Problem 1: Logic Gates and Timing Diagrams [25 points]

Consider the following digital logic circuit:

\[ G = \overline{\overline{A} \overline{B}} + \overline{A} + \overline{B} \]

by DeMorgan's Theorem

a) Fill out the truth table for the logic function G. [8 pts]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

b) Write a simple logical expression for the function G. [5 pts]

\[ G = \overline{A} \overline{B} + \overline{A} + \overline{B} \]

c) How many unit gate delays are there between the inputs (A and B) and the output (G)? [2 pts]

(In other words, how many unit gate delays must you wait, after changing A and/or B, before you can trust the value of G to be valid?)

The longest path between the input variables and the output variable is 3 logic gates. Therefore, we need to wait for a period of 3 unit gate delays after an input variable is changed, before we can trust the value of G to be valid.

d) Assume each logic gate has a unit gate delay \( \tau = 100 \) ps.

Draw the timing diagrams for \( t = 0 \) to \( t = 700 \) ps, for the given logic input values A and B. [10 pts]

- Logic value of A
- Logic value of B
- Logic value of C = \( \overline{A} \)
- Logic value of D = \( \overline{B} \)
- Logic value of E = \( \overline{E} \)
- Logic value of F = \( \overline{F} \)
- Logic value of G = \( \overline{G} \)
Problem 2: Resistive Circuits [30 points]

a) Find the equivalent resistance $R_{eq}$ for the following circuit. [6 pts]

\[ R_{eq} = 13 \Omega \]

9.2 ohm resistors in series → 18.2 ohm resistor
10.2 ohm resistors in parallel → 5.1 ohm resistor

b) Suppose you need a 6 kΩ resistor for your Tutebot project, but your TA gives you only a supply of 10 kΩ resistors. Being a clever Cal student, how would you connect several 10 kΩ resistors together, to achieve a 6 kΩ resistance? [7 pts]

- To achieve an equivalent resistance lower than the individual resistors, we should connect resistors in parallel.
- But the parallel combination of 2 10 kΩ resistors is 5 kΩ — too low!
- Need to increase the resistance of one of the parallel branches.
- Try parallel combination of a 10 kΩ resistor and two 10 kΩ resistors in series:

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\begin{align*}
10k & \quad 20 \cdot 10 = 6.7 kΩ \quad \text{too high}\ \\
10k & \quad 20 + 10 = 6.7 kΩ \quad \text{too high}\ \\
10k & \quad 30 k \quad 0 \quad \text{OK}\ \\
\end{align*}
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(see Page 4)

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Problem 2 (continued)

c) Consider the following circuit:

\[ R_1 = 1 \text{kΩ} \\
R_2 = 2 \text{kΩ} \\
R_3 = 2 \text{kΩ} \\
R_4 = 2 \text{kΩ} \\
I_1 = 1 \text{mA} \\
V = 6 \text{V} \]

i) Find $V_{bd}$. [3 pts]

\[ I_3 = 0 \text{ since terminal c is not connected} \]

Thus the current flowing through $R_1$ equals the current flowing through $R_2$, i.e., we have a voltage divider.

\[ V_{bd} = \frac{R_2}{R_1 + R_2} \cdot V = 2 \text{V} \]

ii) Find the power developed/absorbed by the current source, $P_I$. [3 pts]

The voltage across the current source is established by the voltage source and is equal to 6V.

\[ P_I = I V = (1 \text{mA})(6 \text{V}) = 6 \text{mW} \]

Sent positive current is entering the positive terminal of the current source, it is absorbing power.

iii) Indicate in the table below (by checking the appropriate boxes) how various circuit parameters would change if the terminals c and d were to be shorted together. Justify your answers. [6 pts]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value will:</th>
<th>Brief Explanation/Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{bd}$</td>
<td>increase</td>
<td>The resistance between b and d decreases; by the voltage-divider formula, $V_{bd}$ decreases</td>
</tr>
<tr>
<td>$I_1$</td>
<td>✓</td>
<td>Total resistance between a and d decreases; $V_{bd}$ remains 6V; $I_1 = \frac{V}{R_3}$</td>
</tr>
<tr>
<td>Power developed by voltage source</td>
<td>✓</td>
<td>Since $I_3$ increases, the current supplied by the voltage source increases.</td>
</tr>
</tbody>
</table>

iv) What is the value of $I_3$ when the terminals c and d are shorted together? [5 pts]

Equivalent resistance between terminals a and d is

\[ R_{eq} = R_1 + \frac{R_2}{R_3} = 1 + \frac{2}{2 + 2} = 2 \text{kΩ} \]

\[ I_3 = \frac{V}{R_{eq}} = 3 \text{mA} \]

Using current-divider formula,

\[ I_3 = \frac{1}{2} (3 \text{mA}) = 1.5 \text{mA} \]
Problem 3: Nodal Analysis [20 points]

a) In the circuit below, the independent source values and resistances are known.
Use the nodal analysis technique to write 3 equations sufficient to solve for $V_a$, $V_b$, and $V_c$.
To receive credit, you must write your answer in the box below. [10 pts]
DO NOT SOLVE THE EQUATIONS!

