EE40
Lecture 10
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7/17/2010

Logistics and Lab Reminder

- If you have not submitted a spec and want to do a custom Project 2, talk to me right after class
- HW4 due today at 5
- HW5 due Tuesday at 2PM (it will be short, and up by 5 PM today)
- As requested, all reading assignments for next week will be posted tonight
- We expect you to understand lab concepts. For example, the Schmitt Trigger:
  - Do you know what they are and what they do?

HW Clarification

- There are a bunch of hints on the bspace forums
- “Zero state response” and “zero input response” are terms that I haven’t used in lecture, but they’re really easy and they’re in the book
  - Zero input response: The response you get with f(t)=0 [same as homogeneous solution]
  - Zero state response: The response you get with y(0)=0 [complete response with initial condition equal to zero]

To the board…

- For LC and RLC circuits

RLC Circuits

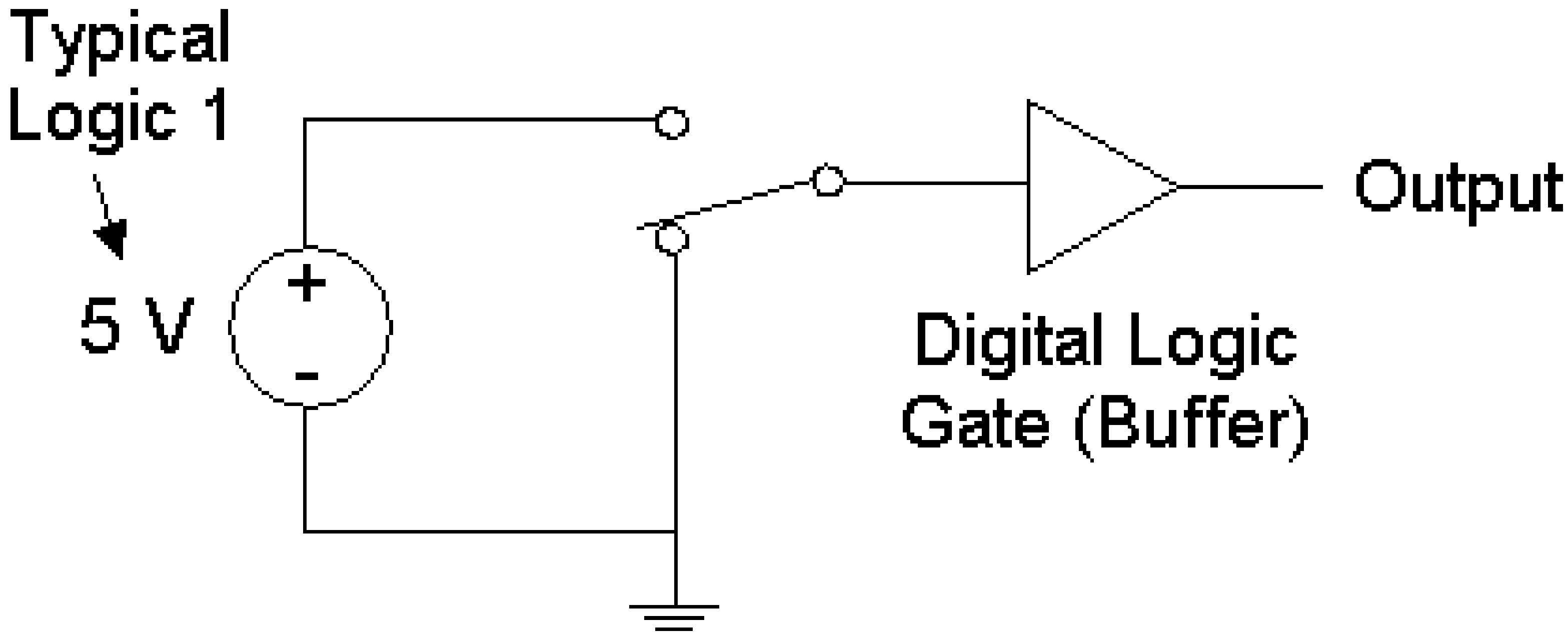
- They are important, but not so much for digital integrated circuit design
- They do play a role in the world of analog circuits, but that's a bit specialized for us to spend a great deal of time
  - Usually care more about "frequency response" than the actual shape of the response in time
- If you want to learn more about analog circuit design (it is hard and probably awesome), see EE

Let’s step back a second

- Earlier this week, I said capacitors are good for
  - Storing energy
  - Filtering
  - Modeling unwanted capacitances in digital circuits
- We’ve discussed the first case pretty heavily now, and filtering will come in great detail next week
- For now, let’s talk about delay modeling

Application to Digital Integrated Circuits (ICs)

When we perform a sequence of computations using a digital circuit, we switch the input voltages between **logic 0** (*e.g.* 0 Volts) and **logic 1** (*e.g.* 5 Volts).

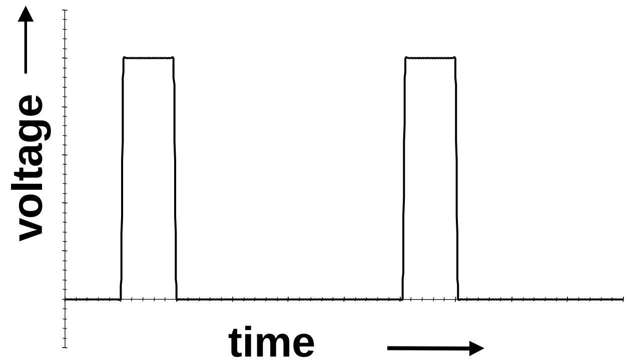


The output of the digital circuit changes between **logic 0** and **logic 1** as computations are performed.

