We have never explored the behaviour of a diode for high negative voltage. A phenomenon called breakdown takes place where the electric field is so intense that the electrons' velocity is very high. An electron, then, hitting a atom can rip another electron from it. The new free electron can do the same and an avalanche phenomenon takes place:

\[ I \]

\[ V \]

\[ I \]

There are diodes that are built to work in the breakdown region. These diodes are called Zener diode and are particularly suited to build power supplies.
To give you the intuition, in breakdown, the V-I characteristic is almost vertical meaning that the voltage is a constant no matter what the current is. This is characteristic for a voltage source!

Let's use load line analysis for the following simple circuit:

\[ V + V_D + V_R = 0 \]
\[ V + V_D + R I_D = 0 \]
\[ \Rightarrow I_D = \frac{-V - V_D}{R} \]

Consider \( V > 0 \):

\[ \text{If } V_D = 0 \Rightarrow I_D = -\frac{V}{R} \]
\[ \text{If } V_0 = -V \Rightarrow I_D = 0 \]
The intersection of the load line with the diode characteristic gives us the quiescent point Q and hence the output voltage \( V_o \) and the current in the diode.

If the input voltage changes, then the load line changes.

We obtain a set of parallel lines because \( R \) is fixed.

The output voltage \( (V_o = -V_d) \) corresponding to the three quiescent points \( Q_1, Q_2, Q_3 \) is always the same. We are regulating the input voltage. The output voltage is then stable and fixed to a specific value.

When you buy a zener diode, it is characterized by a zener voltage which is the voltage at which the breakdown takes place.

It is clear that if the quiescent point is not in the breakdown region, then the output voltage depends on the input voltage. The diode then works as a regulator only if
\[ V > V_z \] where \( V_z \) is the reverse voltage. (\( V_z \) is given on the datasheet as a positive voltage, but the breakdown happens at \(-V_z\) of course).

Let's consider the case where we want to use the output voltage for instance to power on a CD-player. Device can be characterized by its Thevenin equivalent resistor. We denote the device with a load resistor \( R_L \):

\[ \text{This case is not different from the previous one.} \]
\[ \text{We can use the Thevenin equivalent circuit at } A - B. \]

\[ V_{TH} = \frac{V \cdot R_L}{R + R_L} \]

\[ R_{TH} = \frac{R \cdot R_L}{R + R_L} \]
We want the quiescent point to be always in the breakdown region. So it must be

\[-\frac{V_{RL}}{R + R_L} \leq -V_z \Rightarrow \frac{V_{RL}}{R + R_L} > V_z\]

On the other hand, we have to make sure that the zener diode doesn't break. When you buy a zener diode, it is also characterized by a maximum reverse current \( I_{DMAX} \). So we want that \( I_D > -I_{DMAX} \).
The previous inequalities have to hold in all operating conditions.

When we are designing a power supply, the problem is the following:

- We have a voltage source that is not stable. Its value can swing between $V_{\text{min}} \leq V \leq V_{\text{max}}$.

- The load is not fixed, it depends on what kind of device we connect to the power supply. So $R_{\text{min}} \leq R_L \leq R_{\text{max}}$.

In all possible cases we have to make sure the Q point is on the vertical line and doesn't cross the horizontal $I_{\text{Dmax}}$ line. $V_{\text{th}}$ is minimum when $V = V_{\text{min}}$ and $R_L = R_{\text{min}}$. So we have to have:

\[
\frac{V_{\text{min}}}{R + R_{\text{min}}} \geq V_Z
\]

The diode current is maximum when $V = V_{\text{max}}$ and $R_L = R_{\text{max}}$. 
To understand this, we notice that if the diode is in breakdown then the voltage across it is equal to $-V_z$. We can then replace it with a voltage source whose value is $V_z$.

\[
\begin{align*}
V_{TH} & \quad \Rightarrow \quad V_z \\
R_{TH} & \quad \underset{M}{\Rightarrow} \quad I \\
\end{align*}
\]

So,

\[
I = -I_D = \frac{V_{TH} - V_z}{R_{TH}} = \frac{V \frac{R_L}{R + R_L} - V_z}{R R_L} = \frac{V R_L - V_z (R + R_L)}{R (R + R_L) R R_L R R_L}
\]

\[= \frac{V}{R} - \frac{V_z}{R R_L}\]

$I_D$ is maximize for $V = V_{max}$ and $R_L = R_{Lmax}$.

So,

\[
I_D \geq I_{Dmax} \Rightarrow I \leq I_{Dmax}
\]

\[
\frac{V_{max}}{R} - \frac{V_z (R + R_{Lmax})}{R R_{Lmax}} < I_{Dmax}
\]

\[\boxed{\text{2}}\]
Using (1) and (2) we can design our power supply.

