# Lecture 7: September 19th, 2001

# Charging and Discharging of RC Circuits (Transients)

A) Mathematical Method

B) EE42 Easy Method

C) Logic

D) Generalizations

E) Pulse Distortion

The following slides were derived from those prepared by Professor Oldham for EE40 in Fall 01

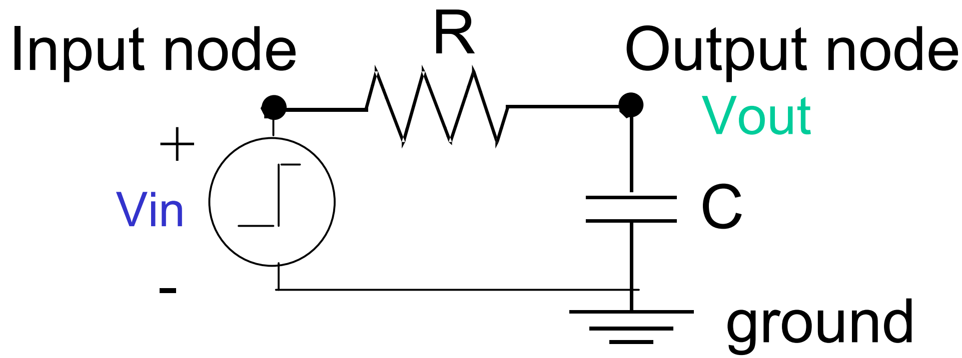
## Reading:

Schwarz and Oldham 8.1 + Handouts

# Charging and discharging in RC Circuits (continued)

**Last Time:**

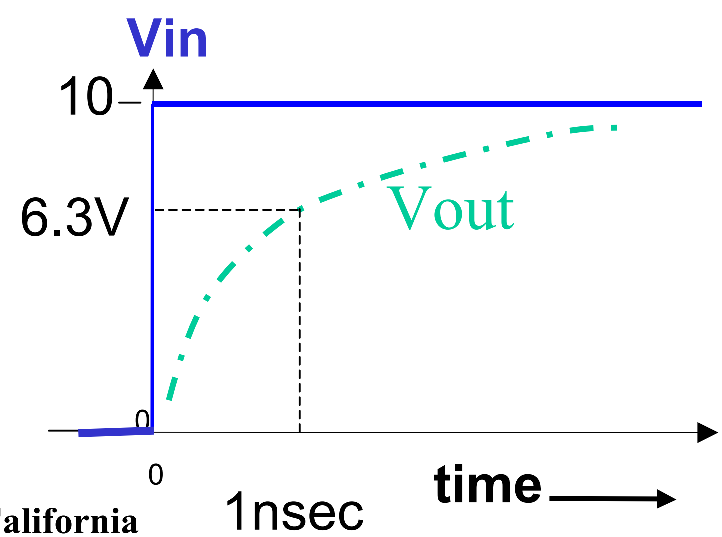
We learned that simple the simple RC circuit with a step input has a universal exponential solution of the form:




$$V_{out} = A + Be^{-t/RC}$$

Example: R = 1K, C = 1pF, Vin steps from zero to 10V at t=0:

1) Initial value of Vout is 0
2) Final value of Vout is 10V
3) Time constant is $10^{-9}$ sec
4) Vout reaches 0.63 X 10 in $10^{-9}$ sec



# RC RESPONSE: Case 1 (cont.)

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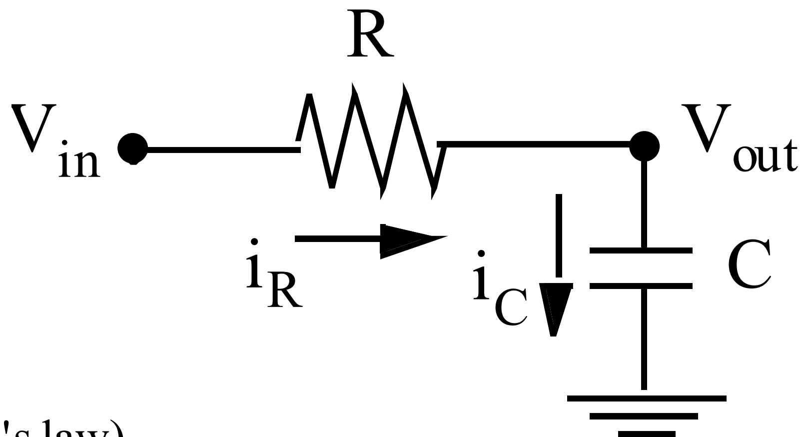
## Proof that $V_{out} = V_1(1 - e^{-t/RC})$




$$i_R = \frac{V_{in} - V_{out}}{R} \quad \text{(Ohm's law)}$$

$$i_C = C\frac{dV_{out}}{dt} \quad \text{(capacitance law)}$$

But $i_R = i_C$ !

Thus, $$\frac{V_{in} - V_{out}}{R} = C\frac{dV_{out}}{dt}$$

**or**

$$\frac{dV_{out}}{dt} = \frac{1}{RC}(V_{in} - V_{out})$$

# RC RESPONSE Case 1 (cont.)

Proof that $V_{out} = V_1(1 - e^{-t/RC})$

We have: $\frac{dV_{out}}{dt} = \frac{1}{RC}(V_{in} - V_{out})$

But $V_{in} = V_1 = \text{constant}$

and $V_{out} = 0$ at $t = 0^+$

I claim that the solution to this first-order linear differential equation is:

$$V_{out} = V_1(1 - e^{-t/RC})$$

Proof by substitution:

$$\frac{dV_{out}}{dt} \stackrel{?}{=} \frac{1}{RC}(V_{in} - V_{out})$$

$$\downarrow$$

$$-\frac{V_1}{RC}e^{-t/RC} \stackrel{?}{=} \frac{1}{RC}(V_1 - V_1(1 - e^{-t/RC}))$$

Exp. Term gives the value of τ.

