Lecture 6: September 17th, 2001

Charging and Discharging of RC Circuits (Transients)

A) Digital pulse environment
B) RC response
C) Review of simple exponentials

The following slides were derived from those prepared by Professor Oldham For EE 40 in Fall 01

Reading:
Schwarz and Oldham 8.1+ Handout

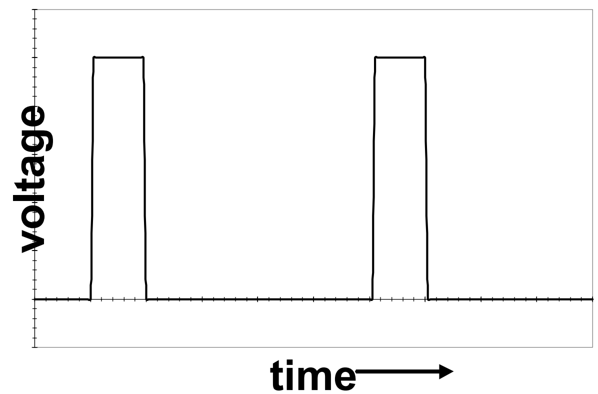
Review of charging and discharging in RC Circuits
(an enlightened approach)

- *Before* we continue with formal circuit analysis - lets review RC circuits
- Rationale: Every node in a circuit has capacitance to ground, like it or not, and it's the charging of these capacitances that limits real circuit performance (speed)

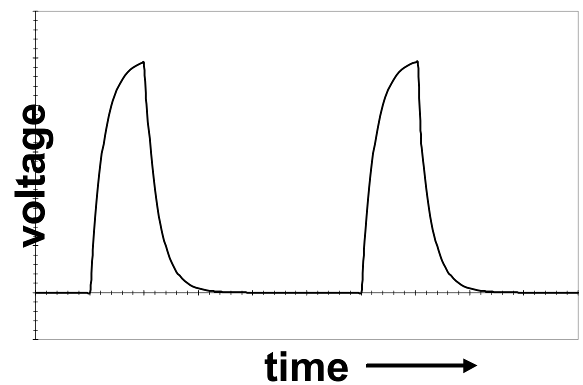
Relevance to digital circuits:

We communicate with pulses

We send beautiful pulses out



But we receive lousy-looking pulses and must restore them



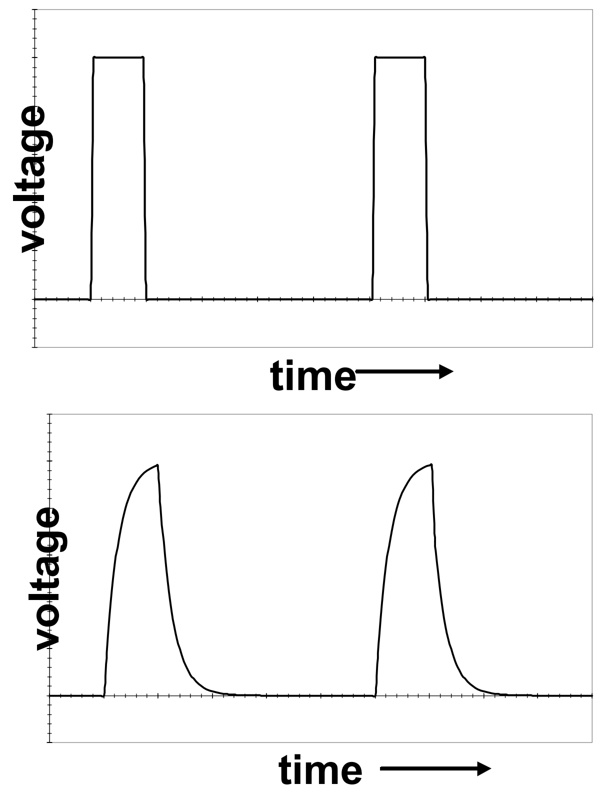
RC charging effects are responsible …. So lets review them.

Simplification for time behavior of RC Circuits

- Before any input change occurs we have a dc circuit problem (that is we can use dc circuit analysis to relate the output to the input).
- Long after the input change occurs things "settle down" …. Nothing is changing …. So again we have a dc circuit problem.

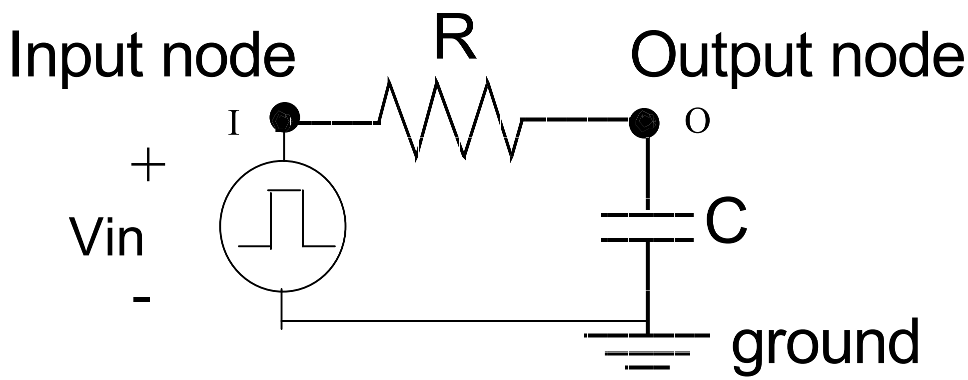
We call the time period during which the output changes the *transient*

We can predict a lot about the transient behavior from the pre- and post-transient dc solutions



What environment do pulses face?

