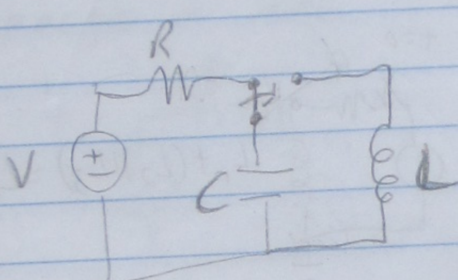
$V_L = V_S - Ri_L$ $\qquad$ $V_L' = V_S' - Ri_L'$

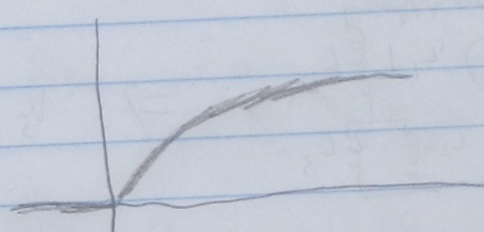
$$\cancel{V_S'(t)} - Ri_L' = \frac{-R\cdot(V_S - Ri_L(t))}{L} + \cancel{V_S'(t)}$$

$$i_L' = -\frac{R}{L} i_L(t) + \frac{V_S(t)}{L}$$

LC circuits

In left position capacitor charges.





In the right position, we have to do some work.

Node voltage gives us: $\qquad$ $i_C = CV_C'$ $\qquad$ $V_L = Li_L'$

$i_L = \int \frac{V_L}{L}$

$$CV_C' + \int \frac{V_L}{L} = 0$$

$$CV_C'' + \frac{V_L}{L} = 0$$

Note $V_L = V_C$ (parallel)

$$CV_C'' + \frac{V_C}{L} = 0.$$

How do we solve? Well, this is a 2nd order ODE so we use a new procedure (see handout).

$$Cs^2 + \frac{1}{L} = 0$$

$$s^2 = \frac{1}{LC}$$

or defining $\omega_0 = \sqrt{\frac{1}{LC}}$,

$$s = \pm\sqrt{-\frac{1}{LC}} \qquad s = \pm j\omega_0 \qquad j = \sqrt{-1}$$

Using our 2^{nd} order ODE procedure, this is just

$$v(t) = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$$

Using the fact that $e^{j\omega_0 t}$ is $\cos(\omega_0 t) + j\sin(\omega_0 t)$, we get:

$$v(t) = A_1(\cos(\omega_0 t) + j\sin(\omega_0 t)) + A_2(\cos(-\omega_0 t) + j\sin(-\omega_0 t))$$
$$+ A_2(\cos(\omega_0 t) - j\sin(\omega_0 t))$$

$$v(t) = (A_1 + A_2)\cos(\omega_0 t) + (A_1 - A_2) j\sin(\omega_0 t)$$

$$v(t) = K_1\cos(\omega_0 t) + K_2\sin(\omega_0 t)$$

Use initial conditions to find K_1, K_2.

$$v(0) = K_1\cos(0) + K_2\sin(0) = K_1 \qquad V(0) = V_C(0)$$
$$v'(0) = \text{''} = K_2 \qquad V'(0) = -\frac{i_L(0)}{C}$$

Can assume without loss of generality that $i_L(0) = 0$, see page 632 for proof. This gives us:

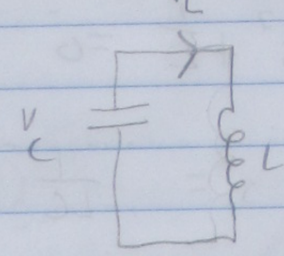
$$v(t) = V_c(0)\cos(\omega_0 t)$$

and $i_L(t)$ is just $-C\frac{dV}{dt}$, so $i_L(t) = C\,V_c(0)\,\omega_0 \sin(\omega_0 t)$

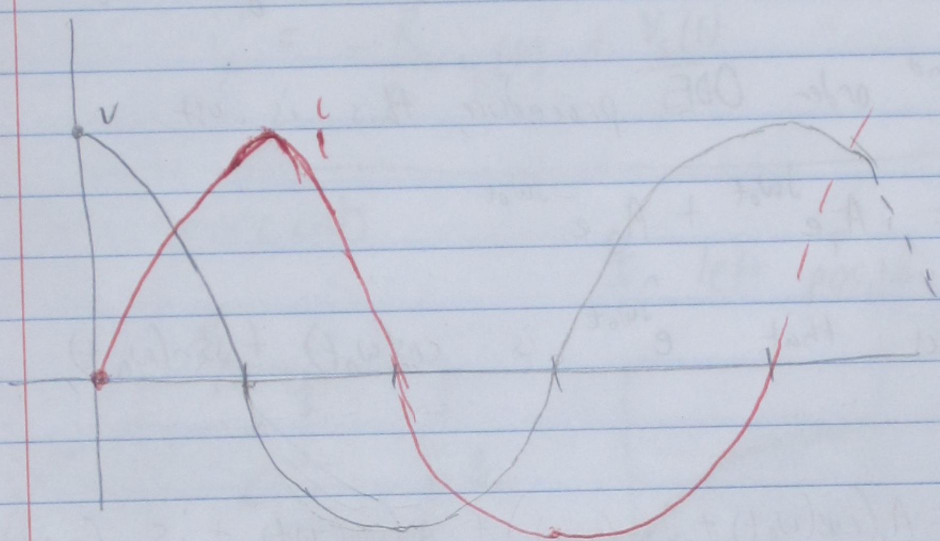
$$= \sqrt{\frac{L}{C}}\, V_c(0)\, \sin(\omega_0 t)$$

