Problem 1

Both amplifiers clearly have negative feedback, so we can use the summing point constraint (spc) in analysis of either op-amp. From the spc we know that $v_1 = v_2$ and $v_3 = 0$. Then we know that there is no current flowing through $R_2$ since the voltage drop across it is $v_3 - 0 = 0$. Therefore, $i_1 = \frac{v_{in}}{R_1} = i_2$. From KCL we know that $i_2 = i_3 + i_4$. Using KVL from the inverting input at the first stage of the amplifier, we find that $i_4 R_4 + v_2 = 0$ and $i_3 R_3 + v_{out} = 0$. Solving for the currents we find that

\[
\begin{align*}
  i_4 &= -\frac{v_2}{R_4} \\
  i_3 &= -\frac{v_{out}}{R_3}
\end{align*}
\]

Substituting into the KCL equation, we get

\[
  i_2 = \frac{v_{in}}{R} = i_3 + i_4 = -\frac{v_2}{R_4} - \frac{v_{out}}{R_3}
\]
From the voltage divider principle,

\[ v_2 = v_1 = \frac{R_5}{R_5 + R_6} v_{\text{out}} \]

\[ \frac{v_{\text{in}}}{R_1} = -\frac{v_2}{R_4} + \frac{v_{\text{out}}}{R_3} \]

\[ \frac{v_{\text{in}}}{R_1} = -\frac{1}{R_4 \frac{R_5}{R_5 + R_6}} v_{\text{out}} - \frac{v_{\text{out}}}{R_3} \]

\[ \frac{v_{\text{in}}}{R_1} = \frac{v_{\text{out}}}{R_1} \left[ -\frac{1}{R_4 \frac{R_5}{R_5 + R_6}} - \frac{1}{R_3} \right] \]

\[ \Rightarrow \frac{v_{\text{out}}}{v_{\text{in}}} = \frac{1}{R_1 \left[ -\frac{1}{R_4 \frac{R_5}{R_5 + R_6}} - \frac{1}{R_3} \right]} \]

\[ = -\frac{R_4 (R_5 + R_6) R_3}{R_1 (R_3 R_5 + R_4 (R_5 + R_6))} \]
Problem 2
Negative feedback is present in the system so the summing point constraint applies. By the SPC, no current flows into the non-inverting input of the opamp, so $R_4$ is inactive. The voltage at the inverting input is 0V, so no current flows through elements $L, C, R_2$ as there is no voltage drop across the terminals of these elements. The simplified circuit is of the form

$$
\begin{align*}
Z_1 & = R_1 + \frac{1}{j\omega C} \\
Z_2 & = Z_{R_3} / \omega C = \frac{R_3}{1 + j\omega CR_3}
\end{align*}
$$

where $Z_1 = R_1 + \frac{1}{j\omega C}$ and $Z_2 = Z_{R_3} / \omega C = \frac{R_3}{1 + j\omega CR_3}$

$$
H(j\omega) = -\frac{Z_2}{Z_1} = -\frac{R_3}{1 + j\omega CR_3} \cdot \frac{1}{R_1 + \frac{1}{j\omega C}} = -\frac{R_3}{1 + j\omega CR_3} \cdot \frac{j\omega C}{\omega R_1 C + 1} = -\frac{j\omega R_3 C}{(1 + j\omega CR_3)(1 + j\omega R_1 C)}
$$

$$
|H(j\omega)| = \frac{R_3 C}{\sqrt{(1 + (\omega CR_3)^2)(1 + (\omega R_1 C)^2)}}
$$

$$
|H(j\omega)|_{dB} = 20\log_{10}|H(j\omega)| = 20\log_{10}(R_3 C) - 10\log_{10}[1 + (\omega R_1 C)^2] - 10\log_{10}[1 + (\omega R_3 C)^2]
$$

$$
\angle H(j\omega) = -\angle[j\omega R_3 C] - \angle[1 + j\omega CR_3] - \angle[1 + j\omega R_1 C] = -\frac{\pi}{2} - \tan^{-1}(\omega CR_3) - \tan^{-1}(\omega CR_1)
$$

In the problem statement we are told that $R_3 >> R_2 >> R_1$, which in terms of the Bode approximation means that $R_3 \approx 100R_1$. The approximate Bode plots are given by
Note that the exact decibel values on the vertical axis of the magnitude plot are dependent on $R_1, R_3$ and are therefore omitted. The slope of the magnitude plot is 20dB/decade where it is non-zero.
Problem 3
Zeroing out the current source (replacing it with an open circuit, we find that the Thevenin resistance is

\[ R_{th} = R + j\omega L - \frac{j}{\omega C} \]
\[ = 10 + j\left(5 \times 10^{-3} \times 10^3 - \frac{1}{200 \times 10^{-6} \times 10^3}\right) \]
\[ = 10 + j(5 - 5) \]
\[ = 10 \Omega \]

