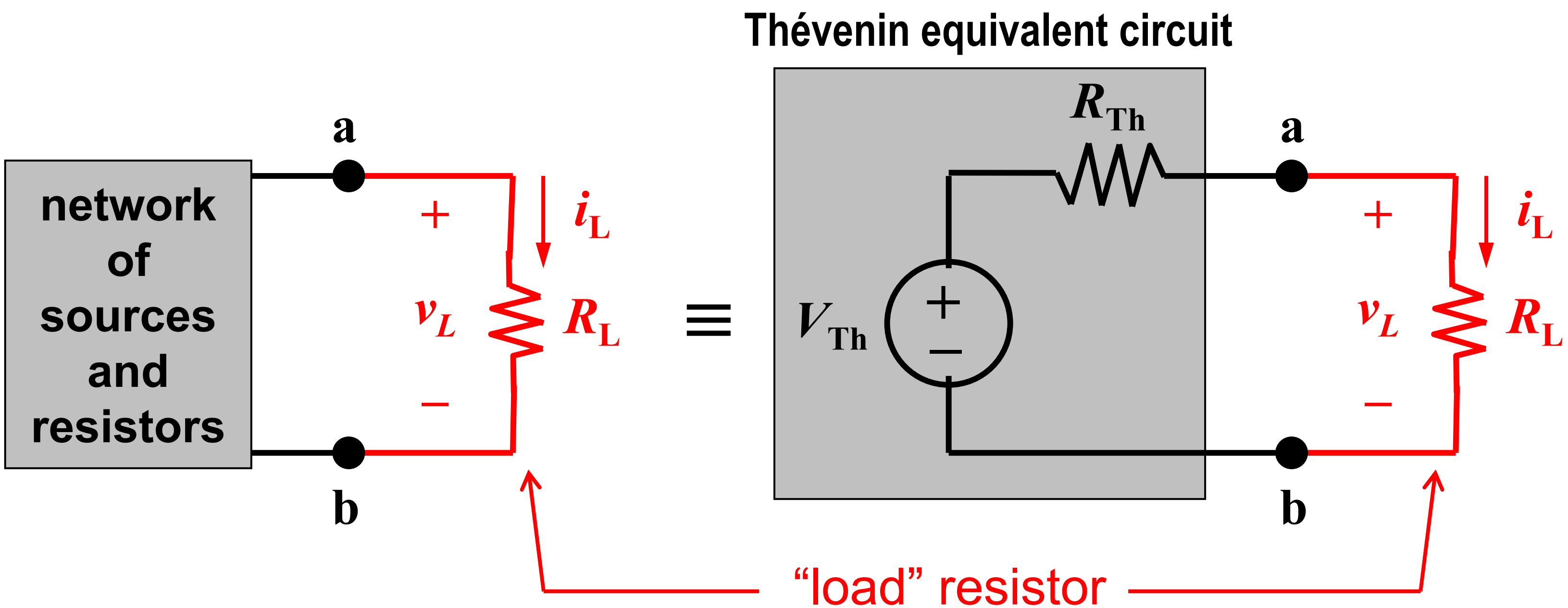
- When viewed across a pair of terminals, a portion of a circuit of voltage sources, current sources, and resistors can be replaced by any ***equivalent circuit*** which has identical terminal properties (*I*-*V* characteristics) without affecting the operation of the rest of the circuit.
- The only restriction is that for dependent sources within the portion being replaced, the variables on which they depend should also be in the portion being replaced.




