Bode plot guide:

Making **Bode Magnitude Plots** from a 1st order function

1. Convert your transfer function into a form such that $j\omega$ stands alone (this will make things easier), e.g. for a low pass filter:

$$|H(j\omega)| = |\frac{k_1}{k_1 + j\omega}|$$

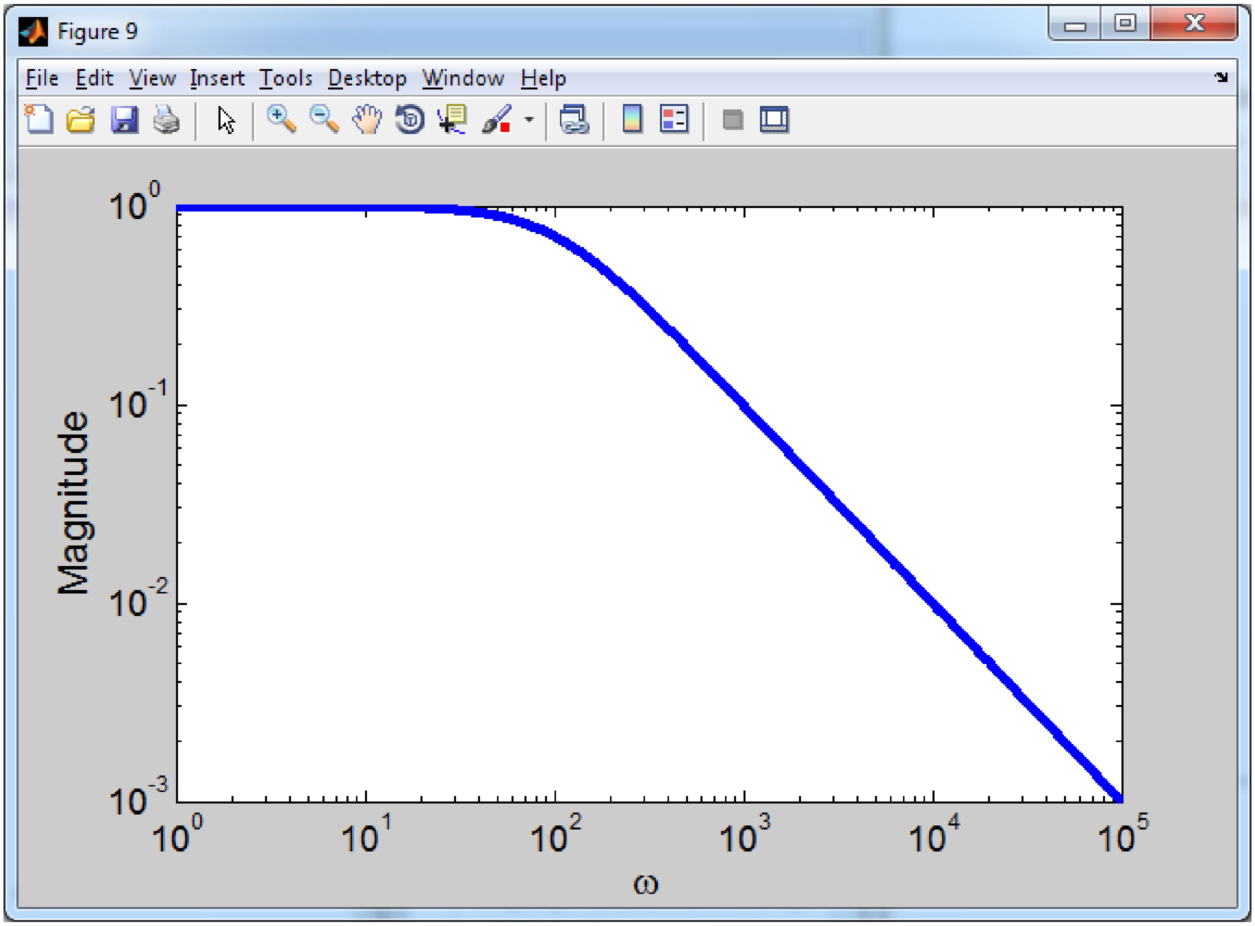
Or for a high pass filter, you'll get:

$$|H(j\omega)| = |\frac{j\omega}{k_1 + j\omega}|$$

   Optionally, you can eliminate the j from your magnitude function, but this isn't necessary and in fact will probably just be a waste of time and most likely confusing.
2. Find the low frequency asymptote
   a. For a low pass filter, this will be a constant
   b. For a high pass filter, this will be a upward sloping line of slope 1 and some offset
   c. In the low pass filter example above, k_1 will dominate the $j\omega$ term for small ω, $|H(jw)| \approx k_1/k_1 = 1$
3. Find the high frequency asymptote, e.g.
   a. For a low pass filter, this will be a downward sloping line of slope- 1 and some offset
   b. For a high pass filter, this will be a constant
   c. In the low pass filter example above, $j\omega$ will dominate k_2 for large frequencies, so $|H(jw)| \approx |k_1/j\omega| = k_1/\omega$
4. Find the intersection of the two asymptotes, this is called the "break frequency", because once you have done this, you can take a break, but also possibly because there is a break in the function behavior
   a. In the low pass example above, $1 = \frac{k_1}{\omega}$ whenever $\omega = k_1$
5. When you are back from your break, find the magnitude AT the break frequency
   a. In the low pass filter example above, $|H(jk_1)| = \left|\frac{k1}{k_1 + jk_1}\right| = \frac{k_1}{\sqrt{2}k_1} = \frac{1}{\sqrt{2}}$

5.5 Draw a loglog axis! Where is zero? There is no zero! It's loglog! Pick values for your axes so that your plot includes the break frequency (so don't plot from 10^{-5} to 10^{-2} if your break frequency is 10^9)

6. Plot the magnitude at the break frequency (just one dot). Then draw low and high frequency asymptotes on the graph, using the dot as a guide for how to make them meet.
   a. You can plug in other points to help guide your plotting. So for example, you can plug in 10^4 and you'll get 10^{-2}.
   b. As an example Bode Plot, if $k_1 = 100$, we'd get:



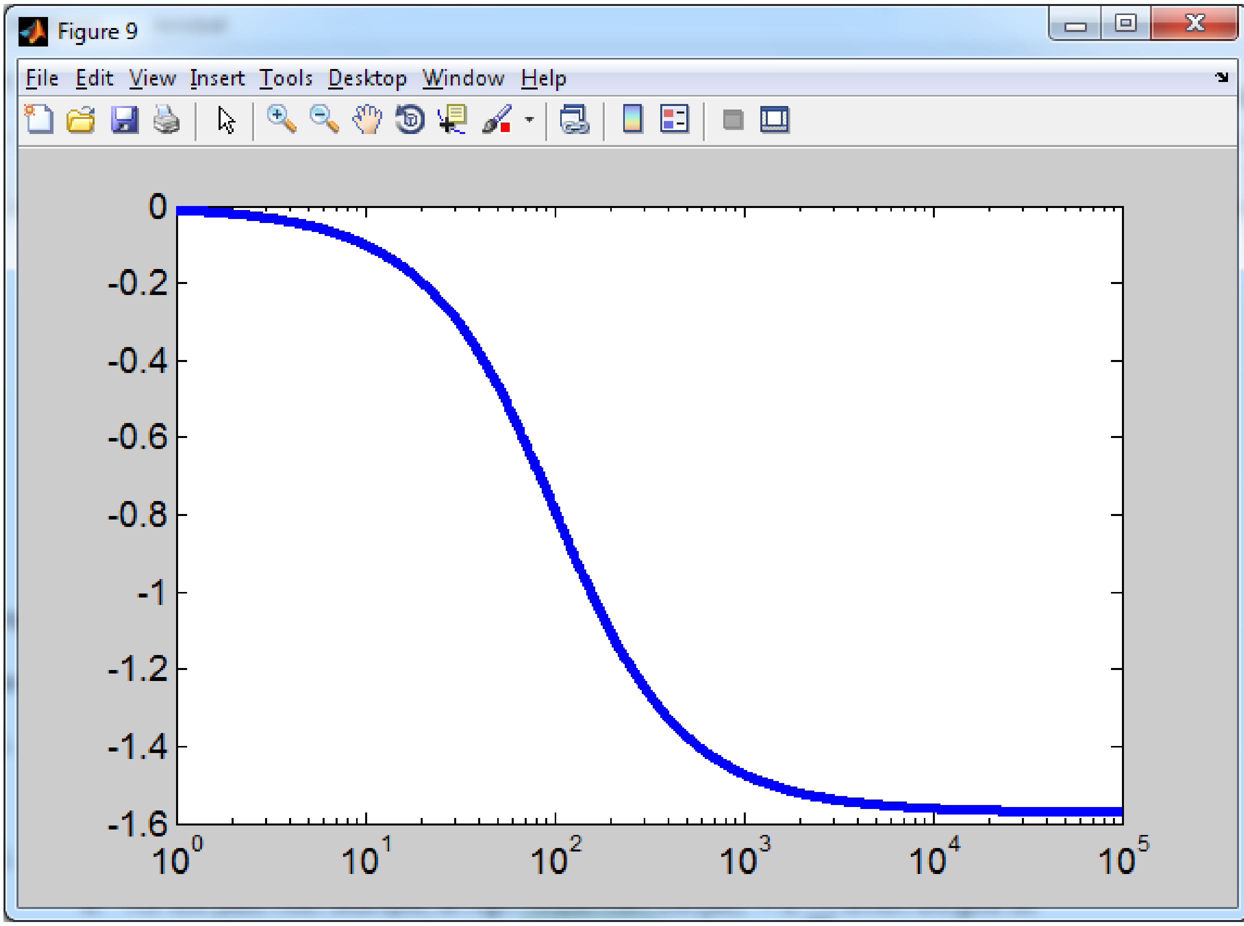
Making **Bode Phase Plots** from a 1st order function

