2nd order circuits

Consider the circuit:

Follow the steps that I tell you to find the answer:

A.1) Pick a reference point like in figure and consider the three modes 1, 2, 3. Pick reference directions and polarities for all branches.

Write KCL for each mode.

A.2) You have to find now the equation for $v_3$ and $v_2$

I) Take derivative of equation at mode 2.

II) From equation at mode 3, write $\frac{dv_2}{dt}$ as function of $v_3$ and $\frac{dv_3}{dt}$ (call this equation 3).

III) Take derivative of equation 4 to find an expression for $\frac{d^2v_2}{dt^2}$
A. 3) We want to study the steady state response when \( v(t) = \sin(w_0t) \) where \( w_0 = \frac{2\pi}{f_0} \).

As usual, assume \( v_3(t) = A \sin(w_0t) + B \cos(w_0t) \).

I) Substitute \( v_3(t) = A \sin(w_0t) + B \cos(w_0t) \) in Eq. 2

II) Compute \( A \) and \( B \)

We know that \( A \sin x + B \cos x = \sqrt{A^2 + B^2} \sin(x + \tan^{-1}(A/B)) \)

We are interested in \( \sqrt{A^2 + B^2} \)

III) Compute \( A^2 + B^2 \)

A. 4) We want to know the cutoff frequencies.

I) Solve the equation:

\[
\sqrt{A^2 + B^2} = \frac{1}{\sqrt{2}}
\]

You should get two solutions for \( w_0 \) meaning:

\[
\sqrt{A^2 + B^2} \approx 1
\]

It is a bandpass filter!!

(cool, isn't it?)
B) Op-Amps: current source

Given the following circuit

\[ \begin{aligned}
 & R_1 & R_2 & R_3 & \text{Vin} \\
 & & & \text{+} & \\
 & & & R_4 & \text{Out} \\
 & & & R_5 & \\
 & & & R_2 & \text{IL} \\
\end{aligned} \]

B.1) Compute \( I_L \) as a function of \( \text{Vin} \)
in the case where
\[
\begin{cases}
R_2 = R_4 + R_5 \\
R_1 = R_2
\end{cases}
\]

C) Op-Amps: non-linear circuit
C.1) Consider \( V_1 = \frac{V_{ac} R_2}{R_1 + R_2} \) and the output to be \( -V_{ac} \).

What is going to happen?

Describe how the circuit works.

C.2) Compute the frequency of the output oscillation.