$$i_A(v_A) = i_B(v_B)$$

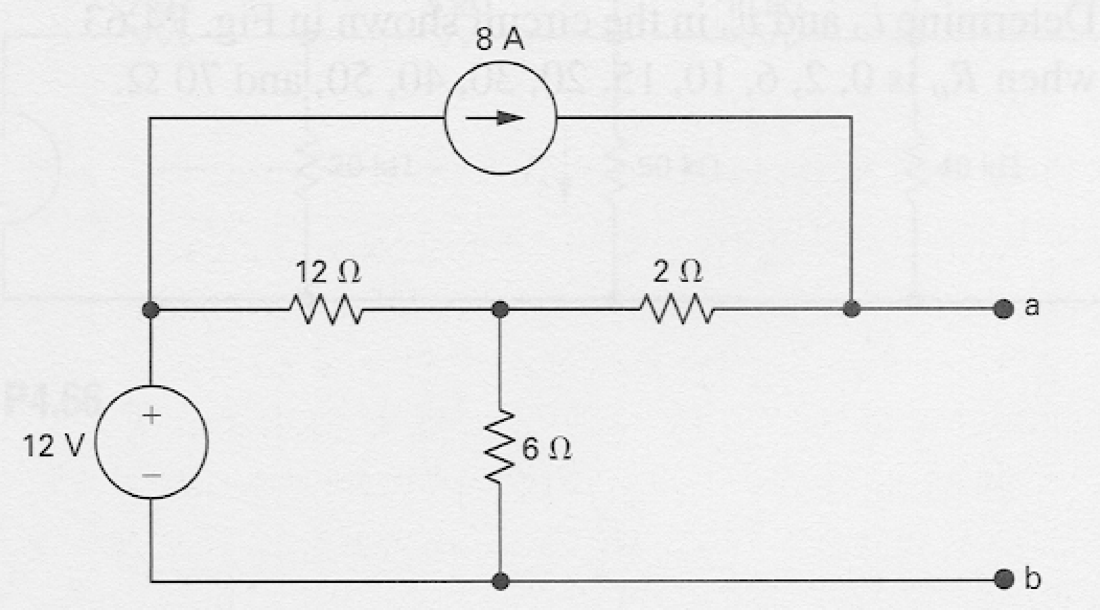
# Thévenin Equivalent Circuit

- Unless it is equivalent to a current source, a *linear* 2-terminal (1-port) network of voltage sources, current sources, and resistors can be replaced by an equivalent circuit consisting of an independent voltage source in series with a resistor without affecting the operation of the rest of the circuit.
  *(This assumes the condition regarding dependent sources mentioned earlier.)*

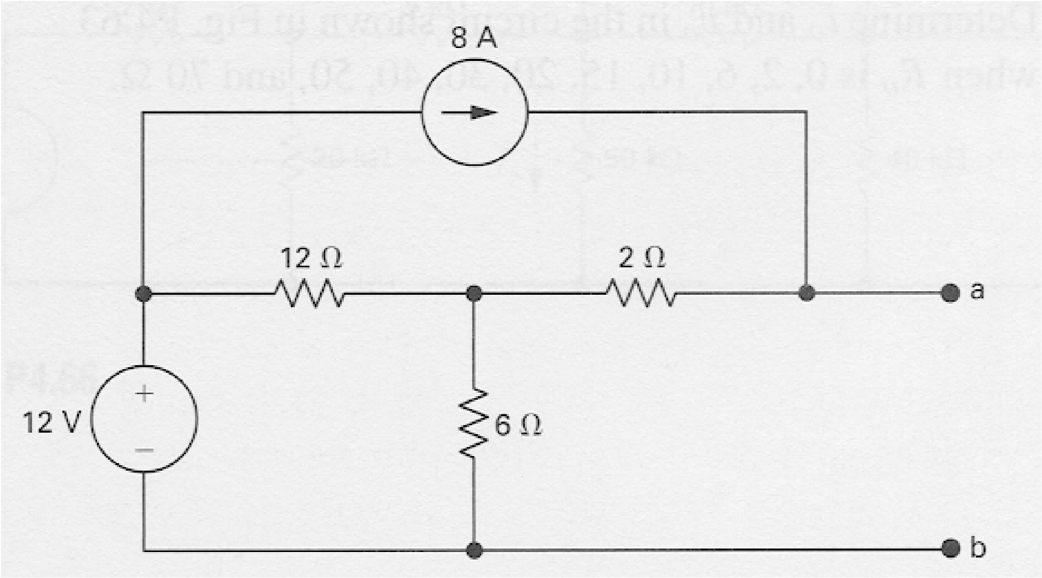


# Thévenin Equivalent: Example 1

Find the Thévenin equivalent with respect to the terminals a,b:



