A) Given the following circuit

\[ \frac{V_o(s)}{V_i(s)} \]

A.1) Compute the transfer function

\[ F(s) = \frac{V_o(s)}{V_i(s)} \]

A.2) Compute the frequency response

\[ F(j\omega) \]

A.3) Compute |F(j\omega)|

A.4 Extra question) Plot |F(j\omega)| and \( \angle F(j\omega) \) as a function of \( \omega \)
B) Given the following circuit:

![Circuit Diagram]

B.1) Considering the input $V_i(t)$ as in figure B.1, plot the output $V_c(t)$. (You have to do the work, don't just plot $V_c(t)$ but compute $V_c(t)$ as a function of time.)

![Graph]

**Fig. B.1**

B.2) Compute $V_c(t_2)$
C) Consider the following circuit

C.1) Using the ideal model for the diode (open-circuit, short-circuit) plot the transfer characteristic (Vo vs. Vi)

C.2) If the input is a sinusoid:

Plot the output

C.3) Extra question) Plot the transfer characteristic using the Dg model
D) Given the circuit:

\[ R_1 \quad R_2 \quad R_3 \quad R_4 \]

\[ V_1 \quad + \quad V_2 \quad - \quad V_0 \]

with \( R_1 = R_4 \), \( R_2 = R_3 \)

D.1) Compute \( V_0 \) as a function of \( V_1, V_2 \)

D.2) What is the input resistance for \( V_1 \) and \( V_2 \)?

D.4 Extra question) If a noise signal \( n(t) \)

is injected in both \( V_1 \) and \( V_2 \),
what is the noise at the output?
Extra of the Extra) Consider a positive logic where the two digits 0 and 1 are encoded by two voltages $V(0), V(1)$ where $V(1) > V(0)$.

Given the following logic device:

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\[ V_A \quad R_s \quad D_1 \quad V(1) \]
\[ R \quad V_0 \]
\[ V_B \quad R_s \quad D_2 \]
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E.1) Write the truth table of this device:

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(Considering the diode ideal and $R \gg R_s$.)
A.1) Using Laplace transform:

\[ V_o(s) = \frac{Z_2}{Y_{sc1} + Z_2} V_i(s) \]

\[ F(s) = \frac{R}{(SC_2R + 1) SC_1} + \frac{(SC_2R + 1) R}{SC_2R + 1} \]

\[ = \frac{SC_1R}{SC_2R + 1 + SC_1R} = \frac{SC_1R}{SR(C_1 + C_2) + 1} \]

4.2) \[ F(jw) = \frac{SC_1R}{jwR(C_1 + C_2) + 1} \]
A 3) \[ |F(jw)| = \frac{wRC_1}{\sqrt{1 + w^2R^2(C_1 + C_2)^2}} \]

A 4) \[ |F(jw)| \]

- Assumimg you don't know anything about Boole diagram

\[
\frac{d |F(jw)|}{dw} = \frac{2wRC_1R^2(C_1 + C_2)^2}{\sqrt{1 + w^2R^2(C_1 + C_2)^2}} - \frac{wRC_1R^2(C_1 + C_2)^2}{1 + w^2R^2(C_1 + C_2)^2}
\]

\[
\frac{d F(jw)}{dw} = 0 \Rightarrow \frac{2wRC_1R^2(C_1 + C_2)^2}{\sqrt{1 + w^2R^2(C_1 + C_2)^2}} = \frac{C_1R}{\sqrt{1 + w^2R^2(C_1 + C_2)^2}}
\]

\[
\Rightarrow 2wRC_1R^2(C_1 + C_2)^2 = C_1R(1 + w^2R^2(C_1 + C_2)^2)
\]

\[
\Rightarrow 2R^2(C_1 + C_2)^2w^2 = 1 + w^2R^2(C_1 + C_2)^2
\]

\[
\Rightarrow \frac{R^2(C_1 + C_2)^2w^2}{1} = 1 \Rightarrow w = \sqrt{\frac{1}{R^2(C_1 + C_2)^2}} = \frac{1}{RC_1R}
\]

\[ \text{This is a max} \]
\[ F(jw) = \frac{\pi}{2} - \frac{1}{\pi w R (C_1 + C_2)} \]

\[ \lim_{w \to 0} \frac{w C_1 R}{\sqrt{1 + w^2 R^2 (C_1 + C_2)^2}} = \lim_{w \to \infty} \frac{C_1 R}{\sqrt{1/w^2 + R^2 (C_1 + C_2)^2}} = \frac{C_1}{C_1 + C_2} \]
B.1) Using N.V.A.

\[ \frac{V_i(t) - V_c(t)}{R} = i_c(t) = C \frac{dV_c(t)}{dt} \]

\[ \frac{dV_c(t)}{dt} + \frac{1}{RC} V_c(t) = \frac{V_i(t)}{RC} \]

If in the example I did using the inductor, you replace \( I(t) \) with \( V_c(t) \), you obtain the same solution.