The current through the source as a cosine function is
\[ I = 2\cos(1000t + \frac{\pi}{2} - \frac{\pi}{2}) = 2\cos(1000t) \]

The Thevenin voltage is the voltage drop across the impedance of the resistor and the impedance of the inductor:

\[ V_{th} = (Z_R + Z_L) \cdot (-I) \]
\[ = (10 + j5) \cdot (-I) \]
\[ = \sqrt{10^2 + 5^2}e^{j\tan^{-1}(0.5)} \cdot (-I) \]
\[ = -2\sqrt{10^2 + 5^2}\cos(1000t + \tan^{-1}(0.5)) \]

\[ I_N = \frac{V_{th}}{R_{th}} = -2\sqrt{10^2 + 5^2}\cos(1000t + \tan^{-1}(0.5)) \]
The short-circuit current \( i_{sc} \) flowing from terminal \( a \) to terminal \( b \) is \( i_{sc} = -I = -2\cos(1000t) \). Therefore, the Thevenin equivalent is

And the Norton equivalent is

\[ I_N = i_{sc} \]
Problem 4

Assume that $D_1$ is conducting and $D_2$ is not. Since we are assuming ideal diode behavior, this means that $D_2$ is replaced with an open circuit and $D_1$ is replaced with a short circuit, as shown below:

![Diode Circuit Diagram]

This assumption is correct if it can be shown that $v_{D_2} < 0$ and $i_{D_1} > 0$. $i_{D_1} = \frac{6V}{12k\Omega} = 0.5mA$. Using the voltage divider principle, $v_{D_2} = \frac{2k\Omega}{2k\Omega + 4k\Omega} \cdot 6V - \frac{8k\Omega}{8k\Omega + 4k\Omega} \cdot 6V = -2V$. Therefore, the assumptions are consistent. The power dissipated through the $8k\Omega$ resistor is given by $i_{D_1}^2 \cdot 8k\Omega = (0.5mA)^2 \cdot 8k\Omega = 2mW$

Many people made the error that the configuration of the circuit should be with both diodes conducting:

![Combined Circuit Diagram]

Combining resistances in parallel and series we get that the total current flowing out of the $6V$ source is equal to $i_1 = \frac{2}{3}mA$. Since $i_2$ and $i_3$ travel through the same resistance to the same voltage, the current must be split evenly, so $i_2 = i_3 = \frac{1}{2}i_1 = \frac{5}{6}mA$. $V_1 = \frac{1.64V}{1.64k\Omega + 2.4k\Omega} \cdot 6V = \frac{3}{3}V$. Then $i_4 = \frac{V_1}{2k\Omega} = \frac{4}{3}mA$ and $i_5 = \frac{V_1}{8k\Omega} = \frac{1}{3}mA$. This indicates that $i_{D_1} = i_4 - i_2 = 0.5mA$ flowing downwards, which contradicts the assumption that the diode is forward biased.
Problem 5
The opamp circuit is of the form of the standard inverting amplifier

\[ v_{out} = -Z_2 Z_1 v_{in} \]

where \( Z_1 = R_2 \) and \( Z_2 = \frac{1}{j\omega C} \). Then the transfer function is given by

\[ H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{Z_2}{Z_1} = -\frac{1}{R_2} \cdot \frac{R_1}{1 + j\omega R_1 C_1} \]

\[ |H(j\omega)|_{dB} = 20\log_{10} \left( \frac{R_1/R_2}{\sqrt{1 + (\omega R_1 C_1)^2}} \right) \]

\[ = 20\log_{10} \left( \frac{R_1}{R_2} \right) - 10\log_{10}[1 + (\omega R_1 C_1)^2] \]

\[ |H(j0)|_{dB} = 20 - 20\log_{10} \left( \frac{R_1}{R_2} \right) \]

\[ \Rightarrow R_2 = \frac{R_1}{10} \]

\[ |H(j\omega_x)|_{dB} = 20\log_{10} \left( \frac{R_1}{R_2} \right) - 10\log_{10}[1 + (\omega_x R_1 C_1)^2] \]

\[ -40 = 20 - 10\log_{10}[1 + (\omega_x R_1 C_1)^2] \]

\[ -60 = -10\log_{10}[1 + (\omega_x R_1 C_1)^2] \]

\[ 10^6 = 1 + (\omega_x R_1 C_1)^2 \]

\[ \Rightarrow C_1 = \frac{\sqrt{(10^6 - 1)}}{R_1 \omega_x} \approx \frac{1000}{R_1 \omega_x} \]