Midterm Exam # 1
March 2, 2004
Time Allowed: 90 minutes

Name: ___________________________  ___________________________
       Last                              First

Student ID #:_____________________,  Signature:_____________________

Discussion Section:_________________________

This is a closed-book exam, except for use of one 8.5 x 11 inch sheet of your notes. Show all your work to receive full or partial credit. Write your answers clearly in the spaces provided.

<table>
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<tr>
<th>Problem #</th>
<th>Points:</th>
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<tr>
<td>1</td>
<td>/20</td>
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<tr>
<td>2</td>
<td>/20</td>
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<td>3</td>
<td>/10</td>
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<td>Total</td>
<td>/50</td>
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a) (2 points)
In the circuit shown in Figure 1(a), the independent source values and resistances are known. Given the indicated reference potential, list the unknown node potentials in the circuit of Figure 1(a).

\[
V_b \quad V_c
\]
b) (8 points)
Write down a complete set of node equations sufficient to solve for the node potentials you listed in part (a). Do not solve! Write your node equations in the box below.

\[ b : -I_A + \frac{V_b - V_A}{R_5} + \frac{V_b - V_c}{R_b} - I_B + \frac{V_b}{R_3} = 0 \]

\[ c : \frac{V_c}{R_4} + I_B + \frac{V_c - V_b}{R_6} = 0 \]
c) (2 points)
How many meshes would be required to solve the circuit of Figure 1(a) by the mesh analysis method?

4.

d) (8 points)

![Figure 1(d)](image)

In the circuit of Figure 1(d), the independent source values and resistances are known. Use the node voltage method to write three equations sufficient to solve for the node potentials $V_a$, $V_b$, and $V_c$. Write your equations in the box below. Do not solve!

\[
\begin{align*}
V_b - V_a &= V_1 \\
V_b - V_c &= V_2 \\
-I_A + \frac{V_b}{R_1} + \frac{V_c}{R_2} &= 0
\end{align*}
\]
2.

![Figure 2(a)](image)

a) (10 points)
Determine the Thevenin equivalent circuit for the circuit in Figure 2(a).

**Hint:** superposition. Write your answer in the box at the bottom of the page.

\[ V_1 = I_1 R_1 = \frac{V_A}{R_1 + R_2} R_1 \]

\[ V_2 = I_2 R_L = \frac{V_B}{R_1 + R_2} R_L \]

\[ V_3 = -I_A (R_1 + R_2) = -\frac{I_A R_1 R_2}{R_1 + R_2} \]

\[ V_{th} = V_1 + V_2 + V_3 = \frac{V_A R_1}{R_1 + R_2} + \frac{V_B R_2}{R_1 + R_2} - \frac{I_A R_1 R_2}{R_1 + R_2} \]

\[ R_{th} : \text{zero all the independent sources} \]

\[ R_{th} = \frac{R_1 R_2}{R_1 + R_2} \]
b) (10 points)

From figure,

when $i_1 = 0$, $V_i = 2V \Rightarrow V_{th1} = 2V$

when $i_1 = -1mA, V_i = 0 \Rightarrow R_1 = 2k\Omega$

\[ V_1 = 2k\Omega \cdot i_1 + 2V \]

\[ V_z = R_2 V_z + V_{th2} \]

From figure

when $i_2 = 0$, $V_2 = -1V \Rightarrow V_{th2} = -1V$

when $i_2 = 1mA, V_2 = 0 \Rightarrow R_2 = 1k\Omega$

\[ V_z = 1k\Omega \cdot i_2 - 1V \]
One-port Networks #1 and #2 are interconnected as shown in Figure 2(b). Each of the one-port networks in Figure 2(b) is characterized by its indicated v-i graph. Determine the Thevenin equivalent network and the Norton equivalent networks for the one-port network shown in the figure by accessing the circuit at the terminals labeled a and b. Write your answer in the box below.

\[ V_{th} = V_{th1} + V_{th2} = 2V + (-1V) = 1V \]

\[ R_{th} = R_1 + R_2 = 2k\Omega + 1k\Omega = 3k\Omega \]

\[ I_N = \frac{V_{th}}{R_{th}} = \frac{1V}{3k\Omega} = \frac{1}{3} mA \]

\[ R_N = R_{th} = 3k\Omega \]

<table>
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<tr>
<th>( V_{th} )</th>
<th>( R_{th} )</th>
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<tbody>
<tr>
<td>1V</td>
<td>3k\Omega</td>
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<table>
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<tr>
<th>( I_N )</th>
<th>( R_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{3} ) mA</td>
<td>3k\Omega</td>
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The op-amp in Figure 3 is ideal. The figure shows a temperature sensor modeled as a temperature-controlled current source. This device senses absolute temperature $T_A$ in the ($^\circ K$) Kelvin scale and delivers a current $kT_A$, where $k = 1 \mu A/^\circ K$.

a) (5 points)
Determine the output voltage as a function of temperature $T_A (^\circ K)$ in terms of the circuit parameters.

As it is negative feedback, we know

$$V_A = V_n = V_p = 0 \quad (\text{virtual short})$$

$$i_n = 0 \quad (\text{virtual open})$$

Write KCL equation for node A:

$$kT_A + \frac{0 - V_{ss}}{R_1} + \frac{0 - V_0}{R_2} = 0$$

$$\Rightarrow V_0 = kR_2 T_A - \frac{R_2}{R_1} V_{ss}$$
b) (5 points)
Determine values for $R_1$ and $R_2$ so that the output voltage sensitivity is 100 $mV/°K$ and the output is zero volts at 300 $°K$. Write your answer in the box below.

\[
\frac{dV_o}{dT_A} = k R_2 = 100 mV/°K
\]

\[
R_2 = \frac{100 mV/°K}{k} = \frac{100 mV/°K}{1 mA/°K} = 10^5 \Omega = 100 k\Omega
\]

$T_A = 300 \, K$, $V_0 = 0$

\[
V_0 = k R_2 T_A - \frac{R_2}{R_1} V_{ss}
\]

$\Rightarrow 0 = 100 mV/°K \times 300 K - \frac{R_2}{R_1} \times 10 V$

\[
10 V \times \frac{R_2}{R_1} = 30 V
\]

\[
\frac{R_2}{R_1} = 3
\]

\[
R_1 = \frac{R_2}{3} = \frac{100 k\Omega}{3} = 33 k\Omega
\]

\begin{align*}
R_1 & = 33 k\Omega \\
R_2 & = 100 k\Omega
\end{align*}