Constant gives A.

Initial condition gives A+B.

$V_{out} = 0$ at $t = 0^+$ OK

# RC RESPONSE (cont.)

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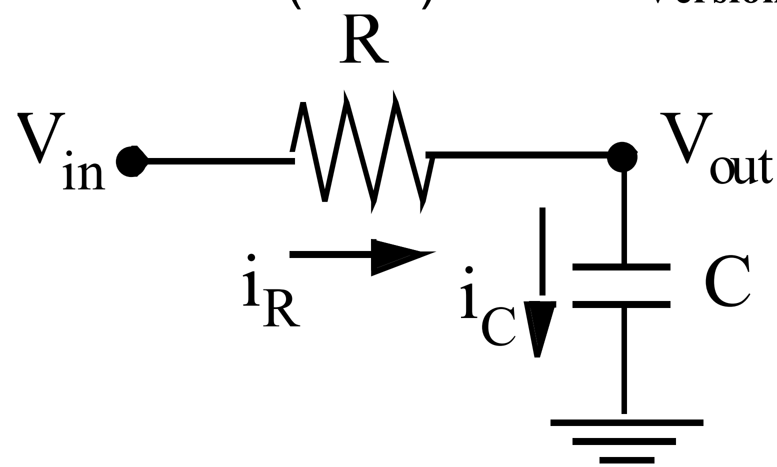



## Generalization

Vin switches at t = 0; then for any time interval t > 0, in which Vin is a constant, Vout is ***always*** of the form:

$$V_{out} = A + Be^{-t/\tau}$$

We determine A and B from the initial voltage on C, and the value of Vin. Assume Vin "switches" at t=0 from Vco to V1:

First, at $t = 0 \quad V_C \equiv V_{Co}$ initial voltage

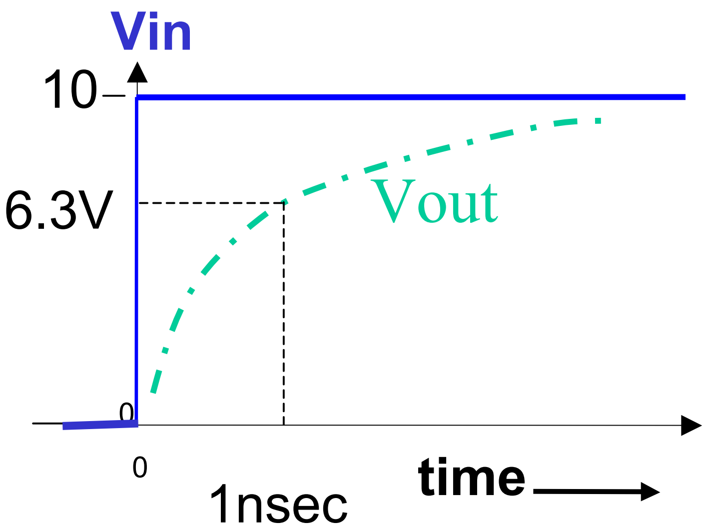
☞ Thus, $A + B = V_{Co}$

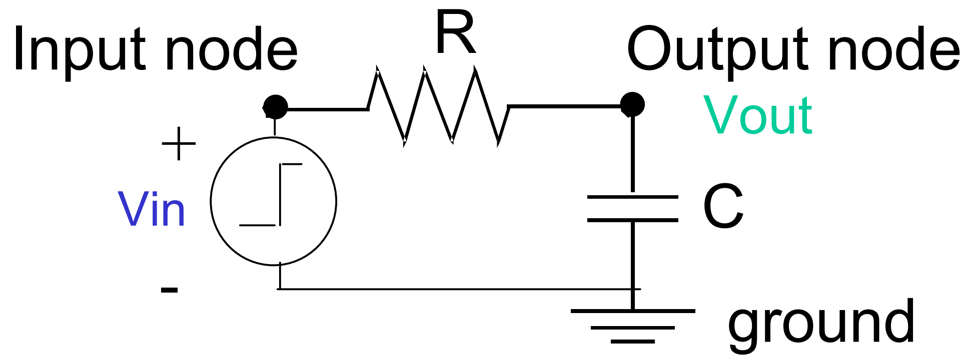
as $t \to \infty, V_C \to V_1$

☞ Thus, $A = V_1 \Rightarrow B = V_{Co} - V_1$

# Charging and discharging in RC Circuits

**For this example:** R = 1K, C = 1pF,
Vin steps from zero to 10V at t=0:




Note that we found this graph without even using the equation

$V_{out} = A + Be^{-t/RC}$ (That is we did not try to evaluate A and B).

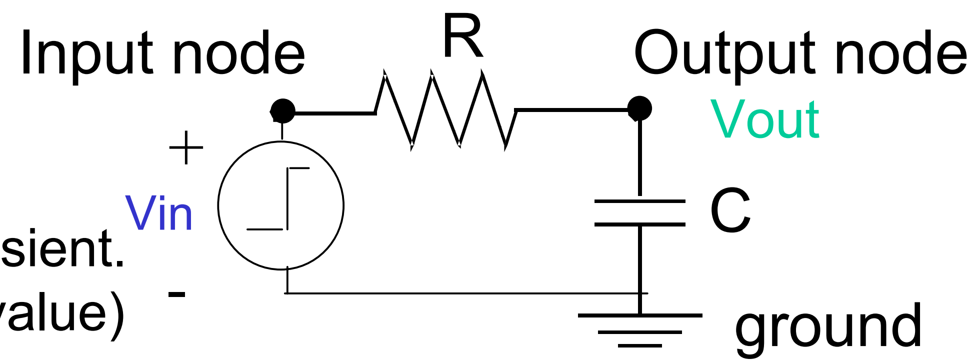
We simply used the dc solution for t<0 and the dc solution for t>>0 to get the limits and we used the time constant to get the horizontal scale. We only need the equation to remind us the solution is an exponential. So this will be the basis of our **easy method**.