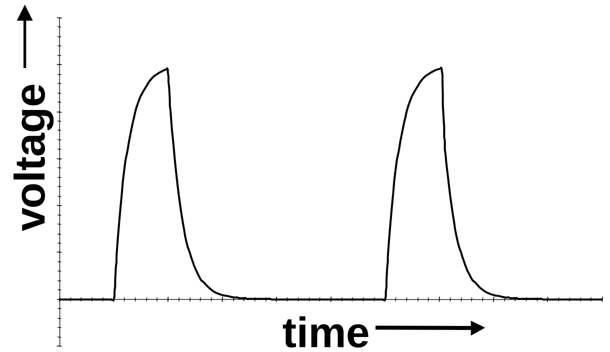
Digital Signals

We compute with pulses.

We send beautiful pulses in:



But we receive lousy-looking pulses at the output:

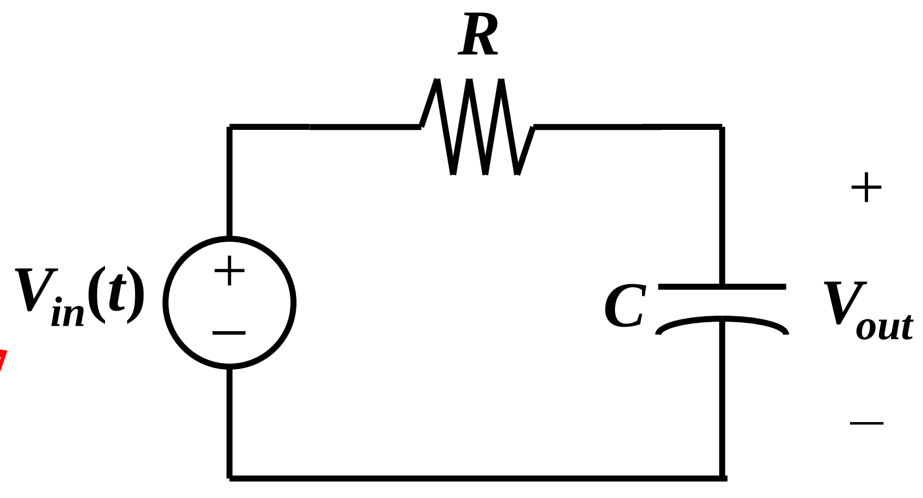


Capacitor charging effects are responsible!

- Every node in a real circuit has capacitance; it's the charging of these capacitances that limits circuit performance (speed)

Circuit Model for a Logic Gate

- As we'll discuss in a couple of weeks, electronic building blocks referred to as "logic gates" are used to implement logical functions (NAND, NOR, NOT) in digital ICs
  - Any logical function can be implemented using these gates.
- A logic gate can be modeled as a simple RC circuit:



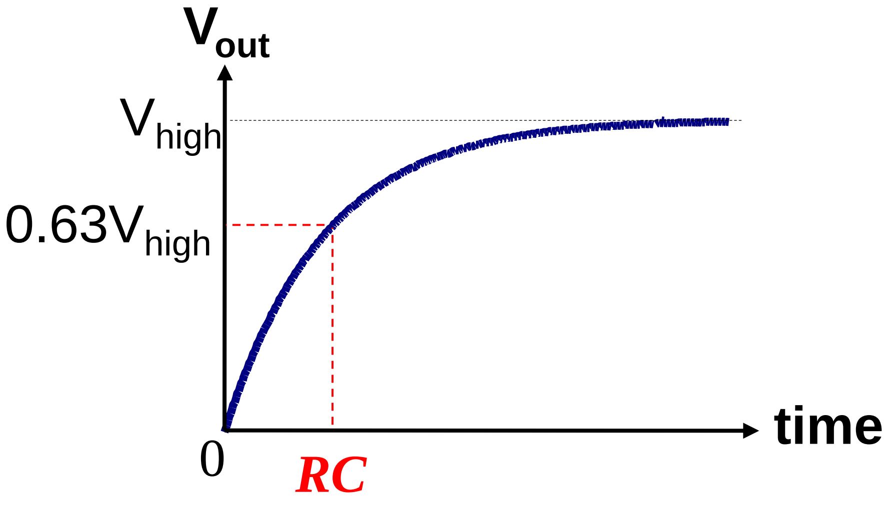
switches between "low" (logic 0) and "high" (logic 1) voltage states

Logic Level Transitions

Transition from “0” to “1”

(capacitor charging)

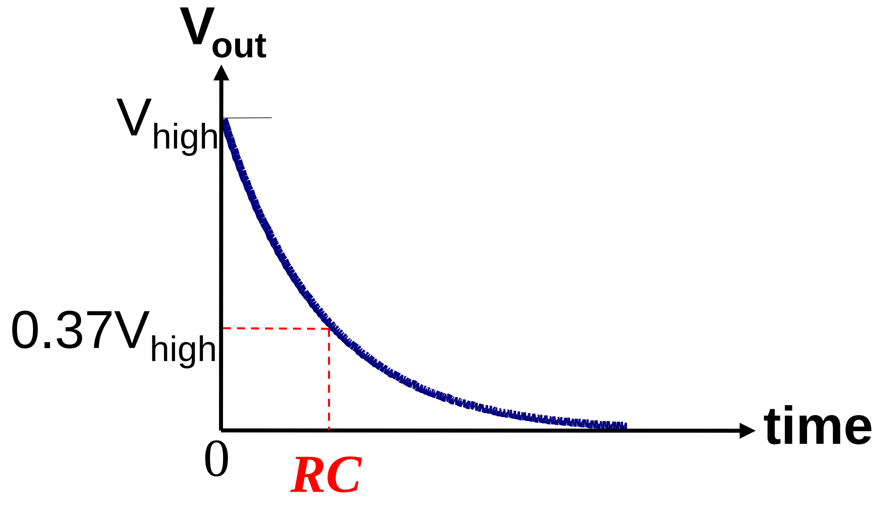
$$V_{out}(t) = V_{high}\left(1 - e^{-t/RC}\right)$$