# $V_{Th}$ and $R_{Th}$ Calculation for Example 1



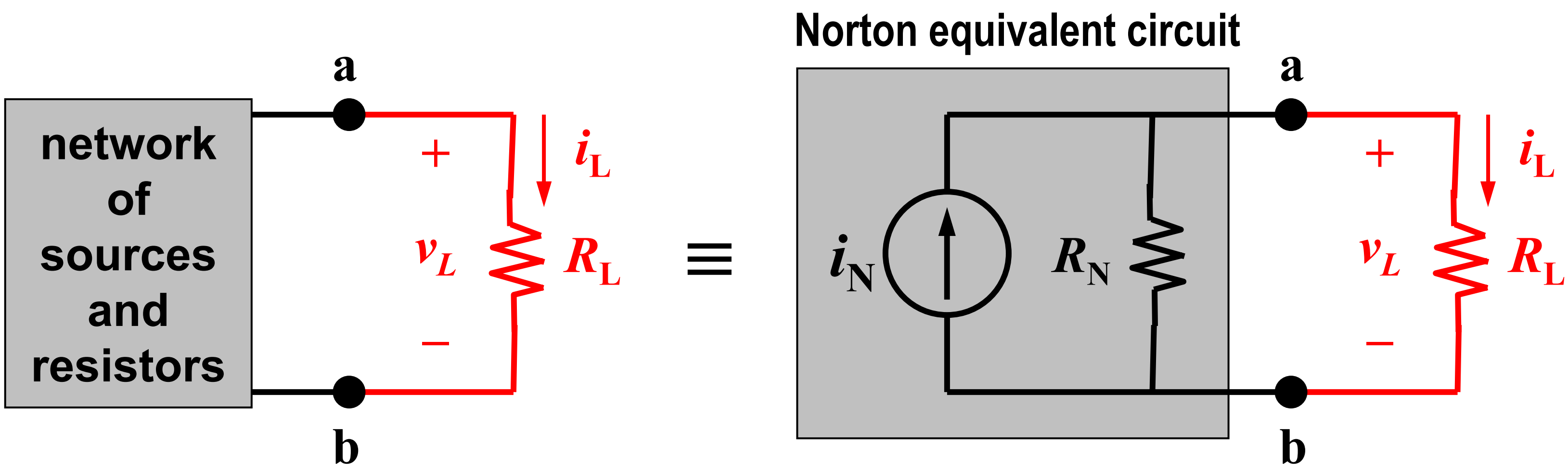

**Find $V_{Th}$ as the open circuit voltage across the terminals.**

**Find $R_{Th}$ by setting all independent sources to 0 and measuring the resistance across the terminals.**
***(Remember that ``setting to 0" shorts an independent voltage source and makes an independent current source an open circuit.)***

Careful: this will not work if the network is equivalent to a current source across its terminals