# Charging and discharging in RC Circuits (The official EE40/EE42 Easy Method)

'01

**Method of solving for any node voltage in a single capacitor circuit.**

1) Simplify the circuit so it looks like one resistor, a source, and a capacitor (it will take another two weeks to learn all the tricks to do this.) But then the circuit looks like this:




2) The time constant of the transient is $\tau$ = RC.

3) Solve the dc problem for the capacitor voltage before the transient. This is the starting value (initial value) for the transient voltage.

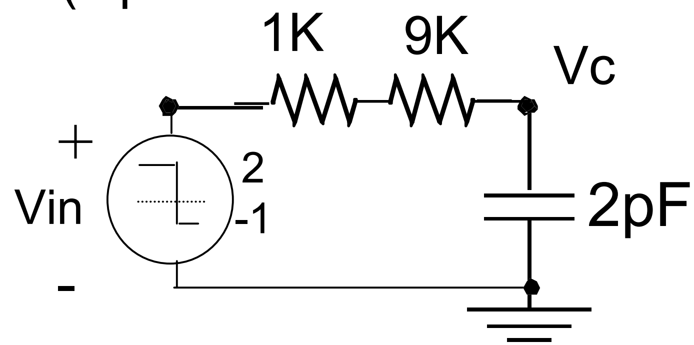
4) Solve the dc problem for the capacitor voltage after the transient is over. This is the asymptotic value.

5) Sketch the Transient. It is 63% complete after one time constant.

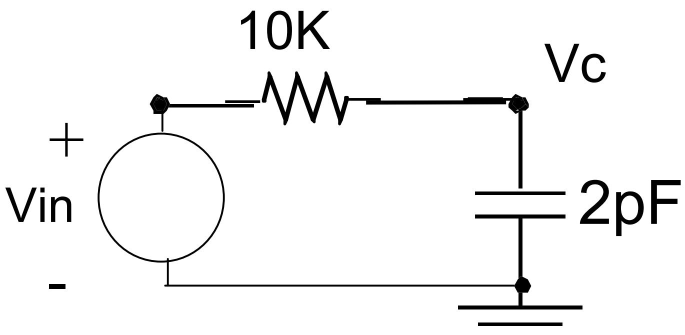
6) Write the equation by inspection.

# Charging and discharging in RC Circuits (Example 1 of the EE42 *Easy Method*)

Find Vc(t) for the following circuit: (input switches from 2V to -1V at t = 0)

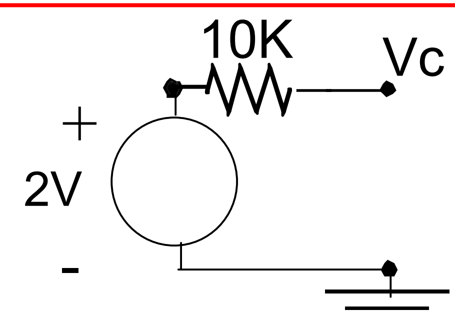



1) Simplify the circuit :

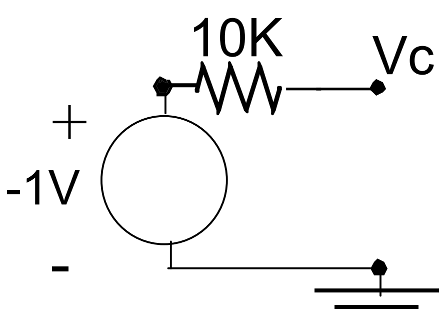



2) The time constant of the transient is $\tau$ = RC = 20nsec

3) Before the transient Vin = 2V so Vc =2V

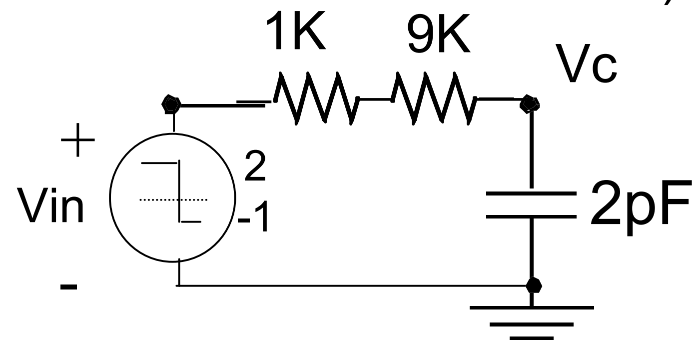



4) After the transient is over Vin = -1V so Vc = -1V. This is the asymptotic value.



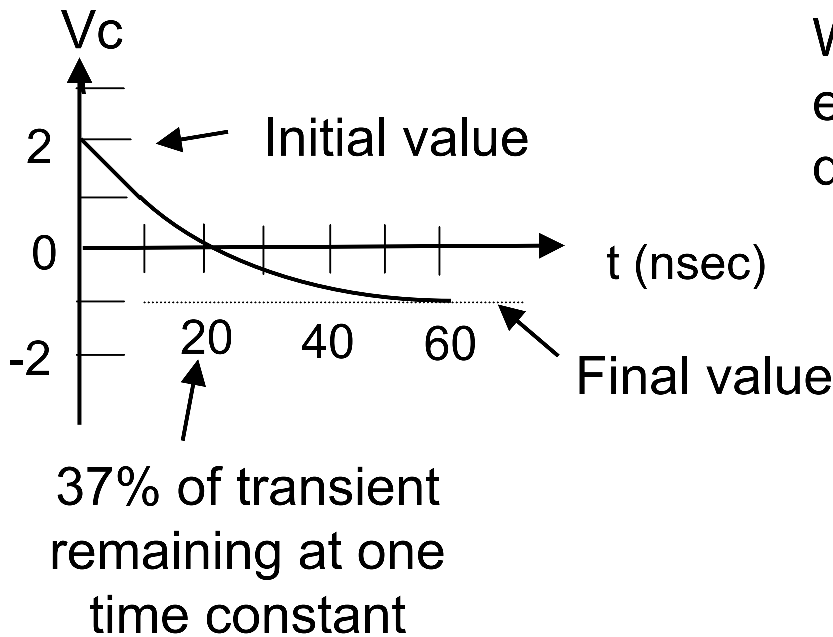
# Charging and discharging in RC Circuits (Example 1 of the EE42 Easy Method)