Transition from “1” to “0”

(capacitor discharging)

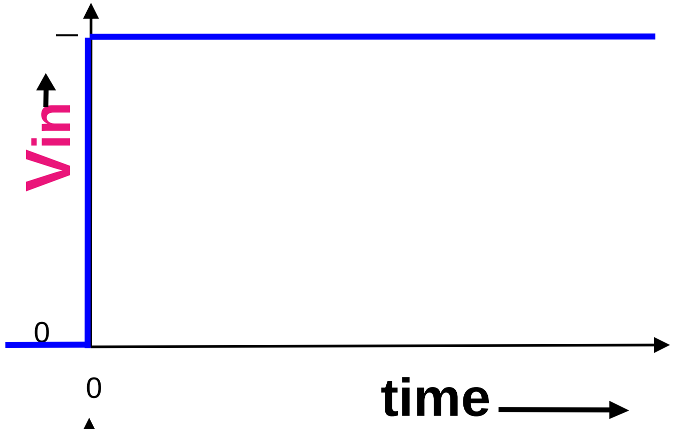
$$V_{out}(t) = V_{high}e^{-t/RC}$$



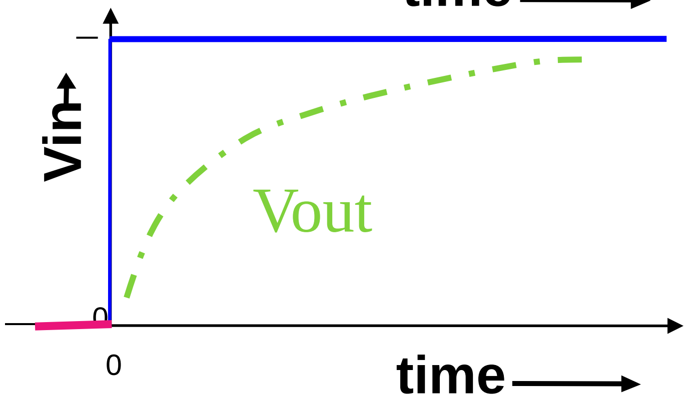
(V_{high} is the logic 1 voltage level)

Sequential Switching

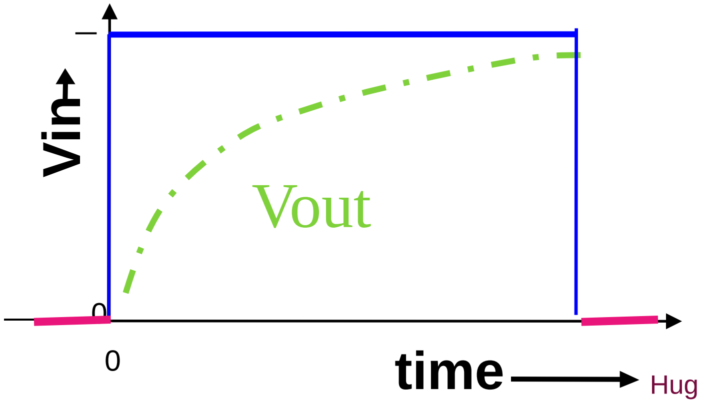
What if we step up the input,



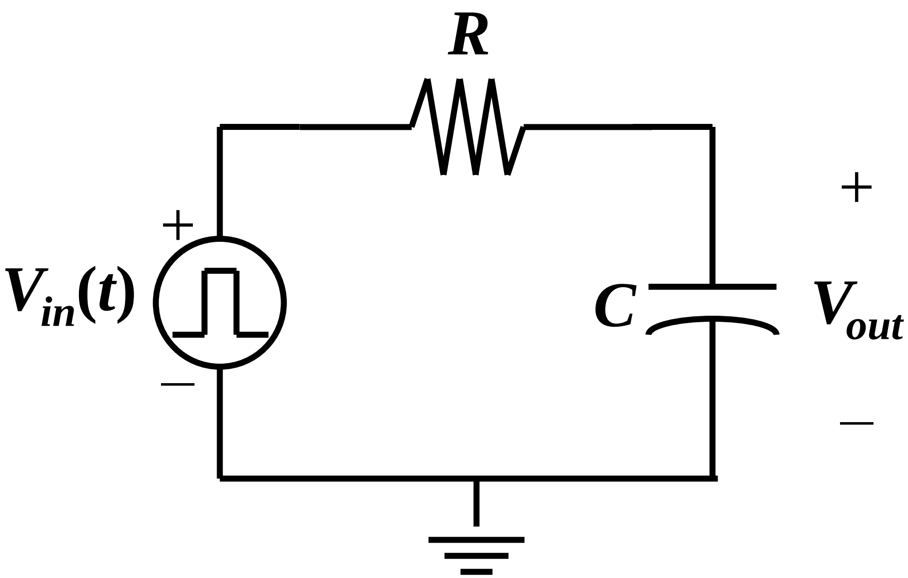
wait for the output to respond,



then bring the input back down?