- Every wire in a circuit has resistance.
- Every junction (called *nodes*) has capacitance to ground and other nodes.
- The active circuit elements (transistors) add additional resistance in series with the wires, and additional capacitance in parallel with the node capacitance.

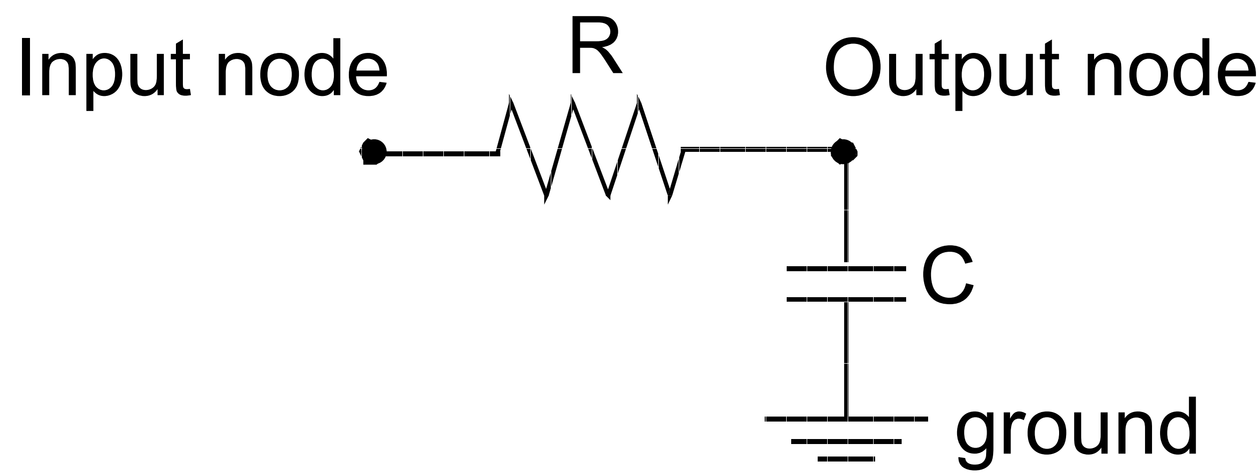


A pulse originating at **node I** will arrive delayed and distorted at **node O** because it takes time to charge C through R

If we focus on the circuit which distorts the pulses produced by Vin, it consists simply of R and C. (Vin is just the time-varying source which produces the input pulse.)

The RC Circuit to Study

(All single-capacitor circuits reduce to this one)



- R represents total resistance (wire plus whatever drives the input node)
- C represents the total capacitance from node to the outside world (from devices, nearby wires, ground etc)

SWITCHING PROPERTIES OF L, C

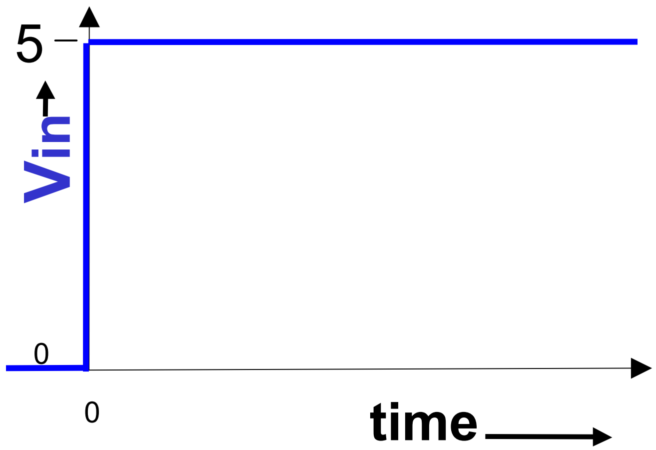
Rule: The voltage across a capacitor must be continuous and differentiable. For an inductor the current must be continuous and differentiable.