These are harder to draw accurately, so we make a much less accurate sketch.

1. Find the low frequency asymptote
   a. For the low pass filter example, at low frequencies, k_1 dominates, and $\angle H(j\omega) \approx \angle 1 = 0$, since 1 is positive and real.
2. Find the high frequency asymptote
   a. For low pass filter example, at high frequencies $\angle H(j\omega) \approx \angle \frac{k_1}{j\omega}$, which will give us $\angle H(j\omega) = -\frac{\pi}{2}$ or $\angle H(j\omega) = \frac{\pi}{2}$ depending on the sign of k_1.
3. Find the break point phase
   a. For low pass filter example, $\angle H(jk_1) = \angle \frac{k_1}{k_1 + jk_1} = \frac{\angle k_1}{\angle(k_1 + jk_1)}$ which will be $-\frac{\pi}{4}$ or $\frac{\pi}{4}$ depending on the sign of k_1

3.5 Draw a semilogx axis. Include your break frequency on the x axis!

4. Draw the low and high frequency asymptotes on a **semilogx scale**, and smoothly connect the asymptotes
   a. For example, if $k_1 = 100$, we'd get [will be in the online version]:



Making **Bode Magnitude Plots** from a 2nd order function:

1. Put your transfer function in a form where the ω^2 term stands alone in the denominator (this will make finding the low and high frequency asymptotes easy). For example:

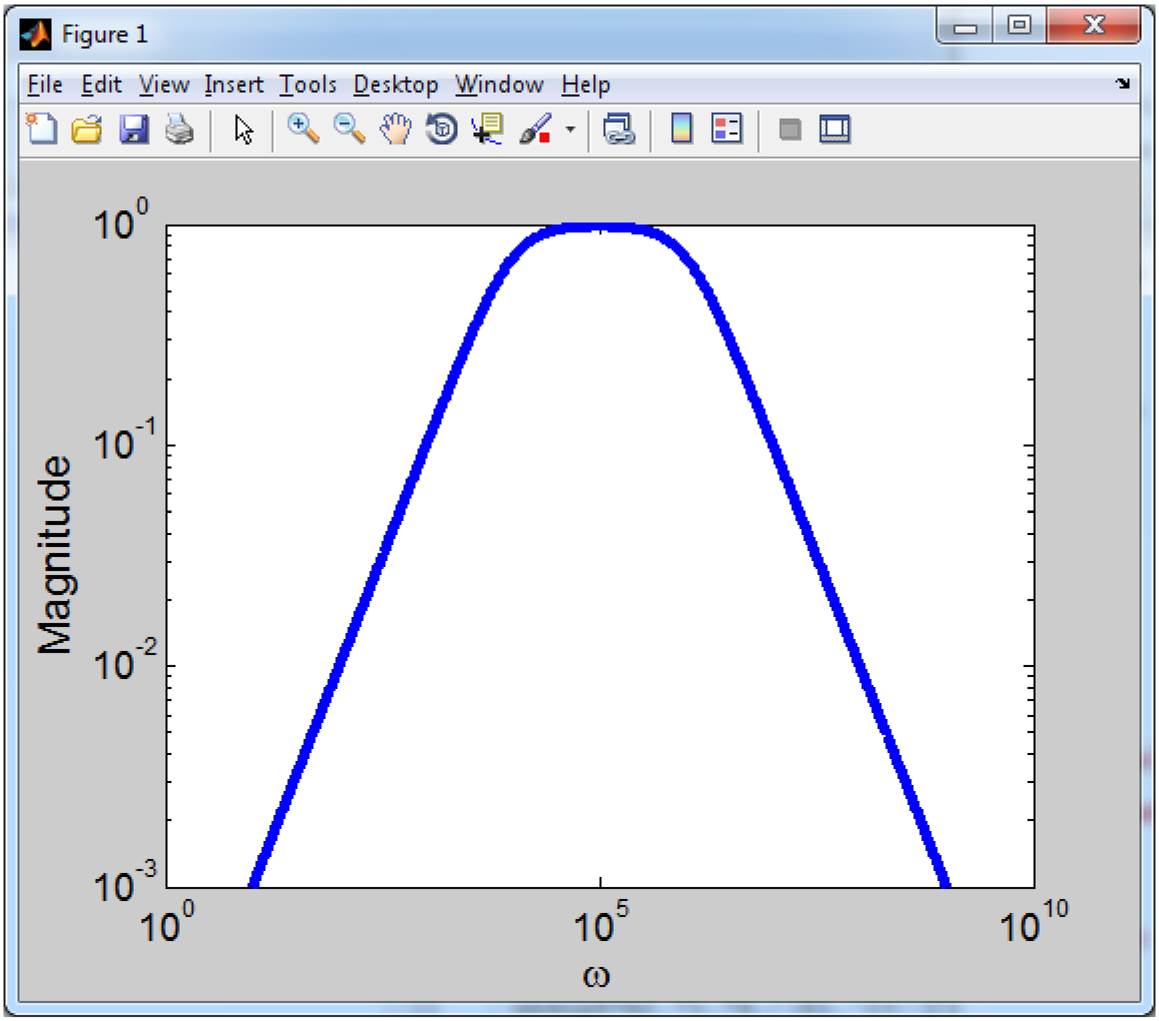
$$|H(j\omega)| = |\frac{k_2 j\omega}{k_1^2 + k_2 j\omega + \omega^2}|$$

2. Find the low frequency asymptote
    a. For the example above, k_1 dominates for low ω, so $|H(j\omega)| \approx \left|\frac{k_2 j\omega}{k_1^2}\right| = \frac{k_2\omega}{k_1^2}$, a line of slope 1 with offset k_2/k_1^2
3. Find the high frequency asymptote
    a. For the example above, ω^2 dominates for high ω, so $|H(j\omega)| \approx \left|\frac{k_2 j\omega}{\omega^2}\right| = |\frac{k_2}{\omega}|$, a line of slope -1 with offset k_2
4. The break point is always the square root of the constant in the denominator.
    a. For the example above, the break point frequency is $\omega^2 = k_1^2$, or $\omega = k_1$
    b. For most bode magnitude plots, this is also the intersection of the low and high asymptotes. However, this does not always work. For a bandstop filter (notch filter), you

will just have to rely on the square root of the constant (because the asymptotes are equal and always intersect, you'll see if you try one).