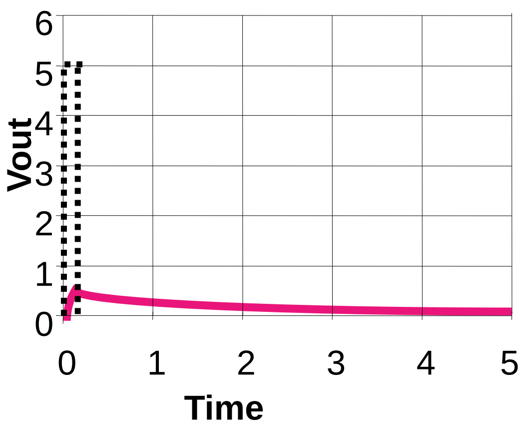
Pulse Distortion



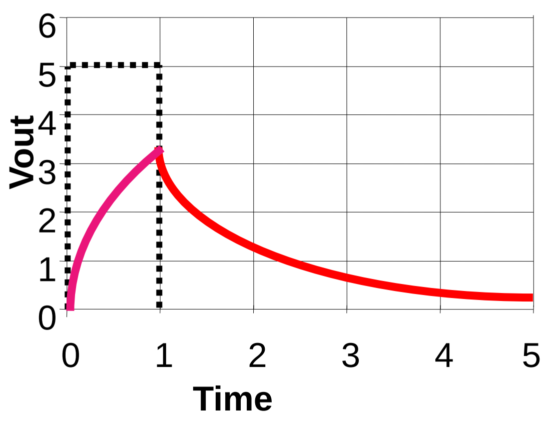
The input voltage pulse width must be long enough; otherwise the output pulse doesn't make it.

(We need to wait for the output to reach a recognizable logic level, before changing the input again.)

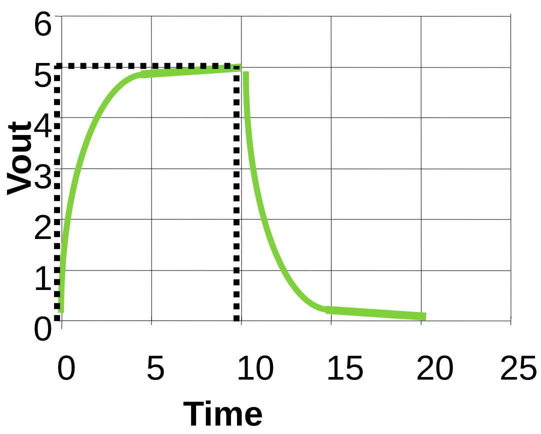
Pulse width = 0.1*RC*



Pulse width = *RC*

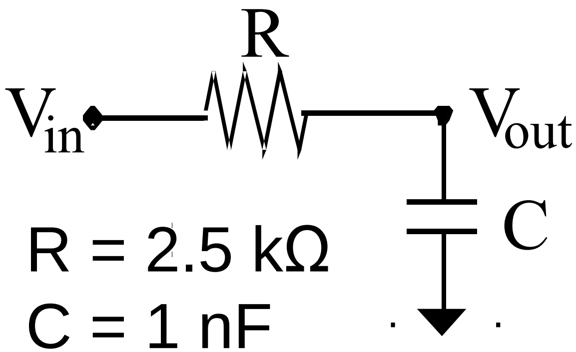


Pulse width = 10*RC*



Example

Suppose a voltage pulse of width 5 μs and height 4 V is applied to the input of this circuit beginning at t = 0:



R = 2.5 kΩ
C = 1 nF

$$\tau = RC = 2.5\ \mu s$$

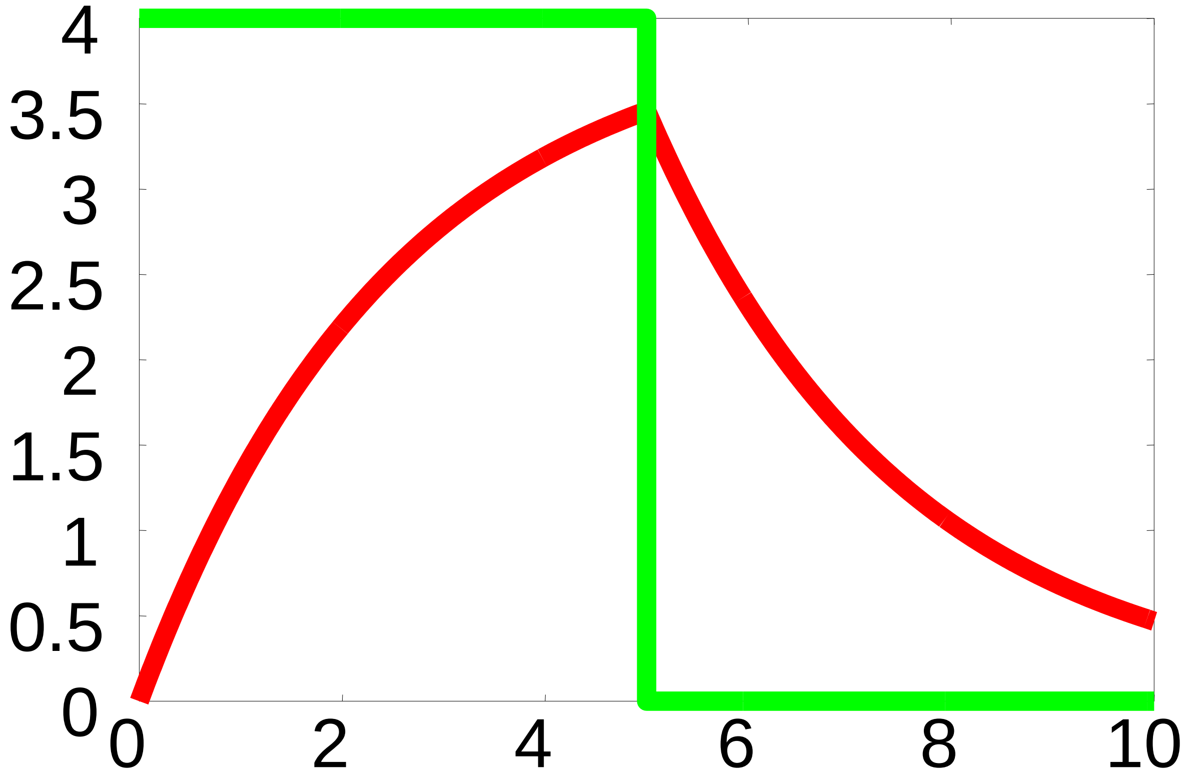
- First, V_{out} will increase exponentially toward 4 V.
- When V_{in} goes back down, V_{out} will decrease exponentially back down to 0 V.