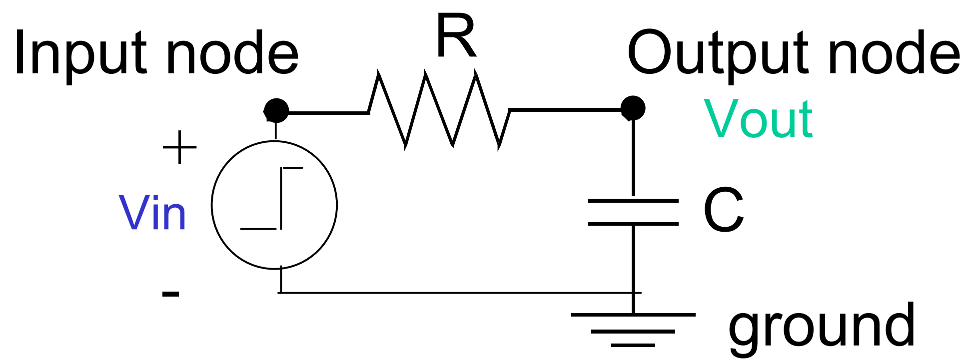
Basis: The energy cannot "jump" because that would require infinite energy flow (power) and of course neither the current or voltage can be infinite. For a capacitor this demands that V be continuous (no jumps in V); for an inductor it demands i be continuous (no jumps in i).

Capacitor	**Inductor**
v is continous	i is continous
I can jump	V can jump
Do not short circuit a charged capacitor (produces ∞ current)	Do not open an inductor with current flowing (produces ∞ voltage)

RC RESPONSE EXAMPLE

Case 1 Capacitor uncharged: Apply + voltage step of 2V

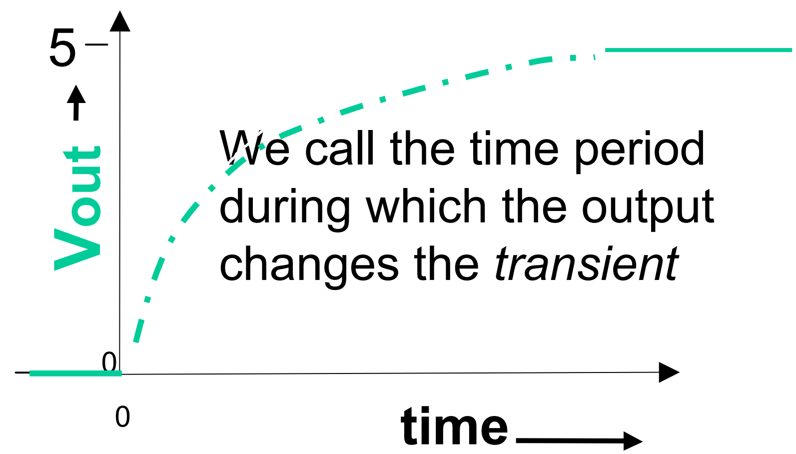




The capacitor voltage, Vout, is initially zero and rises in response to the step in input.

Its initial value, just after the step, is still zero WHY?

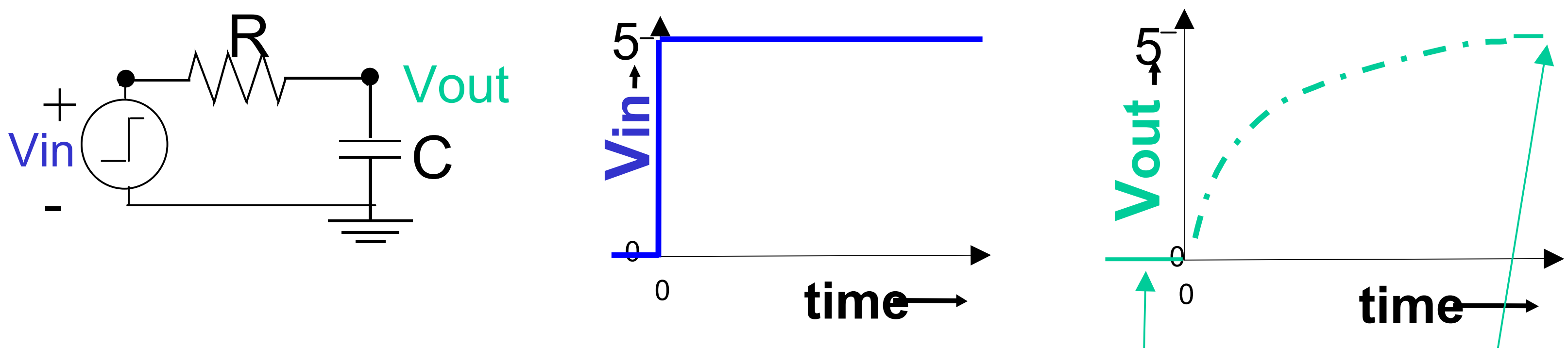
Its final value, long after the step is is 5V WHY?



We call the time period during which the output changes the *transient*

We need to find the solution during the transient.