Thus

$V_L(t) = V_C(t) = V_C(0)\cos(\omega_0 t)$



$i_L(t) = \sqrt{\frac{C}{L}}\, V_C(0)\sin(\omega_0(t))$

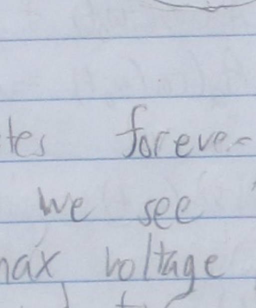


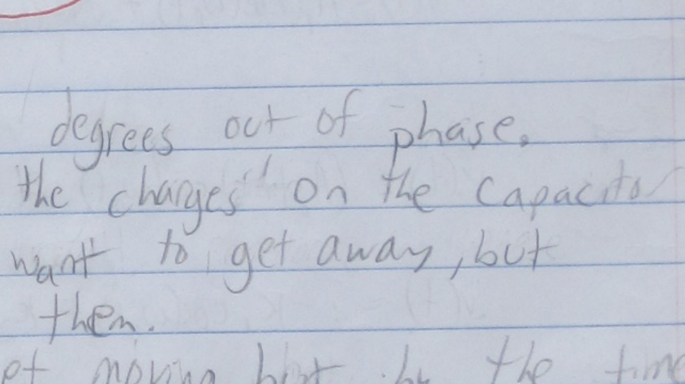
Ok I am a bad artist, you got me

Oscillates forever 90 degrees out of phase.
Above we see that the charges on the capacitor at max voltage really want to get away, but the inductor blocks them.
Charges eventually get moving but by the time they're in balance on the capacitor, the magnetic field has built up and keeps charge moving until the electric field ends up just as strong as it was, but now of opposite sign.

Exercise:
13.1a
13.2 a
13.3

If we drop a small resistor in the mix, then:

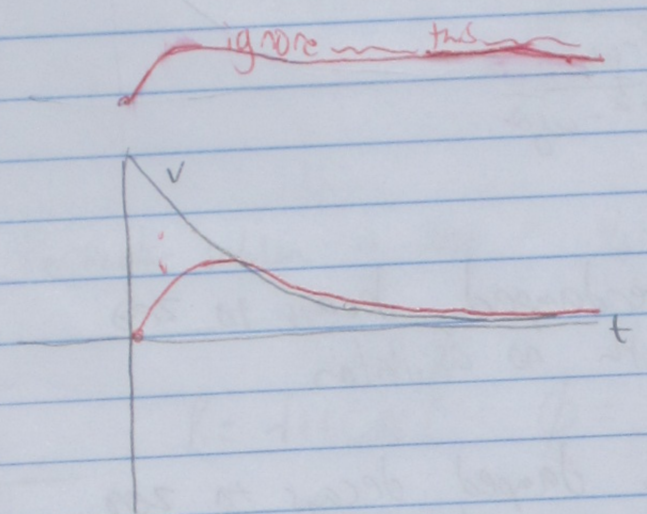




Sinusoid slowly dies.

If R gets big, we see an abrupt qualitative change. At some point, damping becomes so strong that oscillations cease completely.

High R



No oscillation!

Can still overshoot.

How do we show this?

1. Write ODE for whole circuit
2. Solve using 2^{nd} order method.

KVL gives us: [try it yourselfs first]!

$$i_L R + \int \frac{i_L}{C} + L i_L' = 0$$

$$i_L' R + \frac{i_L}{C} + L i_L'' = 0$$

SF: $$i_L'' = -\frac{i_L' R}{L} - \frac{i_L}{LC}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$ – Characteristic polynomial

Rewrite as $s^2 + 2\alpha s + \omega_0^2 = 0$

where $\alpha = R/2L$

$\omega_0 = \frac{1}{\sqrt{LC}}$

Characteristic polynomial then has roots

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

If $\alpha > \omega_0$ - overdamped, decays to zero with no oscillation

If $\alpha = \omega_0$ - critically damped, decays to zero with no oscillation w/minimum decay time

If $\alpha < \omega_0$ - underdamped - decays but able to oscillate on its way to zero

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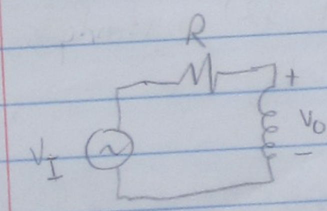
Phasors and Impedances

$v_I = V_i e^{j\omega t}$ $\quad t > 0$

After replacing source



$v_0' = -\frac{v_0}{RC} + \frac{e^{j\omega t}}{RC} V_i$

Homogeneous solution $Ae^{-tR/L}$

Guess $v_{0,p} = K_1 e^{j\omega t}$, giving