What is the peak value of V_{out}?

The output increases for 5 μs, or 2 time constants.

→ It reaches $1-e^{-2}$ or 86% of the final value.

0.86 x 4 V = 3.44 V is the peak value



$$V_{out}(t) = \begin{cases} 4-4e^{-t/2.5\mu s} & \text{for } 0 \le t \le 5\ \mu s \\ 3.44e^{-(t-5\mu s)/2.5\mu s} & \text{for } t > 5\ \mu s \end{cases}$$

Parasitic Capacitances

- We'll discuss these parasitic capacitances in the context of digital integrated circuits right after midterm 2

AC Inputs

- We've discussed to this point how we deal with constant and weird mathematically ideal inputs $(\mathrm{e.g.} V(t) = t^2)$
- Next we'll discuss sinusoidal inputs or AC inputs, useful for, in order of increasing generality:
  - Finding 60 Hz wall voltage response
  - Finding response to inputs that can be approximated by a sum of sinusoids (e.g. square waves)
  - Finding "frequency response"

Solving Circuits with AC Sources

- In principle, we can use the MPHS to solve the circuit below:

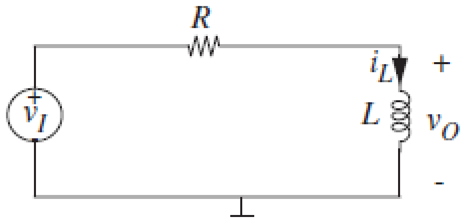


FIGURE 10.48 RL circuit with sine-wave drive.

$$v_I = V\sin(\omega t) \qquad t > 0.$$

- Will finding the homogeneous solution be difficult?

$$i_L = Ae^{-(R/L)t}$$

Solving Circuits with AC Sources

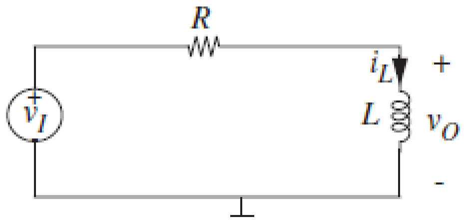


FIGURE 10.48 RL circuit with sine-wave drive.

$$v_I = V\sin(\omega t) \qquad t > 0.$$

- Will finding the particular solution be difficult?

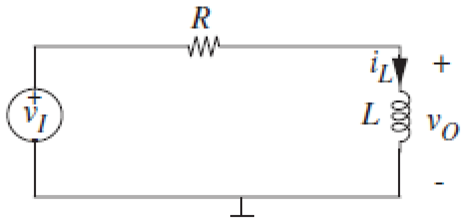
$$i_L = K_1 \sin(\omega t) + K_2 \cos(\omega t).$$

$$K_1 = V\frac{R}{R^2 + \omega^2 L^2}$$

$$K_2 = V\frac{-\omega L}{R^2 + \omega^2 L^2}$$

$$i_L = Ae^{-(R/L)t} + V\frac{R}{R^2 + \omega^2 L^2}\sin(\omega t) - V\frac{\omega L}{R^2 + \omega^2 L^2}\cos(\omega t) \quad t \geq 0$$

Solving Circuits with AC Sources



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$$v_I = V\sin(\omega t) \qquad t > 0.$$

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$$i_L = Ae^{-(R/L)t} + V\frac{R}{R^2+\omega^2L^2}\sin(\omega t) - V\frac{\omega L}{R^2+\omega^2L^2}\cos(\omega t) \quad t \geq 0$$

$$i_L = Ae^{-\frac{R}{L}t} + \sqrt{2}V\frac{\omega L}{R^2+\omega^2L^2}\cos(\omega t + \frac{5\pi}{4})$$

Phasors

- Solving simple resistive circuits
  - Hard way (kitchen sink method)
  - Easy way (node voltage)
- Op-amp circuits
  - Hard way (taking limits as $A \to \infty$)
  - Easy way (summing point constraint)
    - Requires negative feedback, which can be hard to identify
- Circuits with memory
  - Hard way (solving ODE)
  - Easy way (intuitive method)
    - Requires DC sources
  - Next will come an easy method for AC sources

Two Paths

Using Impedances and Phasors

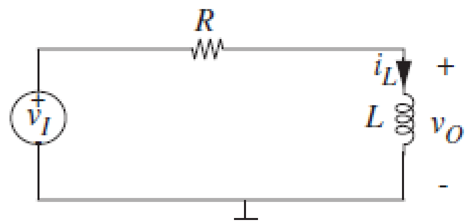
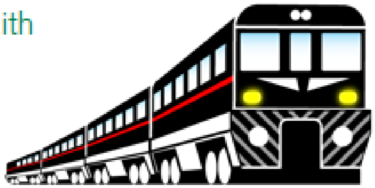
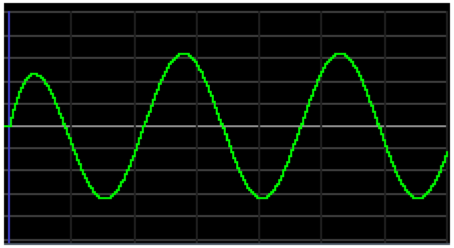


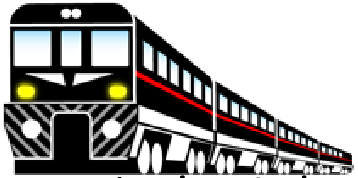
FIGURE 10.48 RL circuit with sine-wave drive.