Apply Kirchhoff's Current Law to nodes a, b, c:
(Sum of currents entering a node = 0)

(a) \[ \frac{V_a - V_b}{R_1} + I_{BB} - I_{CC} + \frac{V_b - V_c}{R_3} = 0 \]

(b) \[ \frac{V_a - V_b}{R_3} + \frac{V_b - V_c}{R_4} + \frac{V_c - V_b}{R_5} = 0 \]

(c) \[ I_{CC} + \frac{V_a - V_c}{R_5} - \frac{V_c - V_b}{R_6} = 0 \]

Problem 3 (continued)

b) Similarly to part (a), use the nodal analysis technique to write 3 equations sufficient to solve for $V_a$, $V_b$, and $V_c$. To receive credit, you must write your answer in the box below. [10 pts]
DO NOT SOLVE THE EQUATIONS!

Current flowing through the voltage source $V_{BB}$ cannot be expressed as a function of the node voltages $V_a$ and $V_b$.

=> use the "supernode" approach.

Applying Kirchhoff's Current Law to the supernode and node c:

supernode: \[ \frac{V_a - V_b}{R_1} + I_{BB} + \frac{-V_c - V_b}{R_3} + I_{CC} = 0 \]

node c: \[ \frac{V_b - V_c}{R_4} + I_{CC} = 0 \]

Need one more equation in order to be able to solve for the 3 unknowns:

$V_b - V_a = V_{BB}$

Write the nodal equations here:

\[
\begin{align*}
\frac{V_a - V_b}{R_1} + I_{BB} - I_{CC} + \frac{V_b - V_c}{R_3} &= 0 \\
\frac{V_a - V_b}{R_3} + \frac{V_b - V_c}{R_4} + \frac{V_c - V_b}{R_5} &= 0 \\
I_{CC} + \frac{V_a - V_c}{R_5} - \frac{V_c - V_b}{R_6} &= 0 \\
\frac{V_a - V_b}{R_1} + I_{BB} - \frac{V_c + V_b}{R_3} + I_{CC} &= 0 \\
\frac{V_b - V_c}{R_4} + I_{CC} &= 0 \\
V_b - V_a &= V_{BB}
\end{align*}
\]
Problem 4: Thevenin and Norton Equivalent Circuits [25 points]

a) Find the Thevenin Equivalent Circuit for the following circuit. [10 pts]

The open-circuit voltage, $V_{oc}$, is equal to $V_{ab}$ which is equal to $V_x$ since no current is flowing through the 2kΩ resistor. Applying KCL to node X (defining node b as the reference node)

\[ \frac{5-V_x}{3} + \frac{-4-V_x}{6} = 0 \]

\[ 6 \cdot 3V_x = 2V \]

\[ V_x = 2V \]

\[ V_{oc} = V_{th} = 2V \]

To find $R_{th}$, set all the independent sources to zero:

b) Use the source transformation method to obtain the Norton Equivalent Circuit for the circuit in part (a). [5 pts]

\[ R_N = R_{th} = 4kΩ \]

\[ I_N = \frac{V}{R} = \frac{2V}{4kΩ} = 0.5 mA \]

Problem 4 (continued)

c) The Thevenin Equivalent Circuit for a certain linear circuit is given below. Plot the current ($I$) versus the output voltage ($V$) for the circuit, labelling the y-intercept and x-intercept. [5 pts]

\[ I = \frac{0-(-6V)}{200} = \frac{-6V}{200} \]

\[ I = 30 mA \]

d) The circuit in part (c) is connected to a 1 kΩ load resistor (placed between the terminals a and b). Find the power absorbed in the load resistor, $P_{lk}$ [5 pts]

\[ P_{lk} = 25 mW \]

Using voltage-divider formula,

\[ V = \frac{1000}{1000+200}(-6) = -5V \]

\[ P = IV = \frac{V}{R}V = \frac{(-5)^2}{1000} = 25 mW \]