Electrical Resistance

where $\rho$ is the resistivity (Units: $\Omega$-cm)
What is a Semiconductor?

- Low resistivity => “conductor”
- High resistivity => “insulator”
- Intermediate resistivity => “semiconductor”

- Generally, the semiconductor material used in integrated-circuit devices is crystalline

  - In recent years, however, non-crystalline semiconductors have become commercially very important

  ![Semiconductor Types](image)

Semiconductor Materials

**Elemental:**

- He
- H
- C
- N
- O
- F
- Ne
- Al
- Si
- P
- S
- Cl
- Ar
- Zn
- Ga
- Ge
- As
- Se
- Br
- Kr
- Rb
- Sr
- Y
- Zr
- Nb
- Mo
- Cs
- Ba
- La
- Ce
- Pr
- Nd
- Pm
- Sm
- Eu
- Gd
- Tb
- Dy
- Ho
- Er
- Tm
- Yb
- Lu
- Ac
- Th
- Pa
- U

**Compound:**

- SiO
- GeO
- AsO
- SeO
- PO
- AsS
- GeS
- AsTe
- SeTe
- POI
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
- AsTe
- GeTe
- AsSe
- SeTe
- PO
The Silicon Atom

- 14 electrons occupying the 1st 3 energy levels:
  - 1s, 2s, 2p orbitals filled by 10 electrons
  - 3s, 3p orbitals filled by 4 electrons

To minimize the overall energy, the 3s and 3p orbitals hybridize to form 4 tetrahedral 3sp orbitals

Each has one electron and is capable of forming a bond with a neighboring atom

The Si Crystal

“diamond cubic” lattice

- Each Si atom has 4 nearest neighbors
- Lattice constant = 5.431 Å
Compound Semiconductors

• “zinc blende” structure
• III-V compound semiconductors: GaAs, GaP, GaN, etc.
  ✓ important for optoelectronics and high-speed ICs

Electronic Properties of Si

• Silicon is a semiconductor material.
  Pure Si has relatively high resistivity at room temperature.

• There are 2 types of mobile charge-carriers in Si:
  *Conduction electrons* are negatively charged.
  *Holes* are positively charged. They are an “absence of electrons”.

• The concentration of conduction electrons & holes in a semiconductor can be affected in several ways:
  1. by adding special impurity atoms *(dopants)*
  2. by applying an electric field
  3. by changing the temperature
  4. by irradiation
Conduction Electrons and Holes

2-D representation

When an electron breaks loose and becomes a *conduction electron*, a *hole* is also created.

Note: A hole (along with its associated positive charge) is mobile!

Definition of Parameters

- $n = \text{number of mobile electrons per cm}^3$
- $p = \text{number of holes per cm}^3$
- $n_i = \text{intrinsic carrier concentration (#/cm}^3\text{)}$

In a pure semiconductor,

\[ n = p = n_i \]
**Generation**
- We have seen that conduction (mobile) electrons and holes can be created in pure (intrinsic) silicon by *thermal generation*.
  - Thermal generation rate increases exponentially with temperature $T$.
- Another type of generation process which can occur is *optical generation*.
  - The energy absorbed from a photon frees an electron from covalent bond.
    - In Si, the minimum energy required is $1.1\text{eV}$, which corresponds to $\sim 1\mu\text{m}$ wavelength (infrared region). $1\text{ eV} = \text{energy gained by an electron falling through 1 V potential} = q_eV = 1.6 \times 10^{-19} \text{C} \times 1\text{ V} = 1.6 \times 10^{-19} \text{J}$.
- Note that conduction electrons and holes are continuously generated, if $T > 0$.