Solving ODEs



MPHS Limited

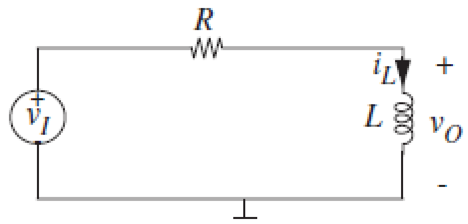




Particular Solution Connector Route

Solution Town

Basic Idea and Derivation of Impedances

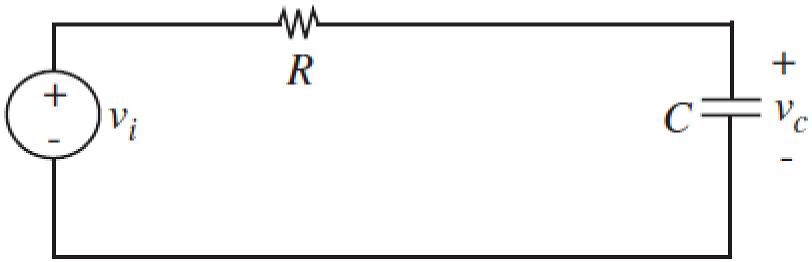


$$v_I = V_i cos(\omega t), \qquad t > 0$$

$$V_O' = -\frac{V_O}{RC} + \frac{V_i cos(\omega t)}{RC}$$

- Naïve way is to pick a particular solution which looks like $v_{o,p} = K_1 \cos(wt + \Phi)$
  - Unnecessary algebra and trigonometry
- Instead, we'll just replace the source by a new source $\tilde{v} = V_i e^{jwt}$ and solve this new problem
- Waittttttttttt, what?
  - Ok this may seem a little weird, we're replacing the voltage source with a new one that we just made up, and sure it is also complex valued, but just trust me.

New Voltage Source Problem



$$v_I = V_i e^{jwt} \quad t > 0$$

$$V_O' = -\frac{V_O}{RC} + V_i \frac{e^{j\omega t}}{RC}$$

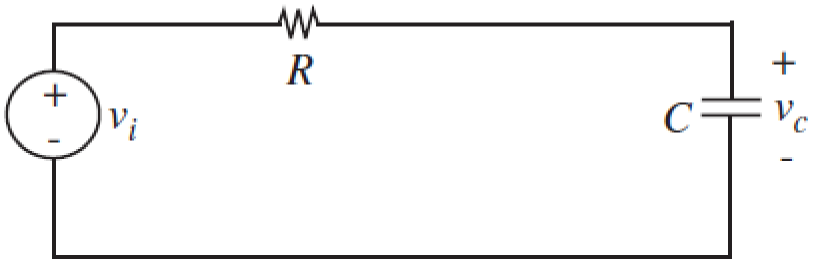
- Homogeneous solution is just $Ae^{-t/RC}$
- Pick particular solution $V_{O,P} = k_1 e^{jwt}$, plug in:

$$k_1 j\omega e^{j\omega t} = -k_1 \frac{e^{j\omega t}}{RC} + V_i \frac{e^{j\omega t}}{RC}$$

- Divide by e^{jwt}

$$k_1 j\omega = -k_1 \frac{1}{RC} + V_i \frac{1}{RC}$$

New Voltage Source Problem



$$v_I = V_i e^{jwt} \quad t > 0$$

$$V_O' = -\frac{V_O}{RC} + V_i \frac{e^{j\omega t}}{RC}$$

- Divide by e^{jwt}

$$k_1 j\omega = -k_1 \frac{1}{RC} + V_i \frac{1}{RC}$$

- Solve for k_1

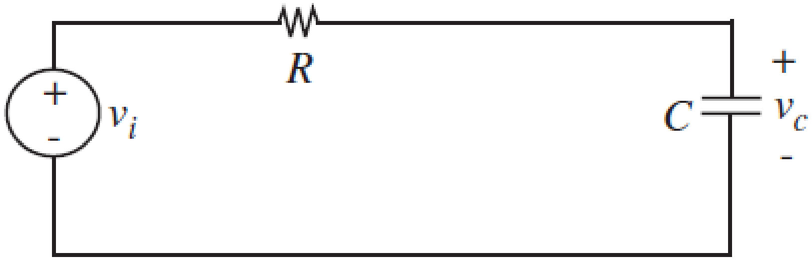
$$k_1 = V_i \frac{1}{1 + j\omega RC}$$

- Particular solution is

$$V_{O,P}(t) = V_i \frac{1}{1 + j\omega RC} e^{jwt}$$

To Recap

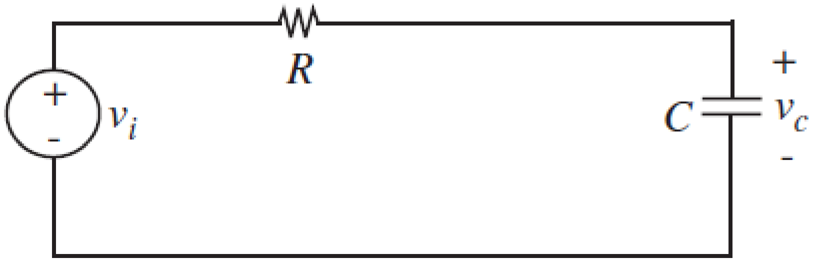
- AC source made it hard to find particular solution:



$$v_I = V_i cos(\omega t), \qquad t > 0$$

$$V_O' = -\frac{V_O}{RC} + \frac{V_i cos(\omega t)}{RC}$$

- So we just replaced the annoying source, giving us:



$$v_I = V_i e^{jwt} \quad t > 0$$

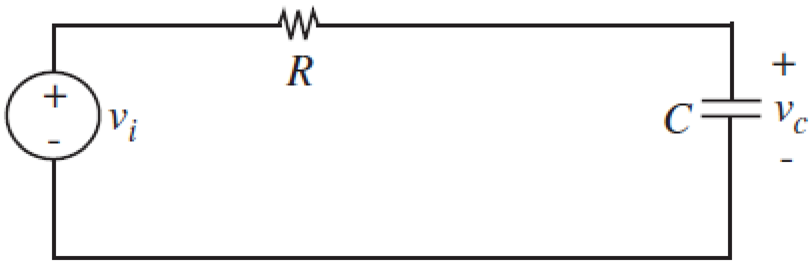
$$V_O' = -\frac{V_O}{RC} + V_i \frac{e^{j\omega t}}{RC}$$