Simplification for time behavior of RC Circuits



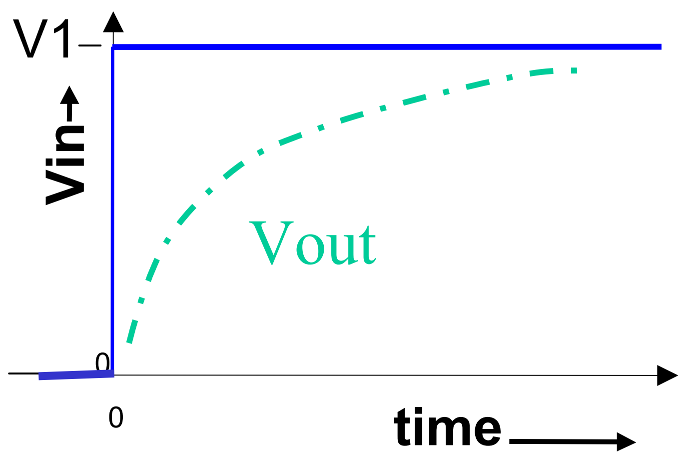
Before any input change occurs (before the transient) we have a dc circuit problem (that is we can use dc circuit analysis to relate the output to the input). That’s because the capacitor current is zero.

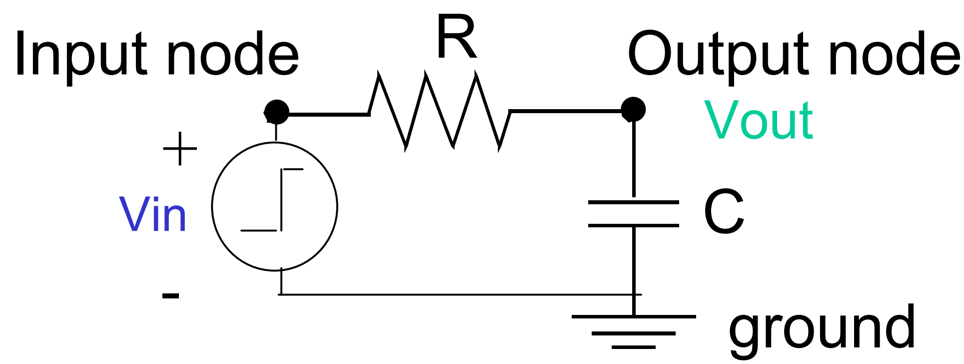
Long after the input change occurs (i.e. after the transient is over) nothing is changing …. So again we have a dc circuit problem. Again that simply because d/dt is zero, hence the capacitor current is zero.

We can predict a lot about the transient behavior from the pre- and post-transient dc solutions

RC RESPONSE

Case 1 – Rising voltage. Capacitor uncharged: Apply + voltage step





- Vin “jumps” at t=0, but Vout cannot “jump” like Vin. Why not?

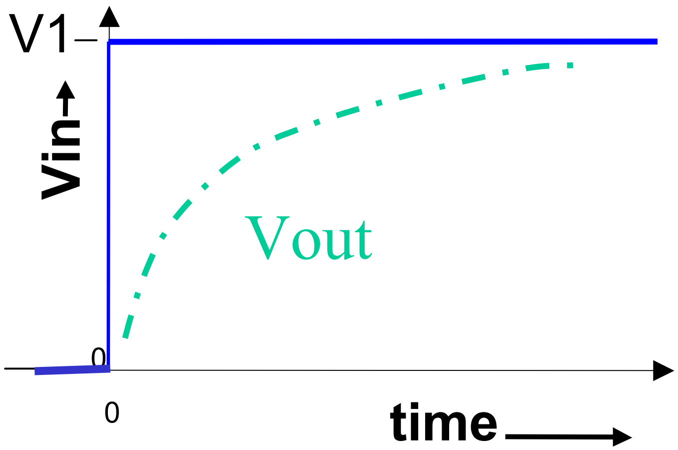
☞ Because an instantaneous change in a capacitor voltage would require instantaneous increase in energy stored ($1/2CV^2$), that is, infinite power. (Mathematically, V must be differentiable: $I=CdV/dt$)

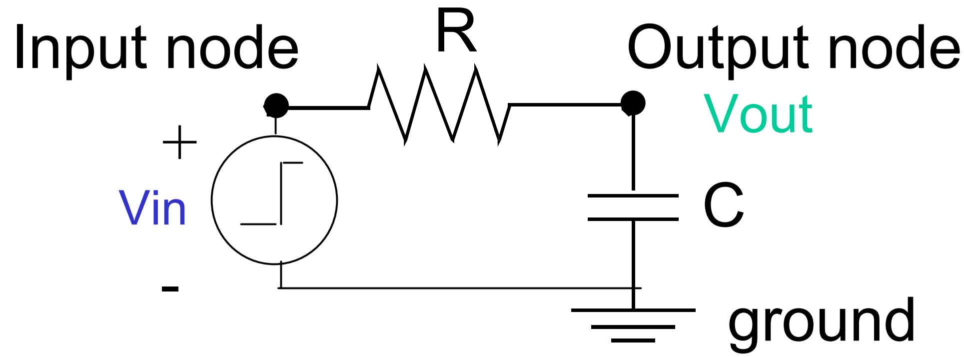
V does not “jump” at t=0 , i.e. $V(t=0^+) = V(t=0^-)$

Therefore the dc solution before the transient tells us the capacitor voltage at the beginning of the transient.