Find Vc(t) for the following circuit: (input switches from 2V to -1V at t = 0)




We have : Initial value of Vc is 2V, final value is -1V and $\tau$ = 20nsec

5) Sketch Vc (t) :

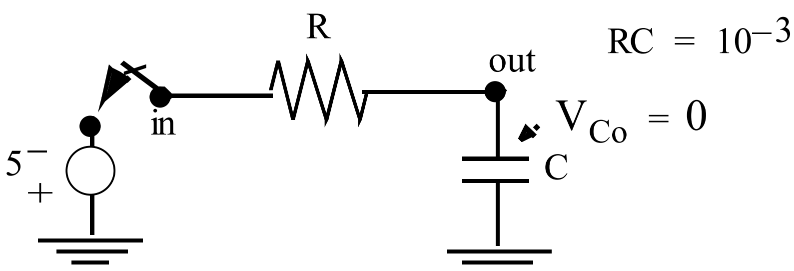


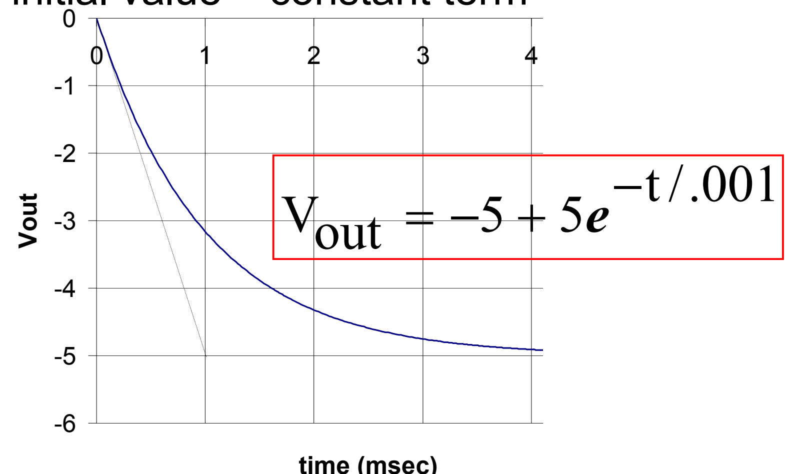

What is the equation for an exponential beginning at 2V, decaying to -1V, with $\tau$ = 20nsec?

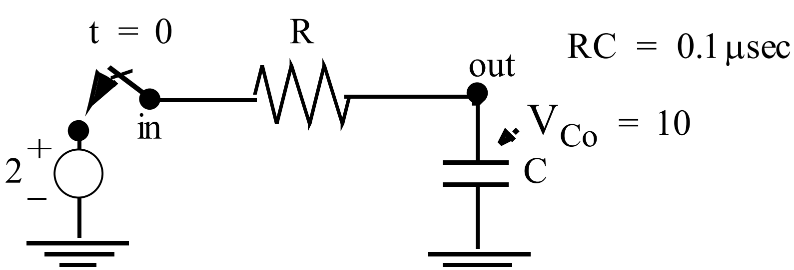
$$V_c(t) = -1 + 3e^{-t/\tau}$$

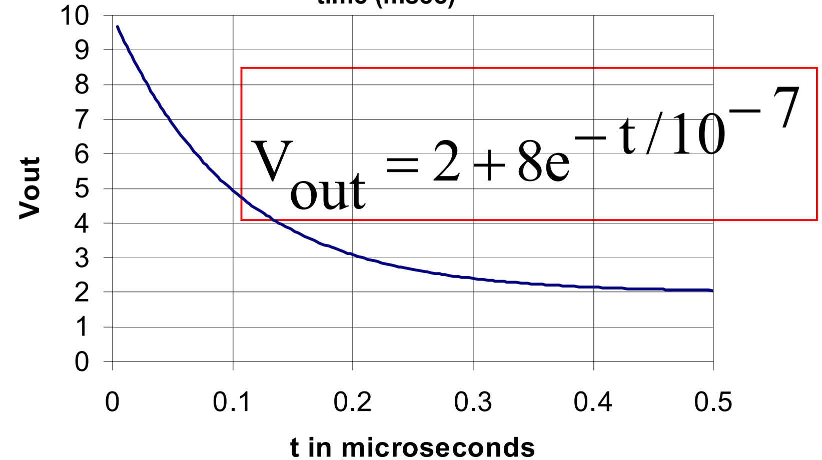
# OUR METHOD AVOIDS ALL MATH!