\[ V_c(t) = k_1 e^{xt} + k_2 \]

\[ t < t_1 \]

\[ V_i(t) = A \]

\[ k_2 = A \] because if \( V_c(t) = k_2 \) then

\[ \frac{dk_2}{dt} + \frac{k_2}{RC} = \frac{A}{RC} \]

\[ \frac{d}{dt} k_1 e^{xt} + \frac{1}{RC} k_1 e^{xt} = 0 \Rightarrow k_1 e^{xt} + \frac{k_1}{RC} e^{xt} = 0 \]

\[ d = -\frac{1}{RC} \]

\[ V_c(t) = k_1 e^{-\frac{1}{RC}t} + A \] but \( V_c(0) = 0 \Rightarrow k_1 = -A \]
\[ V_c(t) = A \left( 1 - e^{-t/RC} \right) \]

In \( t_1 \) we shift the time to \( t_1 \)

\[ t \geq t_1 \quad V_i(t) = 0 \]

\[ \frac{dV_c(t)}{dt} + \frac{1}{RC} V_c(t) = 0 \]

\[ V_c(t_1) = A \left( 1 - e^{-t_1/RC} \right) \]

\[ V_c(t) = K_1 e^{-t/RC} \Rightarrow V_c(t_1) = K_1 e^{-t_1/RC} = A \left( 1 - e^{-t_1/RC} \right) \]

\[ \Rightarrow K_1 = A \left( 1 - e^{-t_1/RC} \right) = \frac{e^{-t_1/RC}}{e^{-t_1/RC}} = A \]

\[ V_c(t) = Ae^{-t/RC} - \frac{e^{-t_1/RC}}{e^{-t_1/RC}} = \frac{(Ae^{-t/RC} - Ae)}{e^{-t_1/RC}} \]

\[ V_c(t) = (Ae^{-t/RC} - Ae) e^{-t/RC} = Ae^{-t_1/RC} - Ae^{-t/RC} \]
B.2) \( V_c(t) = (Ae^{t/vRC} - A) e^{-t/vRC} = Ae^{t-t/vRC} - Ae^{-t/vRC} \) 

C.1) If \( V_i > 0 \) \( \Rightarrow D_1 \) is on and \( D_2 \) is off

\[ V_0 = V_i \]

If \( V_i < 0 \) \( \Rightarrow D_2 \) is on and \( D_1 \) is off

\[ V_0 = -V_i \]

C.2)
(3) If we use D1 model then $V_i$ has to be greater than $V_f$ in order for $O_1$ to be on, and less than $-V_f$ in order for $O_2$ to be on.

If for instance $O_3$ is on then $V_o = V_i - V_f$.

The transfer characteristic is like the following.
By superposition:

\[ V_1 = 0 \quad V_2 \neq 0 \]
\[ V_0' = 0 \quad , \quad V_0 = V_2 \left( 1 + \frac{R_4}{R_3} \right) (\text{non-inverting amp.}) \]
\[ V_2 = 0 \quad V_1 \neq 0 \]
\[ V_0' = V_1 \left( 1 + \frac{R_2}{R_1} \right) \text{ non-inverting amp.} \]
\[ V_0 = -V_0' \left( \frac{R_4}{R_2} \right) \text{ inverting amp.} \]

\[ V_0 = V_2 \left( 1 + \frac{R_4}{R_3} \right) - V_1 \left( 1 + \frac{R_2}{R_1} \right) \frac{R_4}{R_3} = \]
\[ = V_2 \left( 1 + \frac{R_4}{R_3} \right) - V_1 \left( \frac{R_4}{R_3} + 1 \right) = (V_2 - V_1) \left( 1 + \frac{R_4}{R_2} \right) \]
D.2) Since the pump is ideal, \( I_t = 0 \) so
\[
R_{in_{1}} = 0
\]
\[
R_{in_{2}} = 0
\]

D.4) \[
V_o = \left( V'_{2} - V'_{1} \right) \left( 1 + \frac{R_1}{R_2} \right) = \left( V'_2 + m(t) - V'_1 - m(t) \right) \left( 1 + \frac{R_1}{R_2} \right)
\]
\[
= \left( V'_2 - V'_1 \right) \left( 1 + \frac{R_1}{R_2} \right)
\]

So the output noise is zero.
If \( V_A = V_B = V(0) \) then \( D_1 \) and \( D_2 \) are on so

\[
V_o = V(0) + (V(1) - V(0)) \frac{R_s}{R + R_s} = \frac{V(0)}{2R + R_s}
\]

(10) If \( V_A = V(0) \) and \( V_B = V(1) \)
then \( D_1 \) is on and \( D_2 \) is off

\[
V_o = V(0) + (V(1) - V(0)) \frac{R_s}{R + R_s} = V(0)
\]

(10) is symmetric to the previous case, so
\( D_2 \) is on and \( D_1 \) is on.

(11) \( D_1 \) and \( D_2 \) are both off \( \Rightarrow V_o = V(1) \)

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if \( i_o \) is on AND gate