RC RESPONSE

Case 1 – Capacitor uncharged: Apply voltage step





- Vout approaches its final value asymptotically (It never quite gets to V1, but it gets arbitrarily close). Why?

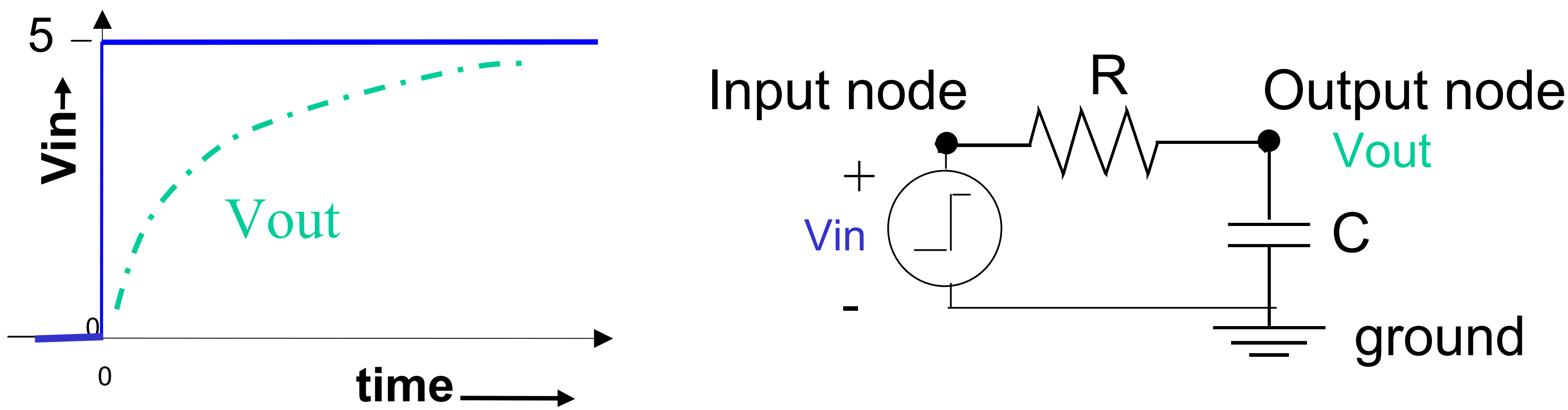
After the transient is over (nothing changing anymore) it means d(V)/dt = 0 ; that is all currents must be zero. From Ohm's law, the voltage across R must be zero, i.e. Vin = Vout.

☞ That is, Vout → V1 as t → ∞. (Asymptotic behavior)

Again the dc solution (after the transient) tells us (the asymptotic limit of) the capacitor voltage during the transient.

RC RESPONSE

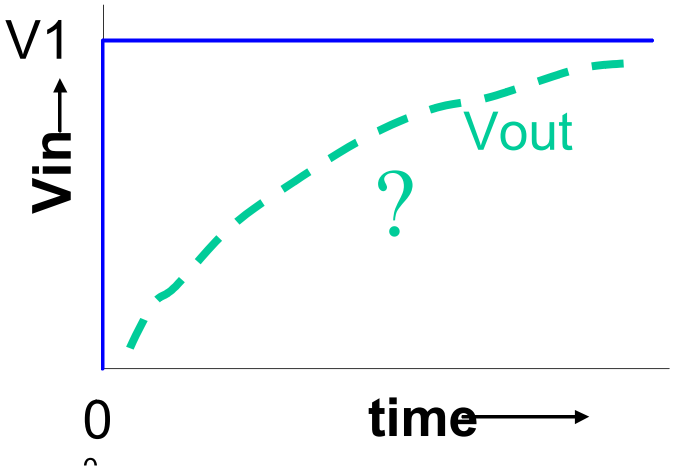
Example – Capacitor uncharged: Apply voltage step of 5 V

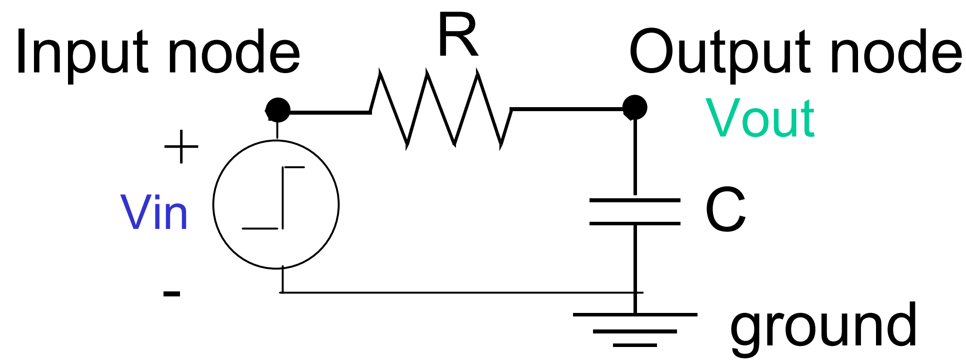


- Clearly Vout starts out at 0V (at t = 0^+) and approaches 5V.
- We know this because of the pre-transient dc solution (V=0) and post-transient dc solution (V=5V).