- ✶ Sketch waveform (starts at Vco, ends asymptotically at V1, initial slope intersects at t = RC or transient is 63% complete at t=RC)
- ✸ Write equation: **2a.** constant term A = limit of V as t → ∞
  **2b.** pre-exponent B = initial value – constant term



# 4 Examples

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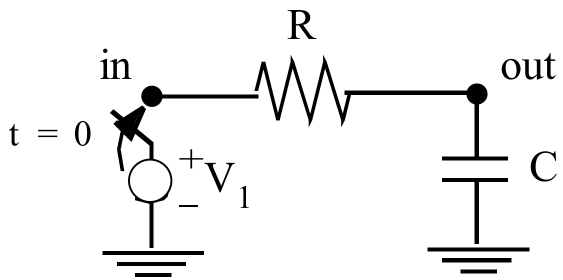


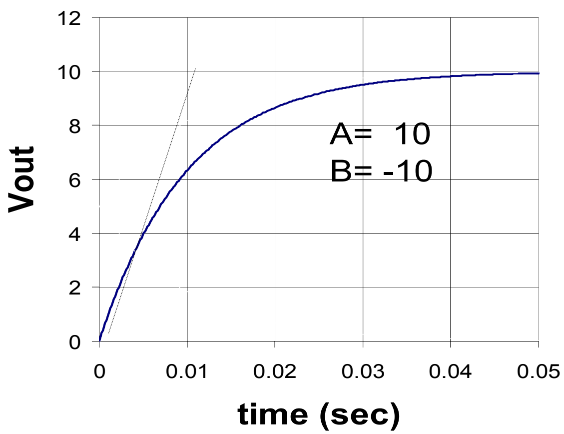

$RC = 0.01\,\text{sec}$

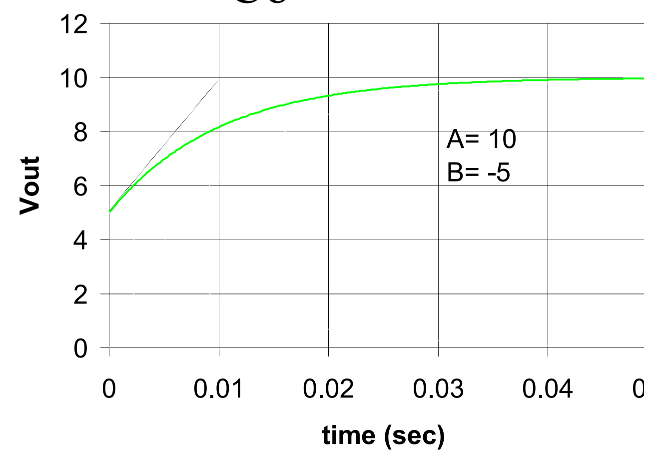
$$V_{out} = A + Be^{-t/RC}$$

$V_{Co} = 0$

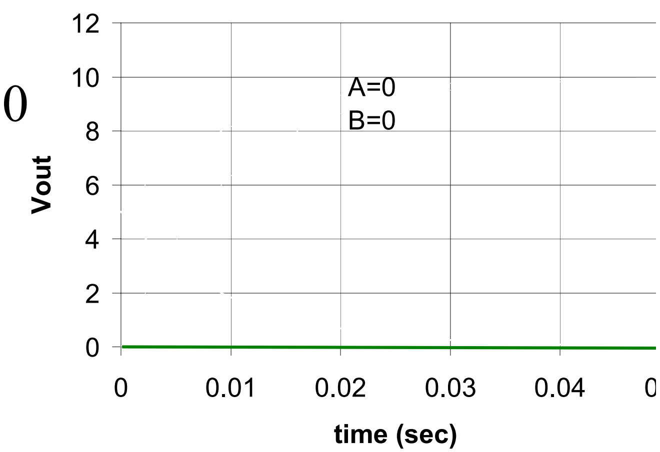
$V_{Co} = 5\text{ V}$

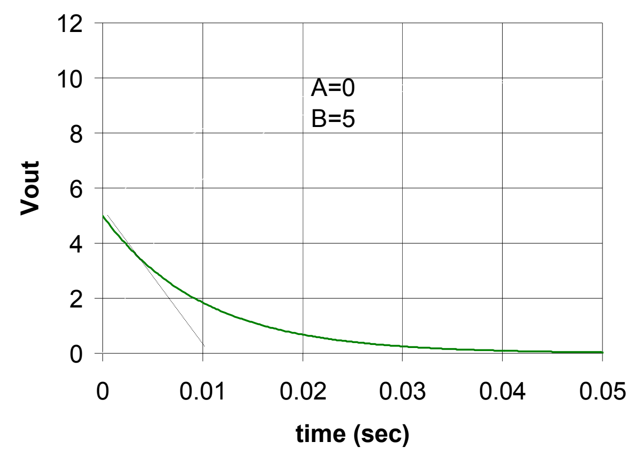
$V_1 = 10$




$V_1 = 0$

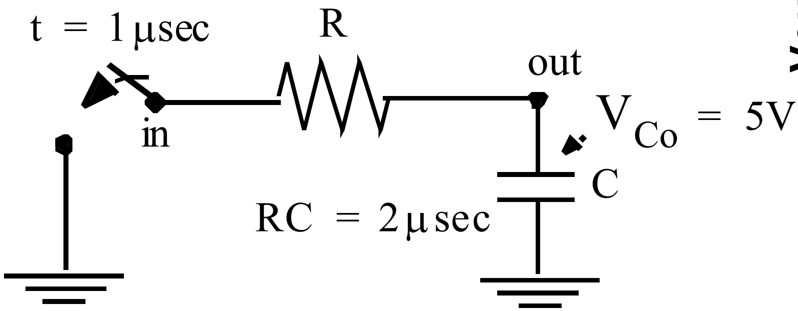


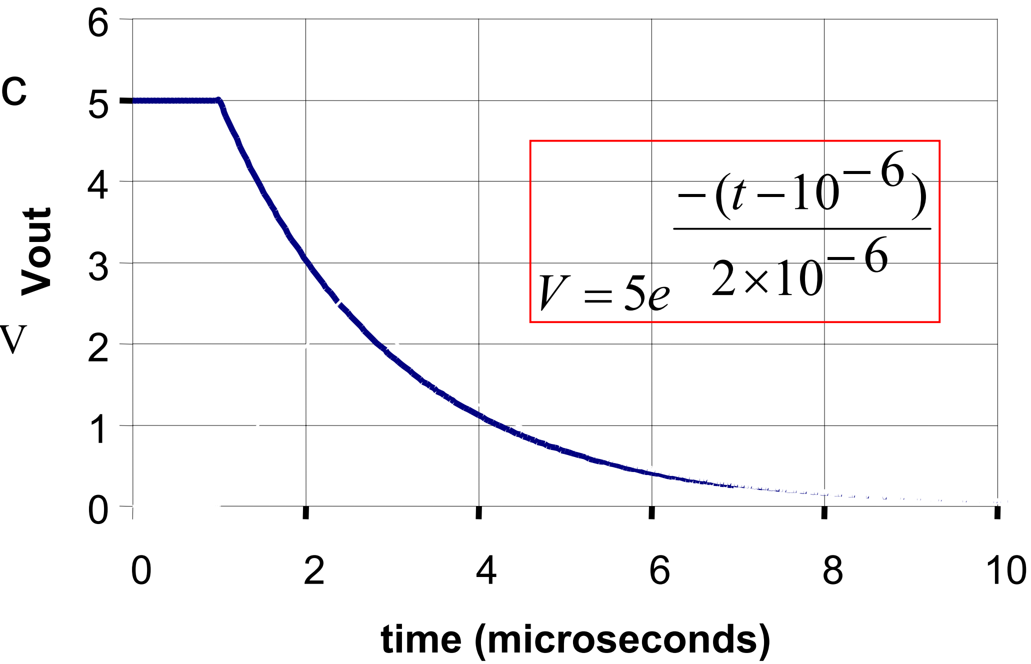



# COMPLICATION: Event Happens at $t \neq 0$

(Solution: Shift reference time to time of event)

Example: switch closes at 1μsec



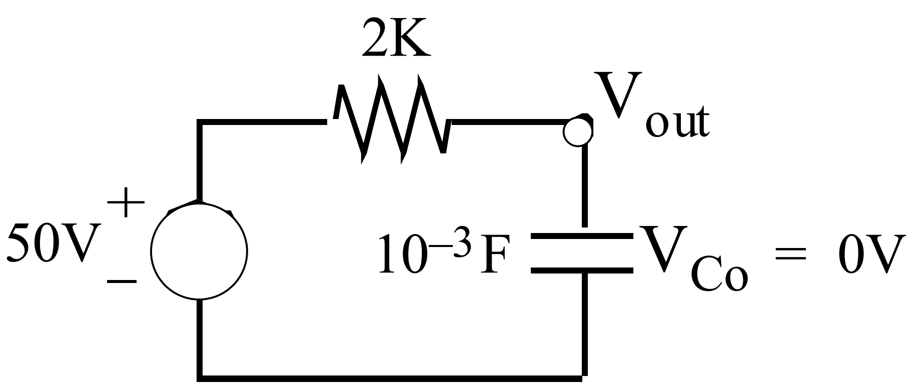



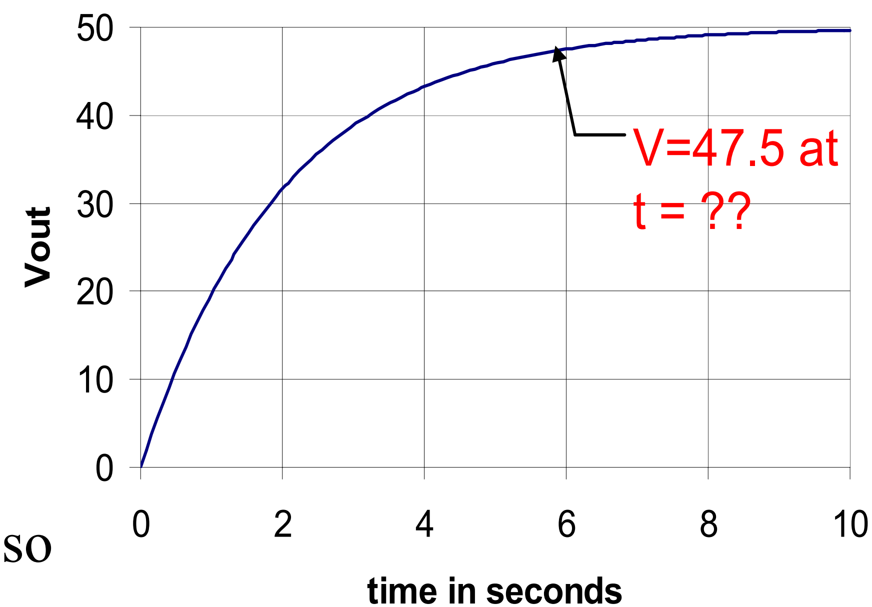

We shift the time axis here by one microsecond, i.e. imagine a new time coordinate t* = t-1μsec so that in the new time domain, the event happens at t* = 0 and our standard solution applies. Of course we replace t* by t -1μsec in the equations and plots. Thus instead of t* =0 we have t = 1μsec, etc.

# FINAL EXAMPLE

Your photo flash charges a 1000μF capacitor from a 50V source through a 2K resistor. If the capacitor is initially uncharged, how long must you wait for it to reach 95% charged (47.5 V)?

Solution: $RC = 2K \times 10^{-3} = 2\ sec$




By inspection: $V_o = 50 - 50e^{-t/2}$, so

$$47.5 = 50(1 - e^{-\frac{t_x}{2}}) \Rightarrow e^{-\frac{t_x}{2}} = (1 - \frac{47.5}{50}) \Rightarrow \boxed{t_x = 6\ sec}$$

# Generalizations

- Switching of circuits with multiple resistors and sources
  - use **$R_T$ seen looking back into the circuit** from the terminals of the capacitor **after the switching changes conditions**.
- **Inductor** circuits use $\tau = L / R_T$

