Version Date 9/23/01

Lecture 8: September 24th, 2001

Nodal Analysis

20 min Quiz on HW 1-4 at start of class on Wed. 9/26

A) Review of KCL and KVL
B) Nodal Analysis Strategy
C) Special Situations and Examples
D) Accounting for All Unknowns

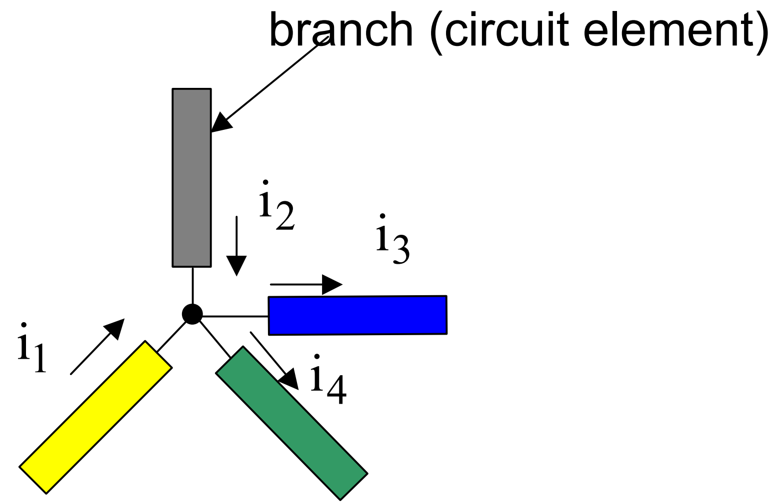
The following slides were derived from those prepared by Professor Oldham For EE 40 in Fall 01

Reading:

Schwarz and Oldham 2.3 pp. 53-58

BRANCHES AND NODES

Circuit with several branches connected at a node:



KIRCHOFF's CURRENT LAW "KCL":

(Sum of currents <u>entering</u> node) – (Sum of currents <u>leaving</u> node) = 0

q = charge stored at node is zero. If charge *is* stored, for example in a capacitor, then the capacitor is a branch and the charge is stored there NOT at the node.

WHAT IF THE NET CURRENT WERE NOT ZERO?

Suppose imbalance in currents is 1μA = 1 μC/s (net current entering node)

Assuming that q = 0 at t = 0, the charge increase is 10–6 C each second

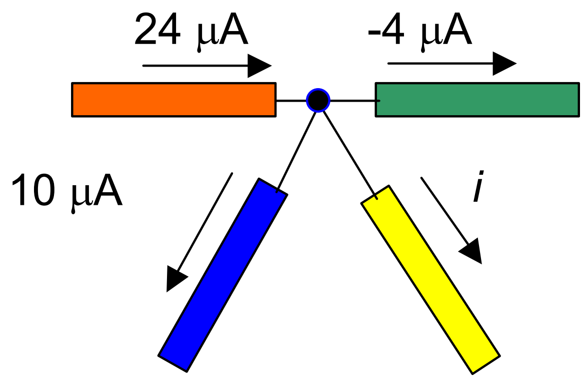
or $10^{-6}/1.6\times10^{-19} = 6\times10^{12}$ charge carriers each second

But by definition, the capacitance of a node to ground is ZERO because we show any capacitance as an explicit circuit element (branch). Thus, the voltage would be infinite (Q = CV).

Something has to give! In the limit of zero capacitance the accumulation of charge would result in infinite electric fields … there would be a spark as the air around the node broke down.

Charge is transported around the circuit branches (even stored in some branches), but it doesn't pile up at the nodes!