For instance:

We want to design a power supply circuit to stabilize an input voltage that can change between 9 V and 10 V. Also the maximum load is $\infty$ (nothing is connected to the p.s.o.) and 10 $\Omega$. The output voltage has to be 5 V.

So:

\[ V_Z = 5 \, \text{V} \quad V_{\text{min}} = 9 \, \text{V} \quad V_{\text{max}} = 10 \, \text{V} \]

\[ R_{\text{min}} = 10 \, \Omega \quad R_{\text{max}} = \infty \]

From (1)

\[ \frac{9 \cdot 10}{R + 10} \geq 5 \Rightarrow 90 \geq 5R + 50 \]

\[ \Rightarrow 40 \geq 5R \Rightarrow R \leq 8 \, \Omega \]

From (2)

\[ \frac{10}{8} - \frac{5}{8} < I_{\text{Dmax}} \Rightarrow I_{\text{Dmax}} > \frac{5}{8} \, \text{A} \]
We computed the properties of both R and the zener diode. We have to consider the power absorbed by the resistor. The voltage across it is \( V - V_Z \) so the power is:

\[
P_R = \frac{(V - V_Z)^2}{R}
\]

It is maximum when \( V = V_{\text{max}} \):

\[
P_{R_{\text{max}}} = \frac{(V_{\text{max}} - V_Z)^2}{R} = \frac{(10 - 5)^2}{8} = \frac{25}{8} \text{ W}
\]

The maximum power shipped to the load is when the resistor \( R_L \) is minimum:

\[
P_{L_{\text{max}}} = \frac{(V_Z)^2}{R_{L_{\text{min}}}} = \frac{25}{10} \text{ W}
\]

so notice that the power dissipated on R is not negligible. The efficiency of this p.s. is:

\[
\eta = \frac{P_{L_{\text{max}}}}{P_{\text{Rmax}} + P_{\text{Zmax}} + P_{D_{\text{Zmax}}}} = \frac{25/10}{25/10 + 23/8 + 5.6} = 28.6\%
\]
The reason is that the original constraints where pretty hard. Usually $V_{\text{max}} - V_{\text{min}}$ is of the order of millivolts.

We know now that this stage is useful to stabilize a voltage for different loads. We have to understand how to transform the 115VAC coming out from the outlet to finally have a 12VDC for example.

We first use a transformer to decrease the amplitude of the 115VAC. A transformer is built using two inductors $L_1$ and $L_2$. The primary voltage (115VAC for instance) is connected to $L_1$ that will generate a magnetic field. The magnetic field lines will pass through the second solenoid $L_2$ an will generate a voltage across $L_2$.

If $L_1$ has $N_1$ turns an $L_2$ $N_2$ turns, then: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$.
By designing $N_1$ and $N_2$ we can choose $V_2$ if we know $V_1$ (in our case 115 VAC).

For instance if we want $V_2 = 11.5$ VAC

$$\Rightarrow \frac{N_1}{N_2} = 10$$

we could use $N_1 = 100$ and $N_2 = 10$.

We need a note here:

When talking about AC signals we use RMS (Root Mean Square) voltage.

Consider a circuit like the following.

\[ V(t) = A \cos(wt) \]

\[ V \text{ and } I \text{ are AC (sinusoidal signals). We want to compute the power that is transferred to } R. \]

\[ p(t) = V(t) I(t) = \frac{V(t)^2}{R} = \frac{A^2 \cos^2(wt)}{R} \]

This power is instantaneous. We can compute the average transferred in a period $T = \frac{2\pi}{w}$

\[ P_R = \frac{1}{R} \frac{1}{T} \int_0^TA^2 \cos^2(wt) \, dt = \frac{1}{R} \frac{1}{T} \int_0^T A^2 \left( \frac{1}{2} + \cos(2wt) \right) \, dt \]

\[ \cos^2(wt) = \frac{1 + \cos(2wt)}{2} \]
Now we ask ourselves what would be the value of a DC voltage source that will make $R$ absorb the same power?

\[ P_R = \frac{A^2}{2R} \]

Then

\[ \frac{V^2}{R} = \frac{A^2}{2R} \Rightarrow V = \frac{A}{\sqrt{2}} \]

This is the RMS voltage.

So an AC voltage whose peak value is $A$ has an RMS voltage equal to $\frac{A}{\sqrt{2}}$.

When you buy a transformer, the output is given in RMS voltage, so for instance an output of 12 V AC means a sinusoid whose peak value is $12\sqrt{2}$ V.
So now we have a sinusoidal voltage whose amplitude is $A$.

We want to get something in closer to a DC voltage in order not to stress the regulator too much.