# Pulse Distortion

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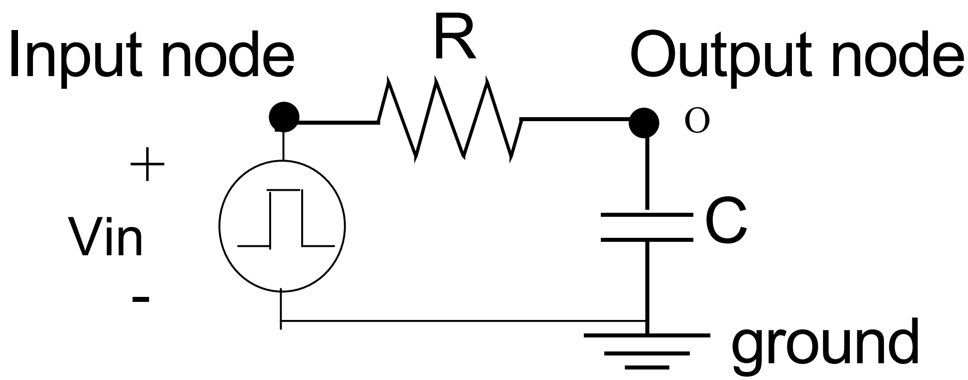
## An example of RC analysis




Rule: The pulse width must be wider than RC to avoid severe pulse distortion.

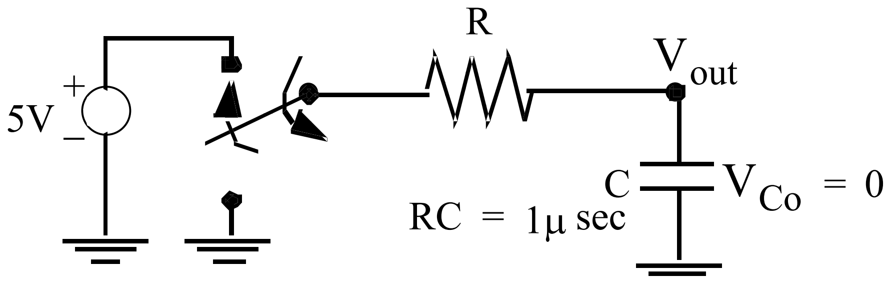
Example: Lets find the shape of the output pulse for pulse width of 0.1, 1, and 10 RC

Method: Use 5V pulse height. Let RC = 1μsec. Replace voltage source with a switch which shorts the input to ground for t<0, switches to 5V at t=0, and switches back to ground at t=0.1μsec , or t=1μsec , t=10μsec .

Thus we have two problems: #1: $V_{out}$ rising from zero and #2: $V_{out}$ falling back toward zero. Cascade solutions!

# PULSE: Output is Rising exponential then Falling exponential

Example: Switch rises at t =0, falls at t = 0.1, 1 or 10μsec (Do 1μsec case)




Solution: for RC = 1μsec: during the first rise V obeys:

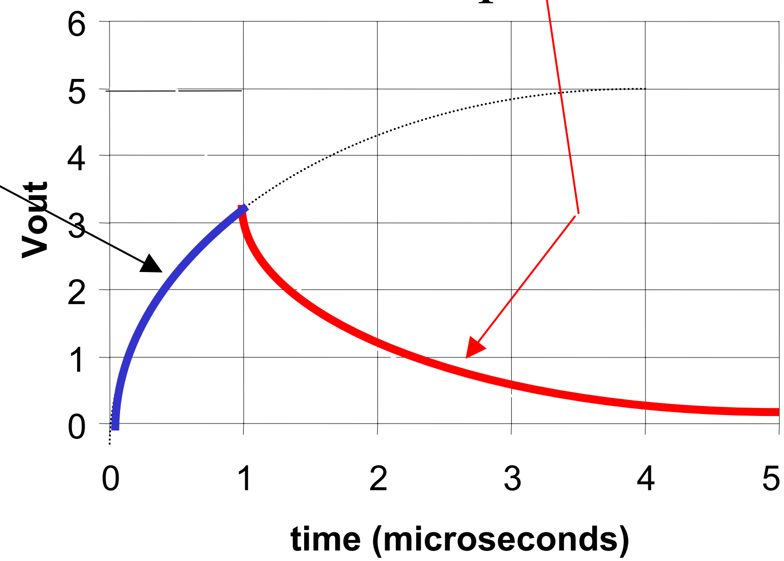
$$V = 5[1 - e^{\frac{-t}{10^{-6}}}]$$

Thus at t = 1μsec, rising voltage reaches

$$5[1 - e^{-1}] = 3.16V$$

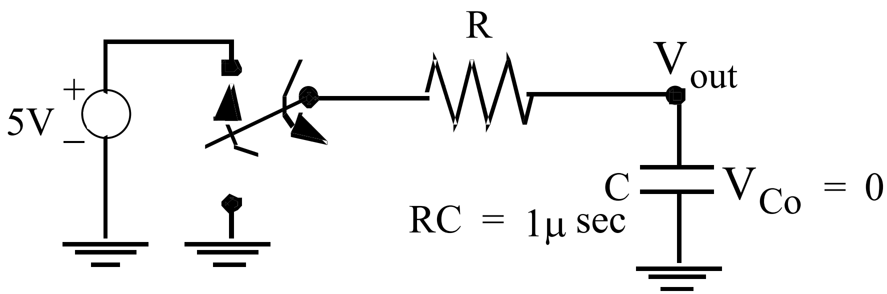
Now starting at 1μsec we are discharging the capacitor so the form is a falling exponential with initial value 3.16 V:

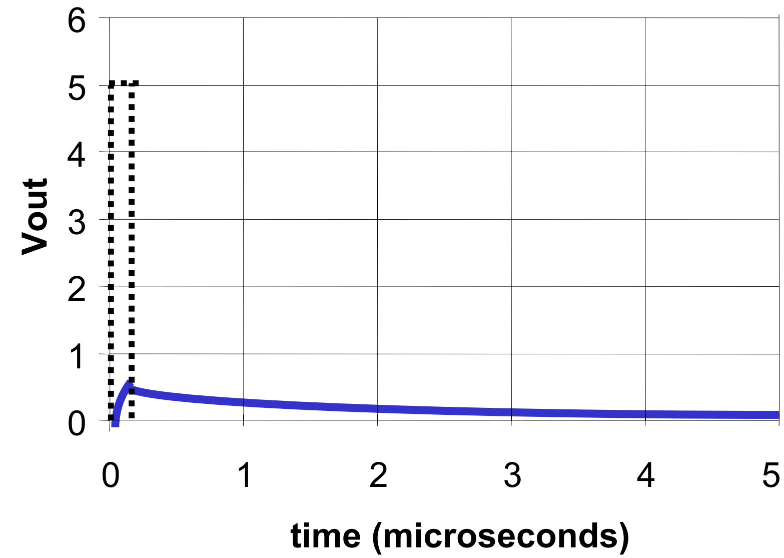
What is equation?



# PULSE DISTORTION – other cases

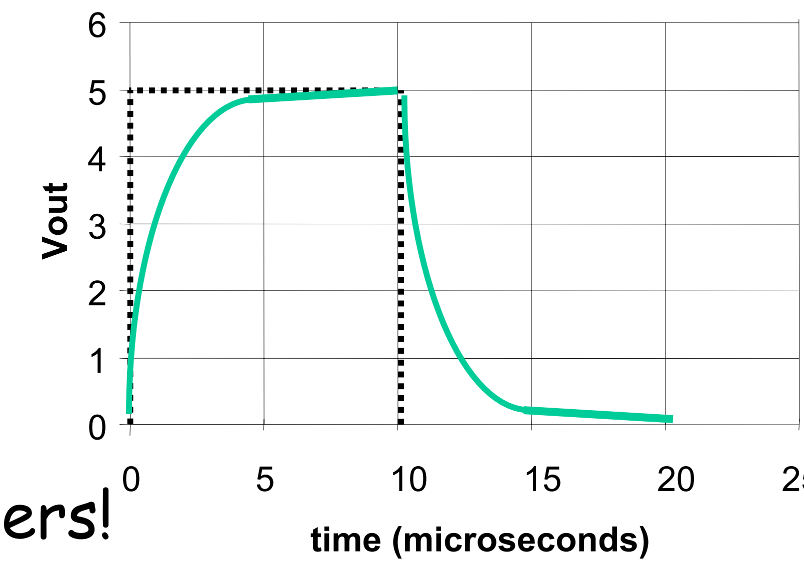
Switch rises at t =0, falls at t = 0.1 or 10μsec (i.e. 0.1 or 10 μsec pulse widths)




Solve for $V_{out}$ in the other two cases (0.1 or 10μsec) just as for 1μsec

At t = 0.1μsec the output has only risen to 0.5V!




Whereas for 10μsec pulse width, output reaches to within 0.995 % of 5V.

You need to verify the numbers!

# Pulse Distortion

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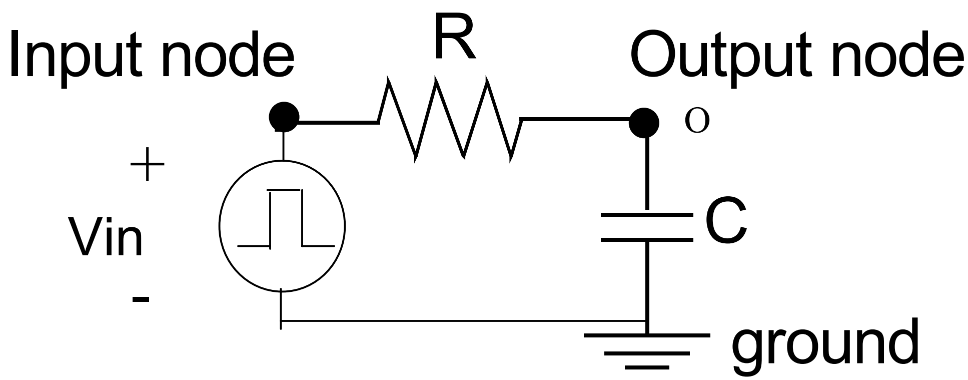
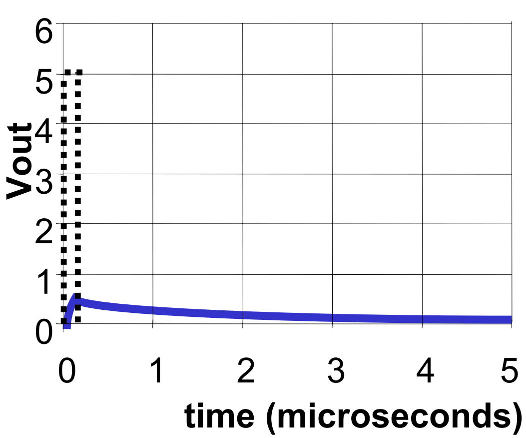
## An example of RC analysis

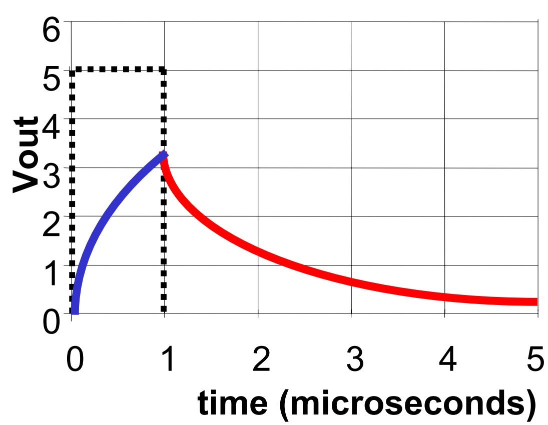



Rule: The pulse width must be wider than RC to avoid severe pulse distortion.

PW = 10RC




PW = RC




PW = 0.1RC