So we know a lot about Vout during the transient - namely its initial value, its final value , *and we know the general shape* .

RC RESPONSE: Case 1 (cont.)

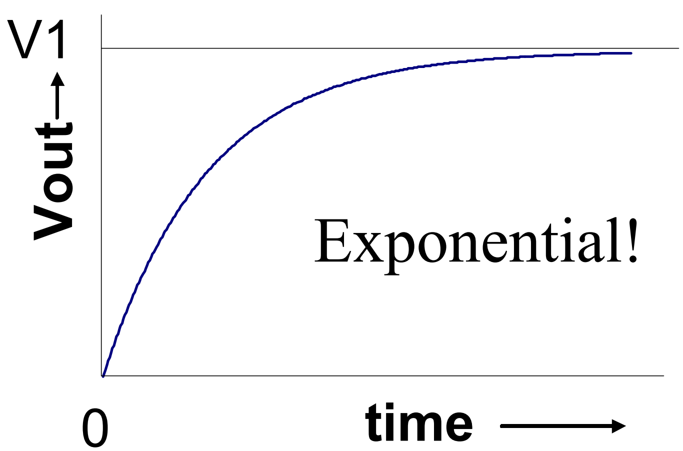


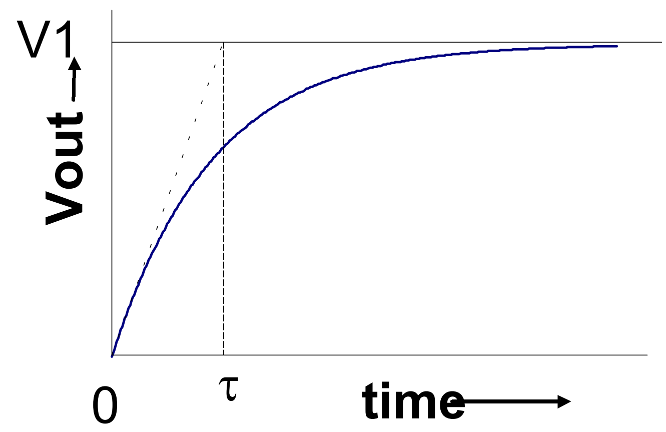


Exact form of Vout?

Equation for Vout: Do you remember general form?

$$V_{out} = V_1(1-e^{-t/\tau})$$





Review of simple exponentials.

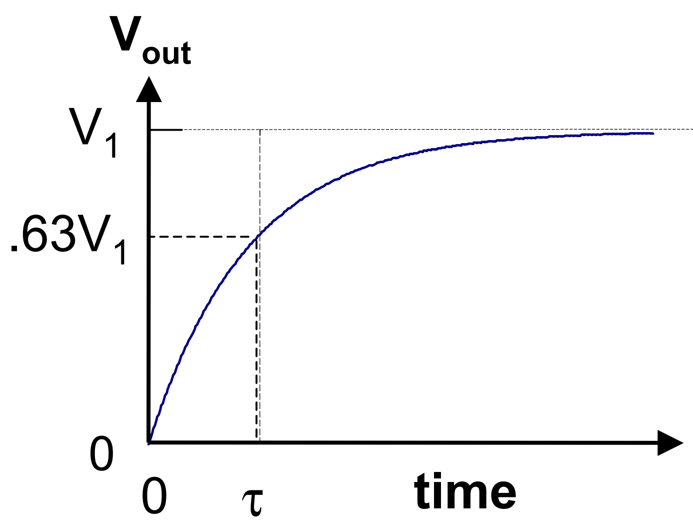
Rising Exponential from Zero

$$V_{out} = V_1(1-e^{-t/\tau})$$

at $t = 0$, $V_{out} = 0$, and

at $t \rightarrow \infty$, $V_{out} \rightarrow V_1$ also

at $t = \tau$, $V_{out} = 0.63\ V_1$



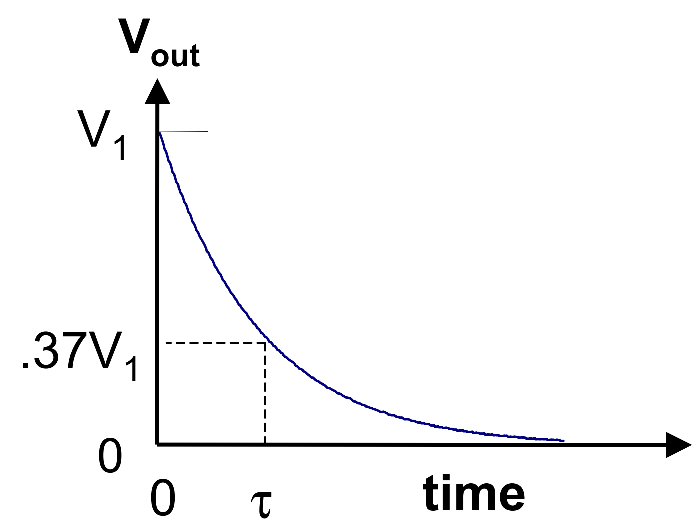
Falling Exponential to Zero

$$V_{out} = V_1 e^{-t/\tau}$$

at $t = 0$, $V_{out} = V_1$, and

at $t \rightarrow \infty$, $V_{out} \rightarrow 0$, also

at $t = \tau$, $V_{out} = 0.37\ V_1$



Further Review of simple exponentials.