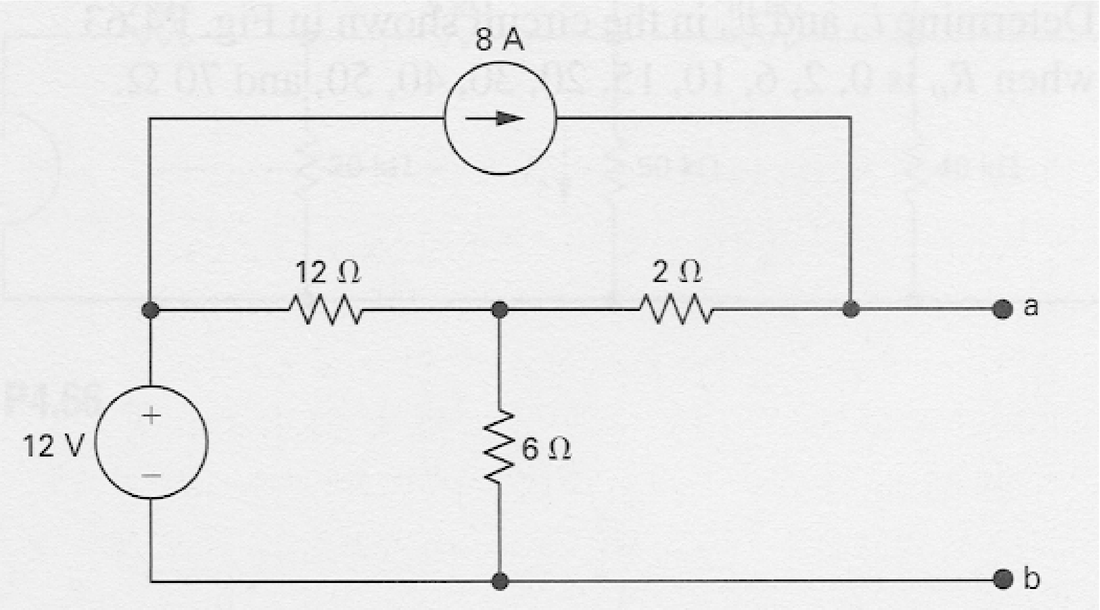
# Norton Equivalent Circuit

- Unless it is equivalent to a voltage source, any *linear* 2-terminal (1-port) network of voltage sources, current sources, and resistors can be replaced by an equivalent circuit consisting of an independent current source in parallel with a resistor without affecting the operation of the rest of the circuit. *(This assumes the condition regarding dependent sources mentioned earlier.)*

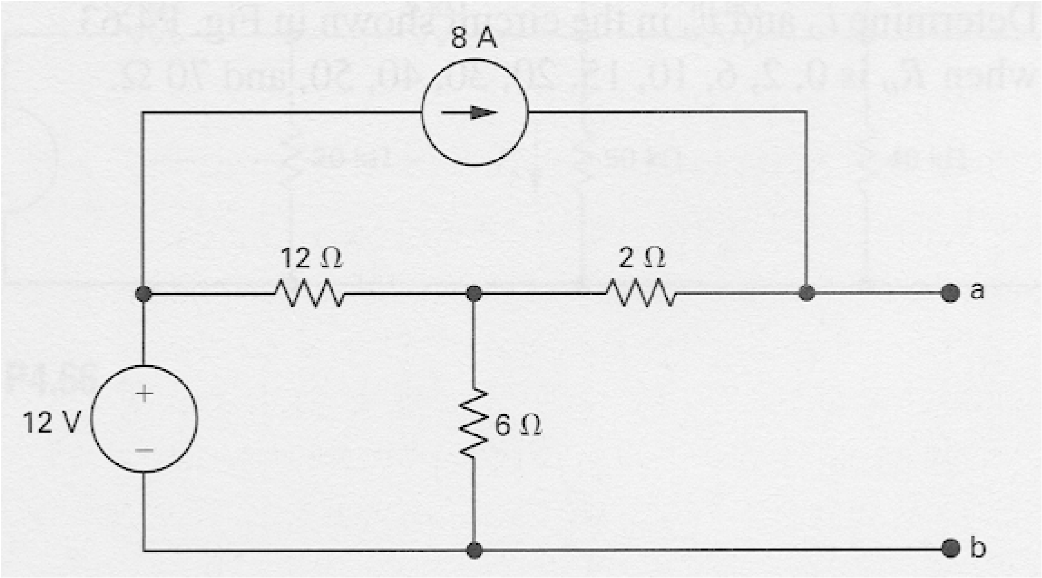


# Norton Equivalent: Example 1

Find the Norton equivalent with respect to the terminals a and b:



# $I_N$ and $R_N$ Calculation for Example 1




**Find $I_N$ as the short circuit current across the terminals.**

**Find $R_N$ by setting all independent sources to 0 and measuring the resistance across the terminals.**
***(Remember that ``setting to 0" shorts an independent voltage source and makes an independent current source an open circuit.)***

Careful: this will not work if the network is equivalent to a voltage source across its terminals