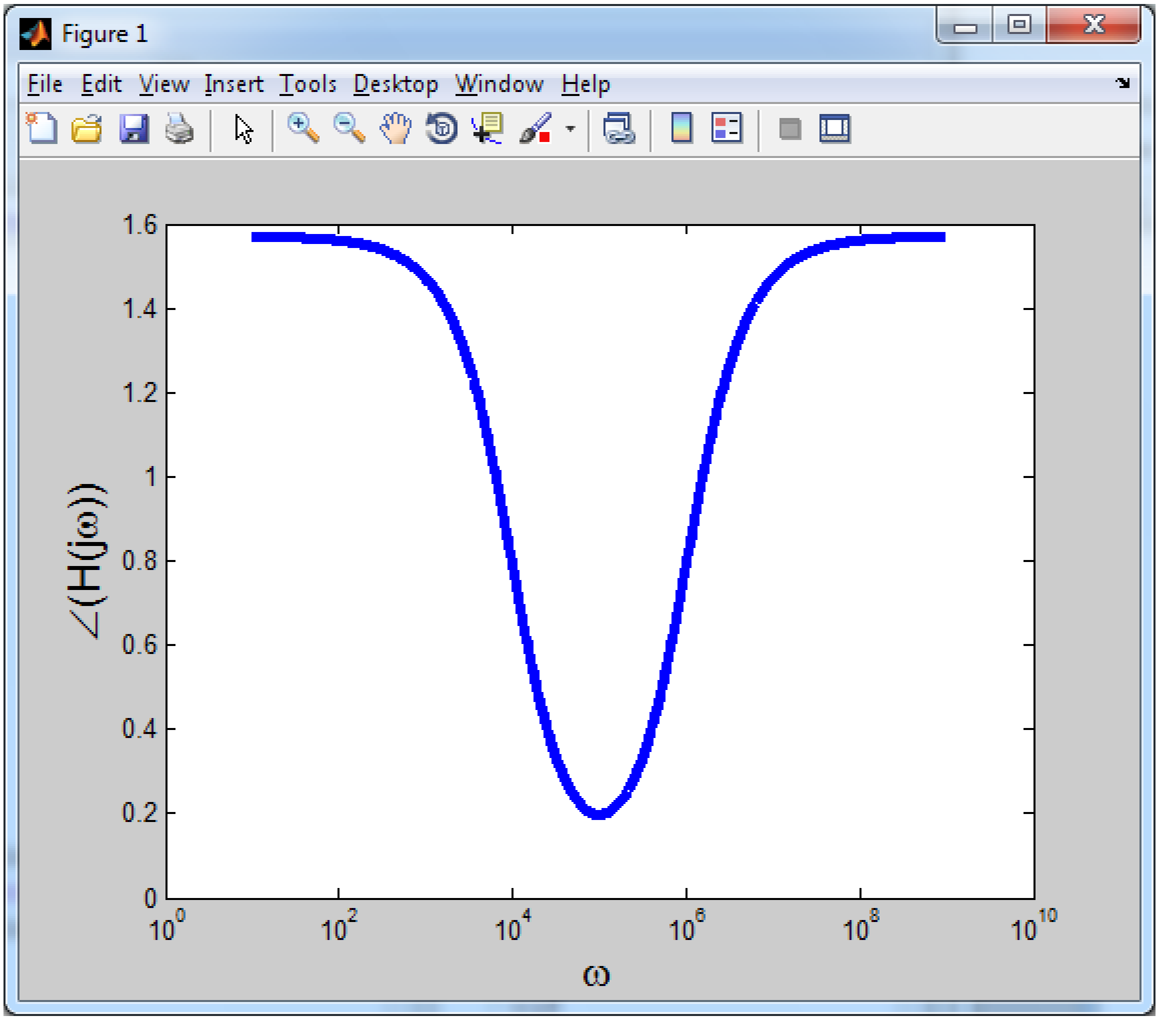
5. Find the breakpoint magnitude
    a. For the example above, $|H(jk_1)| = |\frac{k_2 jk_1}{k_1^2+k_2 jk_1+k_1^2}|$, which will depend on k_1 and k_2

For example, if $k_2 = 10^6$, $k_1 = 10^5$, we'd get [will be in the online version]:



Making Bode Phase Plots from a 2nd order function:

This is exactly the same as the first order case. Note that the procedure can fail for some 2nd order functions, but basically you'll just end up with a slight inaccuracy at the area right next to the break frequency. See EE20's treatment of bode plots if you want a failproof procedure (in terms of "poles" and "zeroes". You will not be penalized if you use the procedure I am using here (which is pretty much the same one as in the book).



Also note: The most common mistake on Bode Phase Plots is ot say that $\angle H(j\omega) = arctan(H(j\omega))$ instead of $\angle H(j\omega) = arctan(\frac{complex(H(j\omega))}{real(H(j\omega))})$. Be careful! If you're getting something weird, go back and make sure you're not doing this!

For example, in the case where $k_2 = 10^6, k_1 = 10^5$, you might be tempted to say the low frequency asymptote is:

$$\angle H(j\omega) = \arctan(\frac{k_2 j\omega}{k_1^2})$$

In this case, you'll hopefully catch your error, because taking the arctan of a complex number isn't something we do in this class. However, in other problems, the low frequency asymptote may be a purely real value, in which case you might just blithely decide to take the arctan of it, and you'll get the answer wrong.