- This gave us the particular solution:

$$V_{O,P}(t) = V_i \frac{1}{1 + j\omega RC} e^{jwt}$$

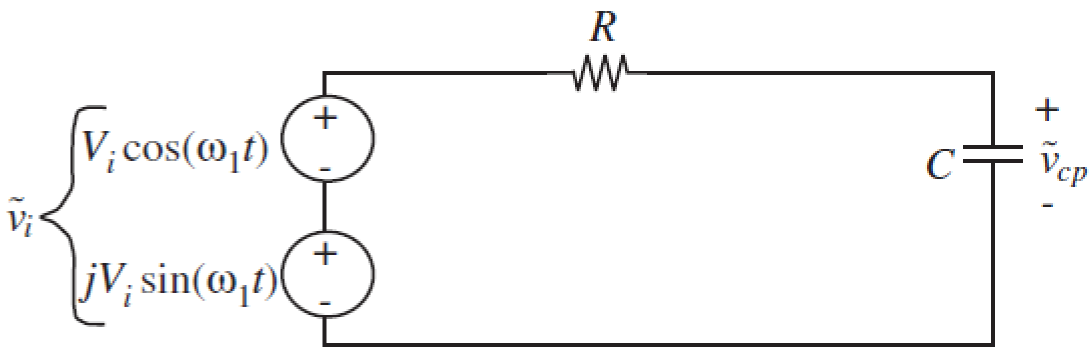
The Inverse Superposition Trick

- Our complex exponential source is actually useful



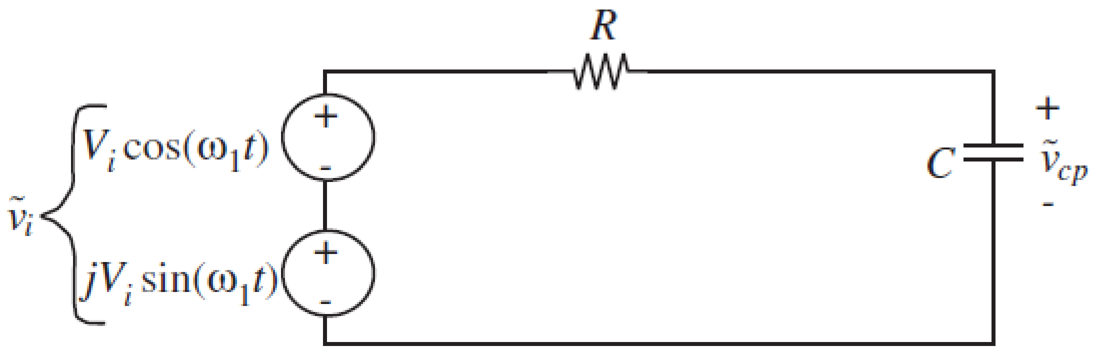
$$v_I = V_i e^{jwt} \quad t > 0$$

$$Ae^{j\omega t} = Acos(\omega t) + jsin(\omega t)$$



- Superposition tells us that our output $V_{O,P}(t)$ will just be the sum of the effect of these two sources

Inverse Superposition



- Superposition tells us that our output $V_{O,P}(t)$ will just be the sum of the effect of these two sources

$$V_{O,P}(t) = V_i \frac{1}{1 + j\omega RC} e^{jwt}$$

- Luckily for us, all complex numbers are the sum of their real and imaginary parts $\mathrm{x} = a + jb$
- Just find real part and we're done!

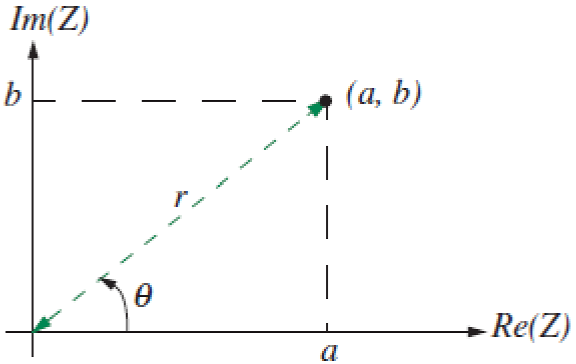
Real Part of Expression

- Finding the real part of the expression is easy, it just involves some old school math that you've probably forgotten (HW5 will have complex number exercises)

$$V_{O,P}(t) = \frac{1}{1 + j\omega RC} V_i e^{jwt}$$

- Key thing to remember is that complex numbers have two representations
  - Rectangular form: $a + jb$
  - Polar form: $re^{j\theta}$

$$r = \sqrt{a^2 + b^2}$$
$$\theta = \arctan\left(\frac{b}{a}\right)$$



Real Part of Expression

- What we have is basically the product of two complex numbers
- Let's convert the left one to polar form

$$V_{O,P}(t) = \frac{1}{1 + j\omega RC} V_i e^{jwt}$$

  - Rectangular form: $a + jb$
  - Polar form: $re^{j\theta}$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \arctan\left(\frac{b}{a}\right)$$

$$V_{O,P}(t) = \frac{1}{Re^{j\phi}} V_i e^{jwt} = V_i \frac{1}{1 + (wRC)^2} e^{\phi j} e^{jwt}$$