# Finding $I_N$ and $R_N = R_{Th}$

1) Find o.c voltage and s.c. current $R_N = R_{Th}$ can be found from

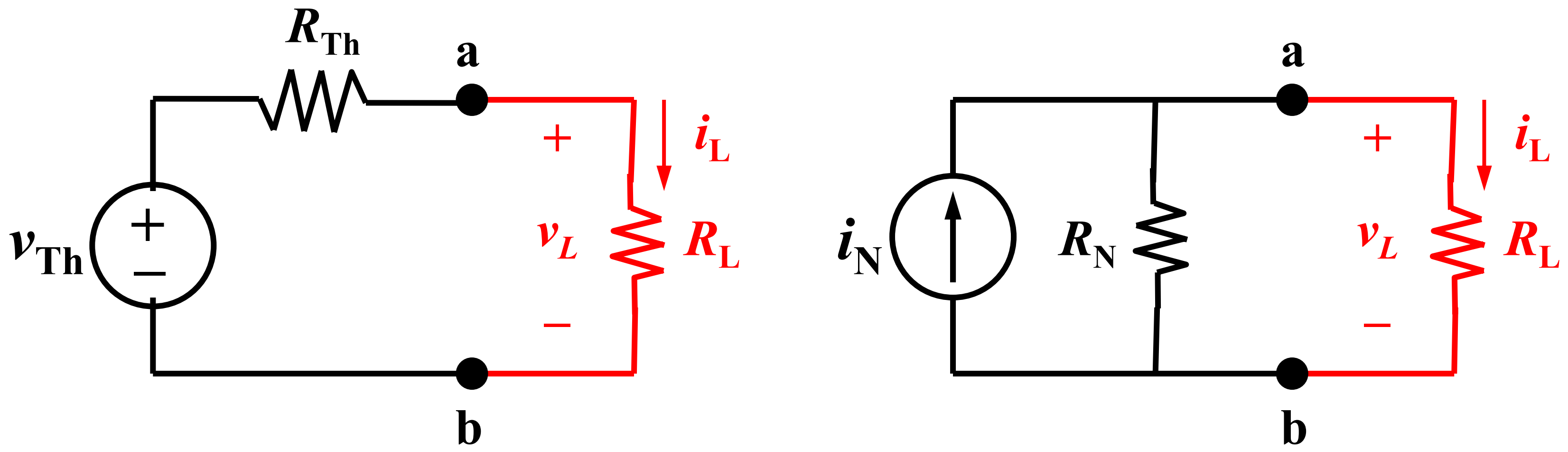
$$I_N \equiv i_{sc} = V_{Th}/R_{Th}$$

2) Or, find s.c. current and Norton (Thev) resistance

***Note***: *To find an equivalent circuit we have to find two distinct points on the I/V characteristic, so trying to find equivalent circuits by finding the open circuit voltage and the short circuit current will not work if the circuit is equivalent to just a resistor across its terminals.*

# Relation between the equivalent circuits

- If both exist, we can derive the Norton equivalent circuit from a Thévenin equivalent circuit and vice versa simply by making a source transformation:




$$R_N = R_{Th} = \frac{v_{oc}}{i_{sc}}; \quad i_N = \frac{v_{Th}}{R_{Th}} = i_{sc}$$

# Comments on Dependent Sources

A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.