Rising Exponential from Zero

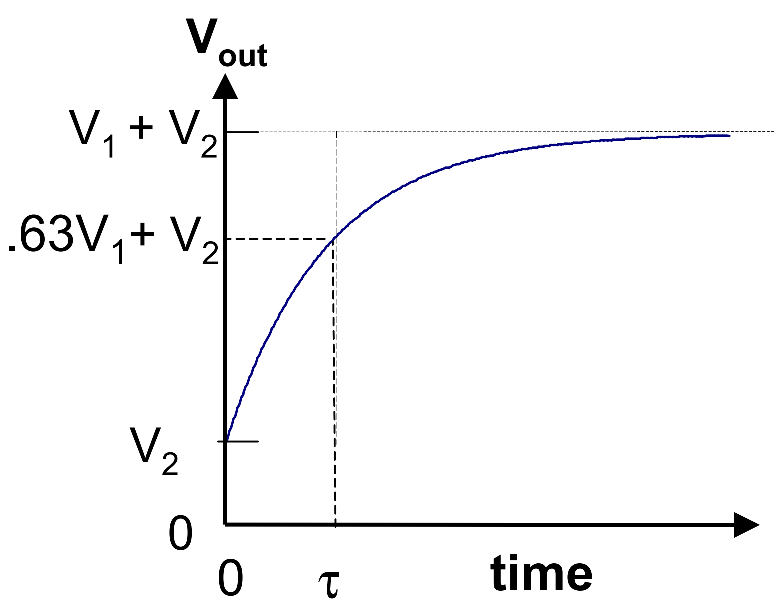
$$V_{out} = V_1(1-e^{-t/\tau})$$

Falling Exponential to Zero

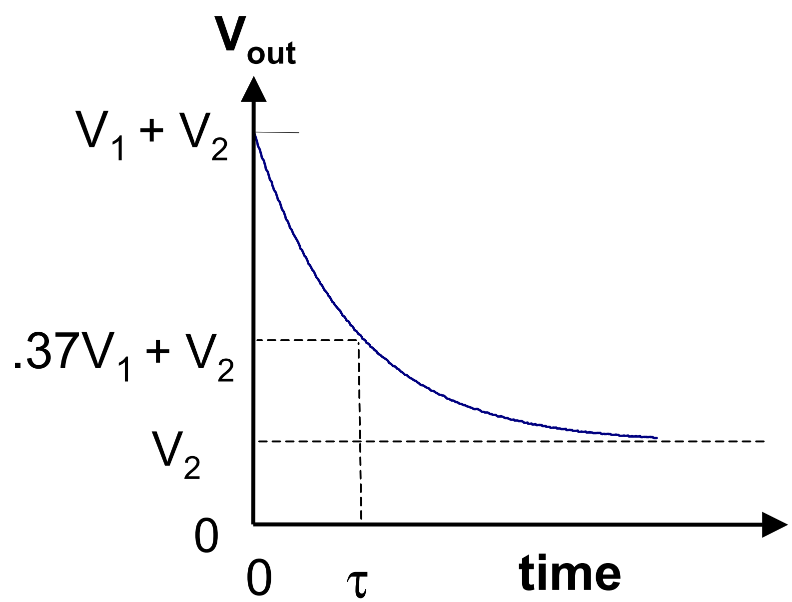
$$V_{out} = V_1 e^{-t/\tau}$$

We can add a constant (positive or negative)

$$V_{out} = V_1(1-e^{-t/\tau}) + V_2$$



$$V_{out} = V_1 e^{-t/\tau} + V_2$$



Further Review of simple exponentials:

Rising Exponential

$$V_{out} = V_1(1-e^{-t/\tau}) + V_2$$

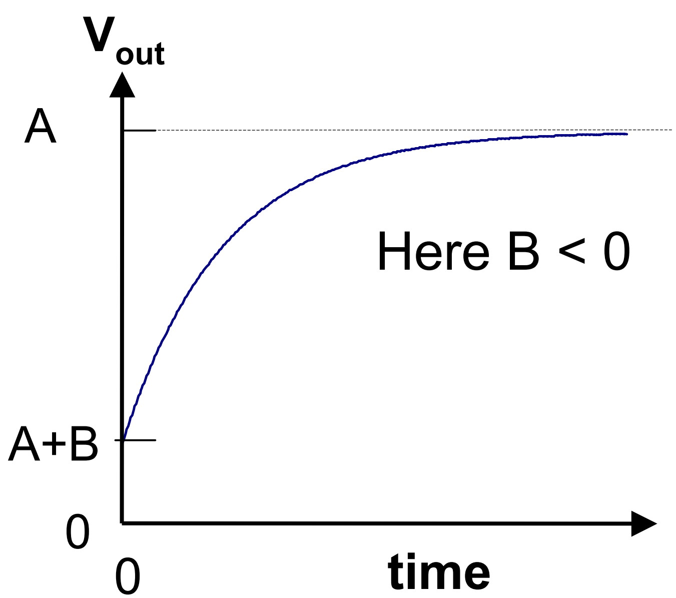
Falling Exponential

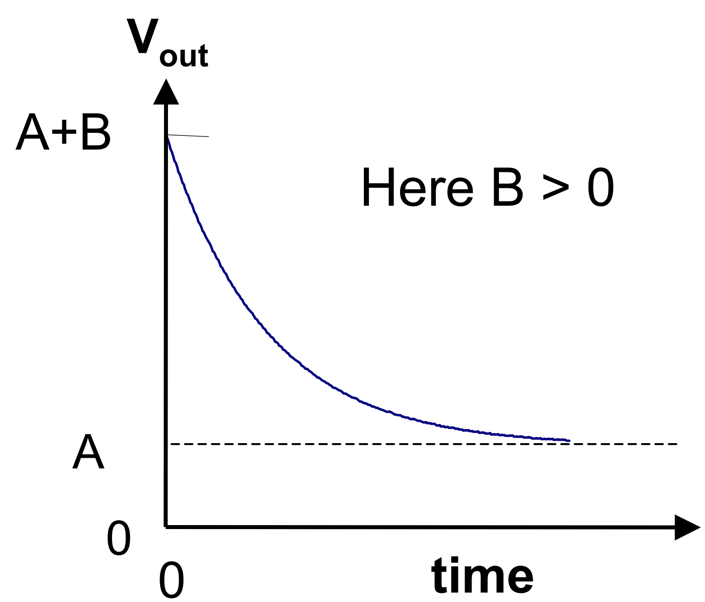
$$V_{out} = V_1 e^{-t/\tau} + V_2$$

Both equations can be written in one simple form: $V_{out} = A + Be^{-t/\tau}$

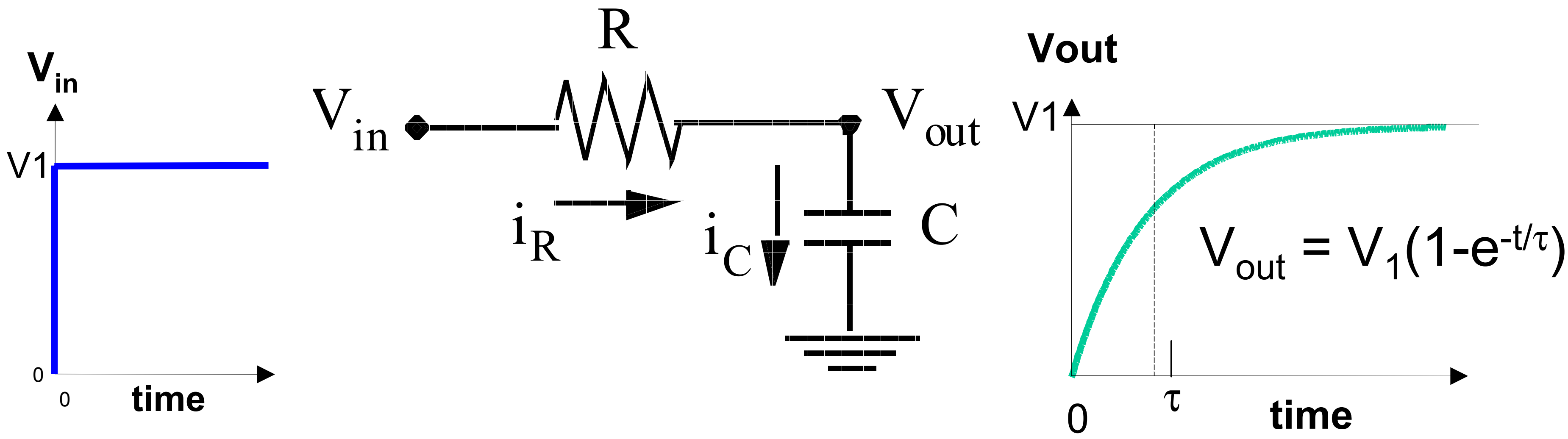
Initial value (t=0) : $V_{out} = A + B$. Final value ($t \gg \tau$): $V_{out} = A$

Thus: if B < 0, rising exponential; if B > 0, falling exponential





RC RESPONSE: Case 1 (Rising exponential)



- How is τ related to R and C ?
  - If C is bigger, it takes longer ($\tau\uparrow$).
  - If R is bigger, it takes longer ($\tau\uparrow$).
  - ✍ Thus, τ is proportional to RC.

☞ In fact, τ = RC !

✍ Thus, $V_{out} = V_1(1-e^{-t/\tau})$