We can use a peak detector with a diode and a capacitor:

The diode acts as an half wave rectifier. The capacitor charges to the peak value of the voltage and discharges when the diode is off (during the negative half of the sinusoid). The capacitor discharges with a time constant equal to $R_L C$. 
Where R<sub>2</sub> is a model of our regulator. The residual voltage oscillation is called ripple. We can compute it very easily:

The capacitor is charged at \( A \) and the time constant is \( R_c C \). Also, the input frequency is 60 Hz so we need to compute the voltage for \( t = \frac{n}{60} \).

\[
\Delta V = A - A e^{-\frac{t}{60C}} = A - A e^{-\frac{n}{60C}}
\]

If \( R_c = 10 \Omega \) and we want a ripple which is 1% of the voltage \( A \), then:

\[
\frac{\Delta V}{A} = 0.01 = 1 - e^{-\frac{1}{600C}} \Rightarrow -0.99 = -e^{-\frac{1}{600C}}
\]

\[
\Rightarrow -\frac{1}{600C} = \ln(0.99) \Rightarrow C = -\frac{1}{600 \ln(0.99)} = 0.166 \text{F}
\]

which is a huge capacitor.

We can use a smaller capacitor in a voltage regulator using Zener diode.
We have the following specifications:

- $R_{\text{min}} = 10 \, \Omega$, $V_{\text{out}} = 5 \, V$

Assuming the zener diode is ideal, we can use the following method:

$$V_z = 5 \, V$$
we fix $\eta = \frac{P_{\text{load}}}{P_{\text{load}} + P_{\text{zener}} + P_{\text{diode}}} = 80\%$

$$\frac{(V_z)^2}{R_{\text{min}}} = 0.8$$

$$\frac{(V_{3\text{max}} - V_z)^2}{R} + \frac{(V_{2z})^2}{R_{\text{min}}} + V_z I_{\text{D,MAX}}$$

$$2.5 = 0.9 \left[ \frac{(V_{3\text{max}} - V_z)^2}{R} + 2.5 + 5 I_{\text{D,MAX}} \right] \Rightarrow$$

$$\frac{0.8}{R} (V_{3\text{max}} - 5)^2 + 5 I_{\text{D,MAX}} = 2.5 \times 0.2 = \frac{1}{2}$$
\[
\frac{V_{3\text{max}}}{R} - \frac{V_{3\text{avg}}}{R} < I_{3\text{max}}
\]

From (1)

\[
\frac{V_{3\text{min}}}{R+10} \geq 5
\]

We have 3 eq. in 4 unknowns. We could now fix the input ripple which is \(V_{3\text{max}} - V_{3\text{min}}\) to obtain another eq.

We notice that \(I = \frac{V_3 - V_2}{R}\) and can be approximately considered constant so the voltage decreases as a ramp because \(V = \int i \, dt\). So:

\[
V_{3\text{max}} - V_{3\text{min}} = \frac{V_3 - V_2}{RC} \Delta t = \frac{V_3 - V_2}{RC} \frac{1}{60}
\]

This is also \(I_{3\text{max}}\) because when \(R = \infty\) this current passes entirely through the diode.
\[
\begin{align*}
\frac{V_{z,\max} - V_z}{R} & \leq I_{D,\max} \\
\frac{V_{z,\min}}{R + 10} & \geq 5 \\
V_{z,\max} - V_{z,\min} & = \frac{I_{D,\max}}{C} \frac{1}{60} \\
0.8 \left( V_{z,\max} - 5 \right)^2 + 5I_{D,\max} & = \frac{1}{2} \\
\text{If we fix } C & = 2200 \mu F \\
V_{z,\min} & = V_{z,\max} - \frac{I_{D,\max}}{6 \cdot 2200 \cdot 10^{-5}} = V_{z,\max} - \frac{I_{D,\max}}{13.2 \cdot 10^{-3}} \\
\left\{ \begin{align*}
V_{z,\max} - 5 & \leq RI_{D,\max} \\
10V_{z,\max} & - \frac{I_{D,\max}}{13.2 \cdot 10^{-3}} \geq 5R + 50 \\
0.8 \left( V_{z,\max} - 5 \right)^2 + 5I_{D,\max} & = \frac{1}{2}
\end{align*} \right.
\end{align*}
\]
This is a non-linear system of inequalities. It can have of course one solution (one) many solution (and you have to pick the most convenient) or \( \varnothing \) solutions (meaning that your choice for \( c \) of \( f \) are bad choices).

After solving the system, you will find \( V_{3\text{max}} \) and \( \frac{V_{3\text{max}}}{\sqrt{2}} \) will be the RMS value of your transformer output.