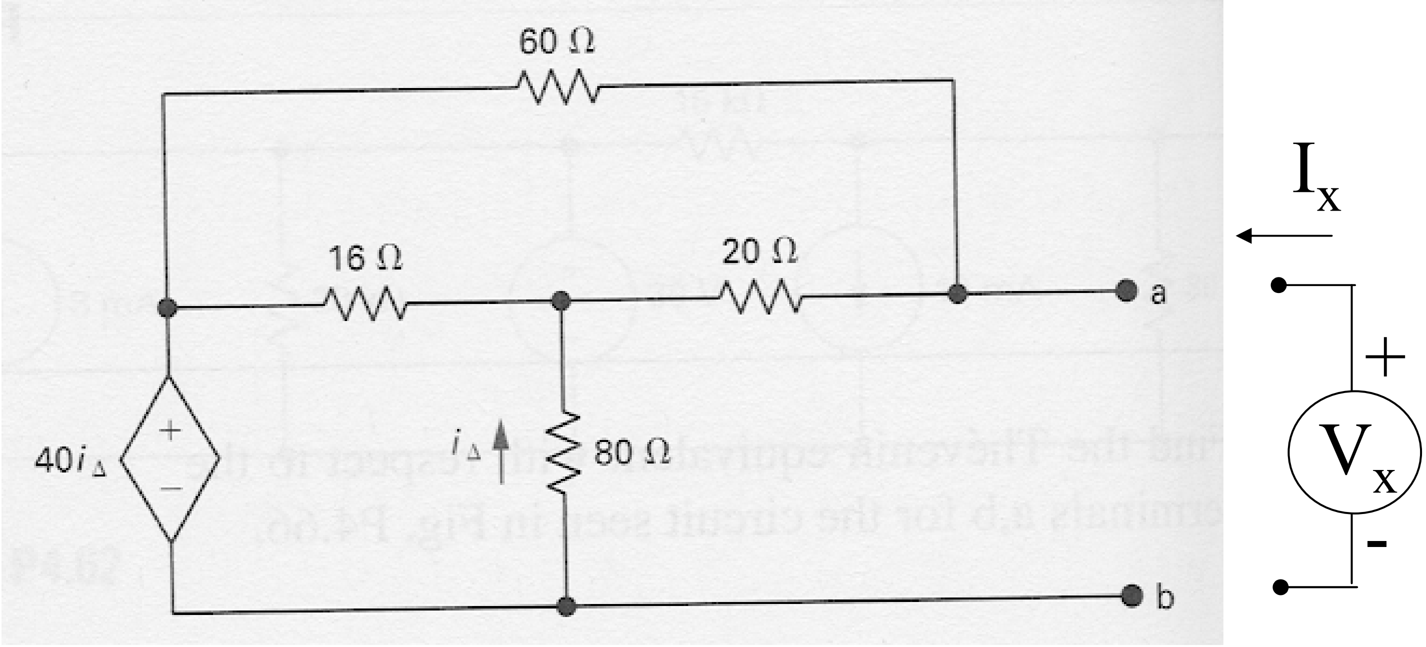
**The relationship between the dependent source and its reference cannot be broken!**

– Dependent sources cannot be turned off when we are finding the Thévenin resistance or the Norton resistance.

**If there are no independent sources in the network being replaced across a pair of terminal by an equivalent we need a different technique for finding the second point that determines the I/V characteristic. We can do this by applying another circuit across the terminals.**

# $R_{Th}$ Calculation: Example 2

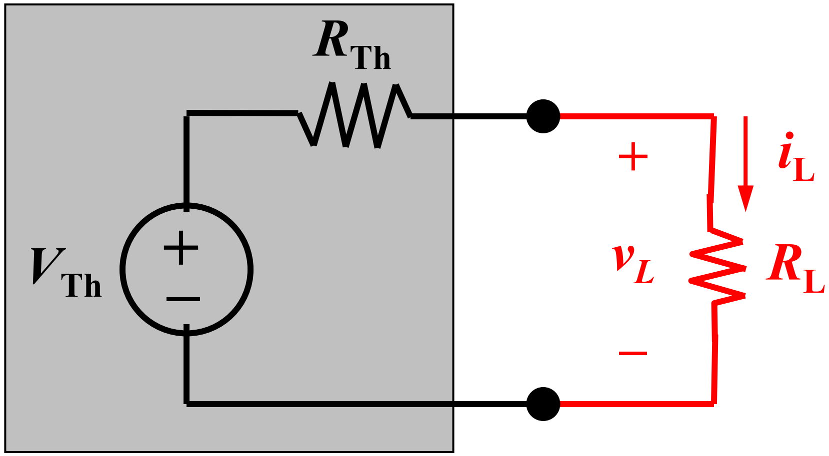
Find the Thévenin equivalent with respect to the terminals a,b:




Since there is no independent source and we cannot arbitrarily turn off the dependence source, we can add a voltage source $V_x$ across terminals a-b and measure the current through this terminal $I_x$ . $R_{th} = V_x / I_x$

# Maximum Power Transfer Theorem

**Thévenin equivalent circuit**




Power absorbed by load resistor:

$$p = i_L^2 R_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

To find the value of $R_L$ for which $p$ is maximum, set $\frac{dp}{dR_L}$ to 0:

$$\frac{dp}{dR_L} = V_{Th}^2 \left[ \frac{(R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$