**Recombination**
- When a conduction electron and hole meet, each one is eliminated, a process called “recombination”. The energy lost by the conduction electron (when it “falls” back into the covalent bond) can be released in two ways:
  1. to the semiconductor lattice (vibrations)
     - “thermal recombination” $\rightarrow$ semiconductor is heated
  2. to photon emission
     - “optical recombination” $\rightarrow$ light is emitted
- Optical recombination is negligible in Si. It is significant in compound semiconductor materials, and is the basis for light-emitting diodes and laser diodes.
Pure Si

Covalent (shared e\textsuperscript{\textminus}) bonds exist between Si atoms in a crystal. Since the e\textsuperscript{\textminus} are loosely bound, some will be free at any T, creating hole electron pairs.

\[ n_i = 3.9 \times 10^{16} T^{3/2} e^{-\frac{0.605 eV}{kT}} \text{ cm}^{-3} \]

\[ n_i \approx 10^{10} \text{ cm}^{-3} \text{ at room temperature} \]

Doping

By substituting a Si atom with a special impurity atom (Column V or Column III element), a conduction electron or hole is created.

**Donors: P, As, Sb**

**Acceptors: B, Al, Ga, In**

Dopant concentrations typically range from $10^{14}$ cm\textsuperscript{-3} to $10^{20}$ cm\textsuperscript{-3}
**Charge-Carrier Concentrations**

\( N_D \): ionized donor concentration (cm\(^{-3}\))

\( N_A \): ionized acceptor concentration (cm\(^{-3}\))

Charge neutrality condition: \( N_D + p = N_A + n \)

At thermal equilibrium, \( np = n_i^2 \) ("Law of Mass Action")

\[
\begin{align*}
n &= \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} \\
p &= \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}
\end{align*}
\]

Note: Carrier concentrations depend on net dopant concentration \((N_D - N_A)\)!

**N-type and P-type Material**

If \( N_D >> N_A \) (so that \( N_D - N_A >> n_i \)):

\[
\begin{align*}
n &\approx N_D - N_A \\
p &\approx \frac{n_i^2}{N_D - N_A}
\end{align*}
\]

\( n >> p \rightarrow \text{material is "n-type"} \)

If \( N_A >> N_D \) (so that \( N_A - N_D >> n_i \)):

\[
\begin{align*}
p &\approx N_A - N_D \\
n &\approx \frac{n_i^2}{N_A - N_D}
\end{align*}
\]

\( p >> n \rightarrow \text{material is "p-type"} \)
Terminology

**intrinsic** semiconductor: “undoped” semiconductor
electrical properties are native to the material

**extrinsic** semiconductor: doped semiconductor
electrical properties are controlled by the added impurity atoms

**donor**: impurity atom that increases the electron concentration
group V elements (P, As)

**acceptor**: impurity atom that increases the hole concentration
group III elements (B, In)

**n-type** material: semiconductor containing more electrons than holes

**p-type** material: semiconductor containing more holes than electrons

**majority carrier**: the most abundant carrier in a semiconductor sample

**minority carrier**: the least abundant carrier in a semiconductor sample

---

Carrier Scattering

- Mobile electrons and atoms in the Si lattice are always in random thermal motion.
  - Average velocity of thermal motion for electrons in Si:
    - \( \sim 10^7 \text{ cm/s} @ 300\text{K} \)
  - Electrons make frequent “collisions” with the vibrating atoms
    - “lattice scattering” or “phonon scattering”
  - Other scattering mechanisms:
    - deflection by ionized impurity atoms
    - deflection due to Coulombic force between carriers
- The average current in any direction is zero, if no electric field is applied.
Carrier Drift

- When an electric field (e.g., due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:

- Electrons drift in the direction opposite to the $E$-field → Current flows

- Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as classical particles moving at a constant average drift velocity.

Drift Velocity and Carrier Mobility

Mobile charge-carrier drift velocity is proportional to applied $E$-field:

$$|v| = \mu E$$

$\mu$ is the mobility (Units: cm$^2$/V$\cdot$s)

Note: Carrier mobility depends on total dopant concentration ($N_D + N_A$)!
**Current Density**

The current density $J$ is the current per unit area 
($J = I / A$ ; $A$ is the cross-sectional area of the conductor)

If we have $N$ positive charges per unit volume moving 
with average speed $v$ in the $+x$ direction, then the current 
density in the $+x$ direction is just $J = qNv$

*Example:*

2 x $10^{16}$ holes/cm³ moving to the right at 2 x $10^4$ cm/sec

$J = 1.6 \times 10^{-19} \times 2 \times 10^{16} \times 2 \times 10^4 = 64 \text{ A/cm}^2$

Suppose this occurs in a conductor 2 μm wide and 1 μm thick:

$I = J \times A = 64 \times (2 \times 10^{-4} \times 1 \times 10^{-4})$

= 1.28 μA

---

**Electrical Conductivity $\sigma$**

When an electric field is applied, current flows due to 
drift of mobile electrons and holes:

- **electron current density:** 
  $J_n = (-q)nv_n = qn\mu_n E$

- **hole current density:** 
  $J_p = (+q)pv_p = qp\mu_p E$

- **total current density:** 
  $J = J_n + J_p = (qn\mu_n + qp\mu_p)E$

$J = \sigma E$

**conductivity** 
$\sigma \equiv qn\mu_n + qp\mu_p$ (Units: Ω-cm⁻¹)
Electrical Resistivity $\rho$

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

$$\rho \approx \frac{1}{qn\mu_n} \quad \text{for n-type mat'l}$$

$$\rho \approx \frac{1}{qp\mu_p} \quad \text{for p-type mat'l}$$

(Units: ohm-cm)

---

Example

Consider a Si sample doped with $10^{16}$/cm$^3$ Boron. What is its resistivity?

**Answer:**

$$N_A = 10^{16}$/cm$^3$, \quad N_D = 0 \quad (N_A >> N_D \rightarrow \text{p-type})$$

$$\rightarrow p \approx 10^{16}$/cm$^3$ and \quad n \approx 10^4$/cm$^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \approx \frac{1}{qp\mu_p}$$

$$= \left[ (1.6 \times 10^{-19})(10^{16})(450) \right]^{-1} = 1.4 \ \Omega \text{ – cm}$$

*From $\mu$ vs. ($N_A + N_D$) plot*
Example (cont’d)

Consider the same Si sample, doped additionally with $10^{17}$/cm$^3$ Arsenic. What is its resistivity?

*Answer:*

$$N_A = 10^{16}$/cm$^3$, $N_D = 10^{17}$/cm$^3$ \ ($N_D \gg N_A \rightarrow \text{n-type}$)

$$\Rightarrow n \approx 9 \times 10^{16}$/cm$^3$ \ and \ $p \approx 1.1 \times 10^3$/cm$^3$

$$\rho = \frac{1}{qn \mu_n + qp \mu_p} \approx \frac{1}{qn \mu_n}

= \left[1.6 \times 10^{-19}\right] \left(9 \times 10^{16}\right) \left(700\right) \approx 0.10 \Omega \cdot \text{cm}$$

The sample is converted to n-type material by adding more donors than acceptors, and is said to be “compensated”.

---

**Sheet Resistance $R_s$**

$$R = \frac{\rho}{Wt} = R_s \frac{L}{W} \quad \Rightarrow \quad R_s \equiv \frac{\rho}{t} \quad \text{(Unit: ohms/square)}$$

(L, W, t = length, width, thickness) \ $R_s$ is the resistance when $W = L$

- The $R_s$ value for a given layer in an IC technology is used
  - for design and layout of resistors
  - for estimating values of parasitic resistance in a circuit

- $R = R_s$
- $R = R_s/2$
- $R = 2R_s$
- $R = 3R_s$
- $R \approx 2.6R_s$
**Integrated-Circuit Resistors**

The resistivity $\rho$ and thickness $t$ are fixed for each layer in a given manufacturing process.

A circuit designer specifies the length $L$ and width $W$, to achieve a desired resistance $R$

$$R = R_s \left( \frac{L}{W} \right)$$

**Example:** Suppose we want to design a 5 k$\Omega$ resistor using a layer of material with $R_s = 200$ $\Omega/\square$

---

**Summary**

- **Crystalline Si:**
  - 4 valence electrons per atom
  - diamond lattice: each atom has 4 nearest neighbors
  - $5 \times 10^{22}$ atoms/cm$^3$

- **In a pure Si crystal, conduction electrons and holes are formed in pairs.**
  - Holes can be considered as positively charged mobile particles which exist inside a semiconductor.
  - Both holes and electrons can conduct current.

- **Dopants in Si:**
  - Reside on lattice sites (substituting for Si)
  - Group V elements contribute conduction electrons, and are called **donors**
  - Group III elements contribute holes, and are called **acceptors**