$$\phi = \arctan(\omega RC)$$

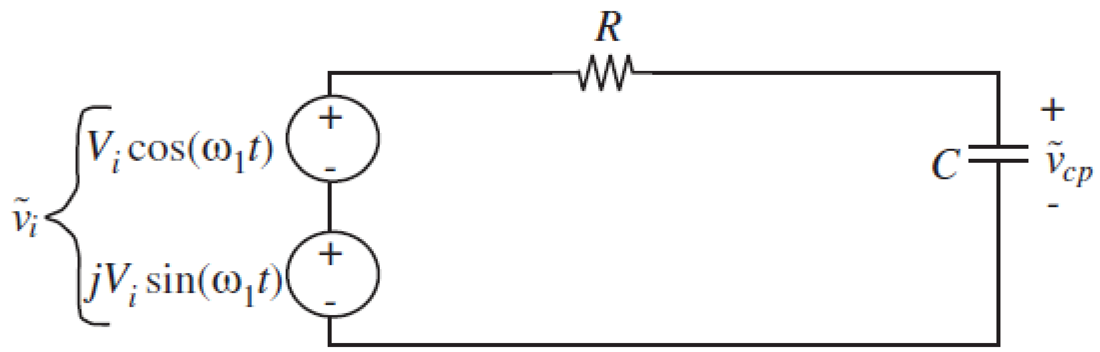
Real Part of Expression

$$V_i \frac{1}{1+(wRC)^2} e^{j\phi} e^{jwt}$$

$$\frac{V_i}{(1+\omega RC)^2} e^{j(\phi+\omega t)}$$

$$\frac{V_i}{(1+\omega RC)^2} (\cos(\omega t+\phi) + jsin(\omega t+\phi))$$

Real Part of Expression



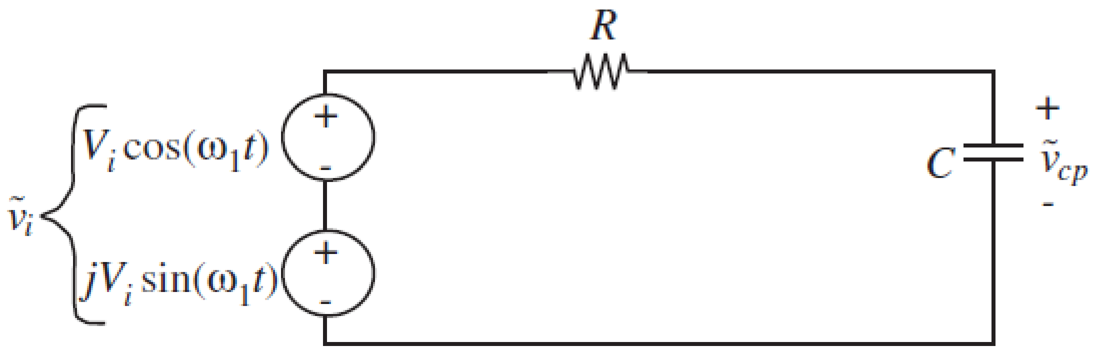
- Superposition tells us that our output $V_{O,P}(t)$ will just be the sum of the effect of these two sources

$$V_{O,P}(t) = \frac{V_i}{(1+\omega RC)^2}(\cos(\omega t + \phi) + j sin(\omega t + \phi))$$

- Thus, particular solution (forced response) of original cosine source is just the real part

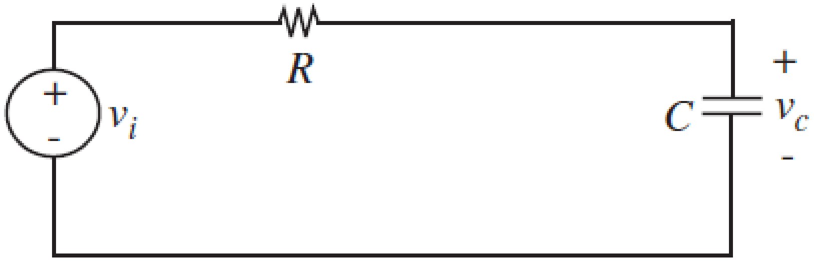
$$V_{O,P}(t) = \frac{V_i}{(1+\omega RC)^2}\cos(\omega t + \phi)$$

Wait…. That was easier?



- What we just did was mostly a derivation
- Only have to do the hard math one time
  - Sort of like intuitive method for DC sources
- What's the "easy way" to find a particular solution, now that we did the hard math one time?

Impedance



For a complex exponential source:

$$V_{C,P}(t) = \frac{1}{1 + j\omega RC} v_I(t)$$

Rewrite as:

$$V_{C,P}(t) = \frac{1/jwC}{1/jwC + R} v_I(t)$$

Let $Z_c = 1/jwC$

$$V_{C,P}(t) = \frac{Z_c}{Z_c + R} v_I(t)$$

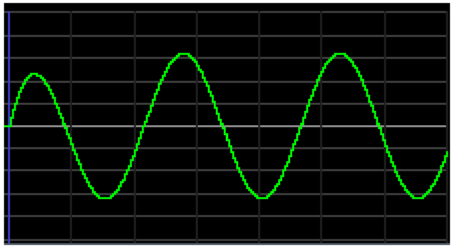
Looks a lot like… voltage divider

Impedance Method for Solving AC Circuits

- With a little more derivation, we can unveil a very powerful technique: impedance analysis:
  - Replace capacitors by $Z_C = \frac{1}{j\omega C}$
  - Replace inductors by $Z_L = j\omega L$
  - Replace resistors with $Z_R = R$
  - Replace source(s) with constant source with same magnitude (phasor representation)
- Then treat the whole thing like a resistive circuit to get "phasor" version of particular solution
- Optionally, convert back into time variable

Impedance Analysis

- Requires sinusoidal source
- Reduces any network of capacitors, inductors, and resistors into a big set of **algebraic equations**
  - Much easier to deal with than ODEs
- Only gives you the particular solution, but we usually don't care about the homogeneous solution



Impedance Analysis Example

- On board

Extra Slides

- Impedance example to help you on HW#5