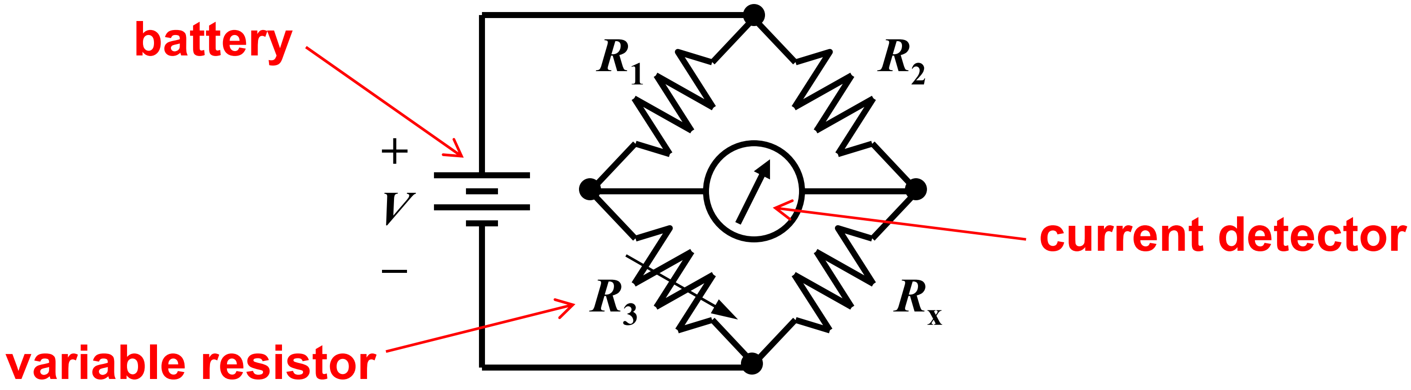
$$\Rightarrow (R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L) = 0$$

$$\Rightarrow \boxed{R_{Th} = R_L}$$

**A resistive load receives maximum power from a circuit if the load resistance equals the Thévenin resistance of the circuit.**

# The Wheatstone Bridge

- Circuit used to precisely measure resistances in the range from 1 Ω to 1 MΩ, with ±0.1% accuracy
  - $R_1$ and $R_2$ are resistors with known values
  - $R_3$ is a variable resistor (typically 1 to 11,000Ω)
  - $R_x$ is the resistor whose value is to be measured

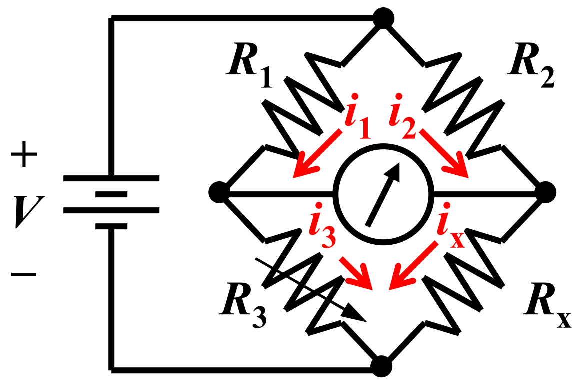


# Finding the value of $R_x$

- Adjust $R_3$ until there is no current in the detector

Then, $$R_x = \frac{R_2}{R_1} R_3$$

Derivation:




Typically, $R_2 / R_1$ can be varied from 0.001 to 1000 in decimal steps

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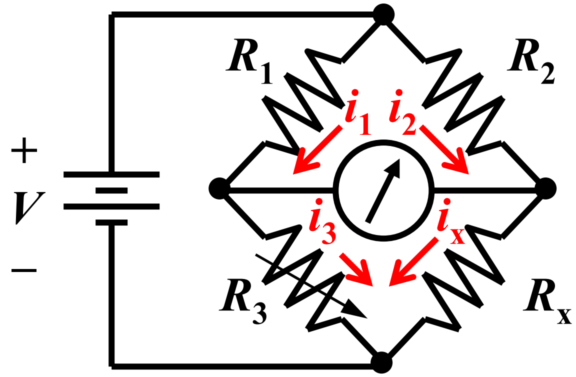


Typically, $R_2 / R_1$ can be varied from 0.001 to 1000 in decimal steps

**Derivation:**

KCL => $i_1 = i_3$ and $i_2 = i_x$

KVL => $i_3 R_3 = i_x R_x$ and $i_1 R_1 = i_2 R_2$

$$i_1 R_3 = i_2 R_x$$

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$