KIRCHHOFF'S CURRENT LAW EXAMPLE



Currents entering the node: 24 μA

Currents leaving the node: –4 μA + 10 μA + i

24 = 10 + (–4) + i

i = 18 μA

Three statements of KCL

$$\sum_{IN} i_{in} = \sum_{OUT} i_{out} \qquad 24 = -4 + 10 + i \quad \Rightarrow \quad i = 18\ \mu A$$

$$\sum_{ALL} i_{in} = 0 \qquad 24 - (-4) - 10 - i = 0 \quad \Rightarrow \quad i = 18\ \mu A$$

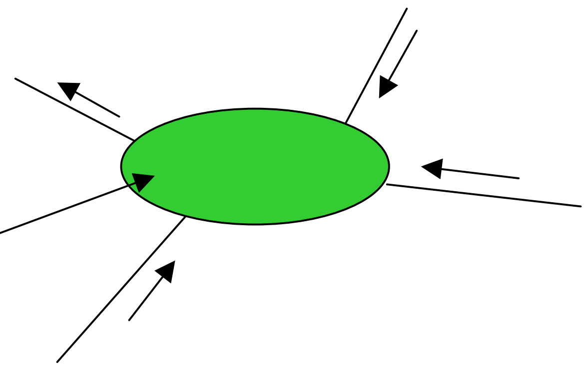
$$\sum_{ALL} i_{out} = 0 \qquad -24 - 4 + 10 + i = 0 \quad \Rightarrow \quad i = 18\ \mu A$$

EQUIVALENT

GENERALIZATION OF KCL

Sum of currents entering and leaving a closed surface is zero

Physics 7B



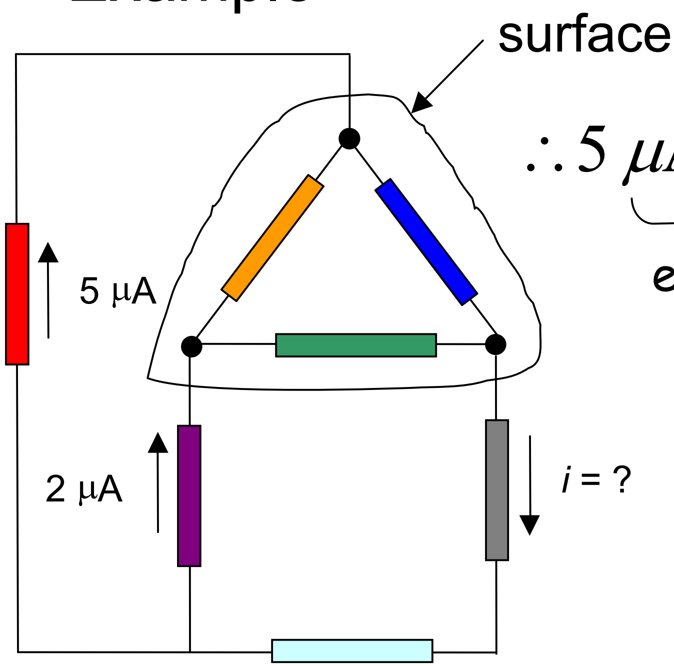
Could be a big chunk of circuit in here, e.g., could be a "Black Box"

Note that *circuit branches* could be inside the surface.

The surface can enclose more than one node

KIRCHHOFF'S CURRENT LAW USING SURFACES

Example

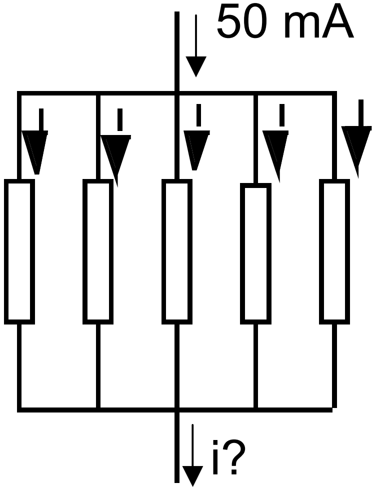


$$\therefore 5\ \mu A + 2\ \mu A = i$$

entering leaving

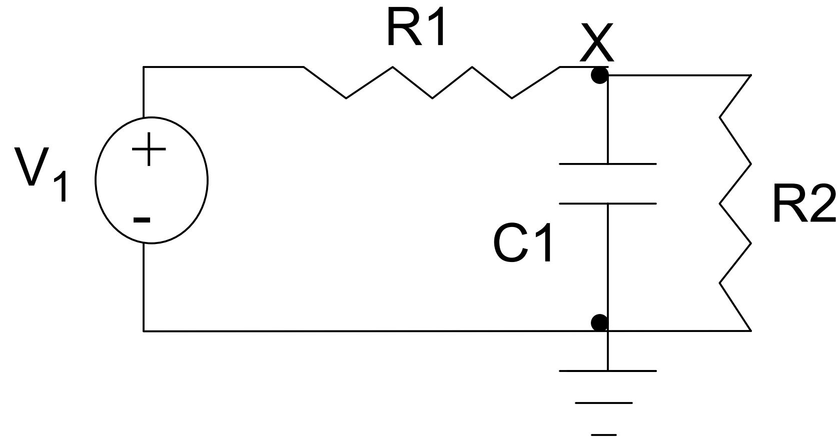
i=7μA

Another example



i must be 50 mA

Example of the use of KCL



At node X:

Current into X from the left:

$(V_1 - v_X)/R1$

Current out of X to the right:

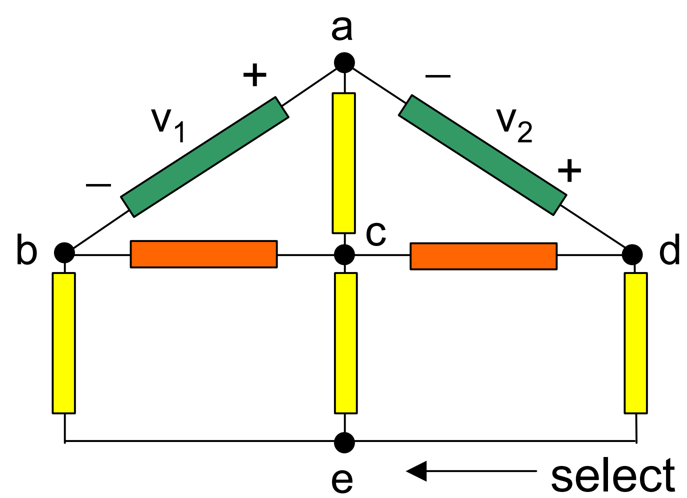
$v_X/R2 + Cdv_X/dt$

KCL: $(V_1 - v_X)/R1 = v_X/R2 + Cdv_X/dt$

Given V_1, This differential equation can be solved for $v_X(t)$.

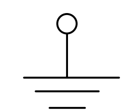
BRANCH AND NODE VOLTAGES

The voltage across a circuit element is defined as the difference between the node voltages at its terminals



$v_2 = v_d - v_a$

$v_e \equiv 0$ (since it's the reference)

select as ref. ⇒  "ground"

Specifying node voltages: Use one node as the implicit reference (the "common" node … attach special symbol to label it)

Now single subscripts can label voltages:

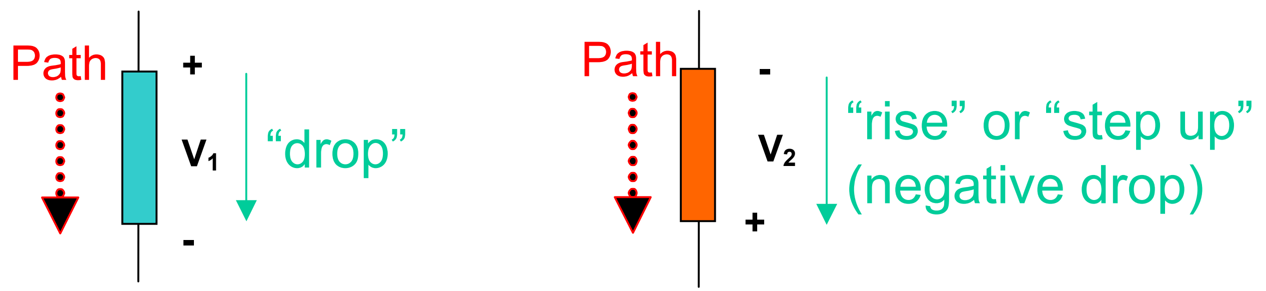
e.g., v_b means $v_b - v_e$, v_a means $v_a - v_e$, etc.

KIRCHHOFF'S VOLTAGE LAW (KVL)

The algebraic sum of the "voltage drops" around any "closed loop" is zero.

Why? We must return to the same potential (conservation of energy).

Voltage drop → defined as the branch voltage if the + sign is encountered first; it is (-) the branch voltage if the – sign is encountered first … important bookkeeping

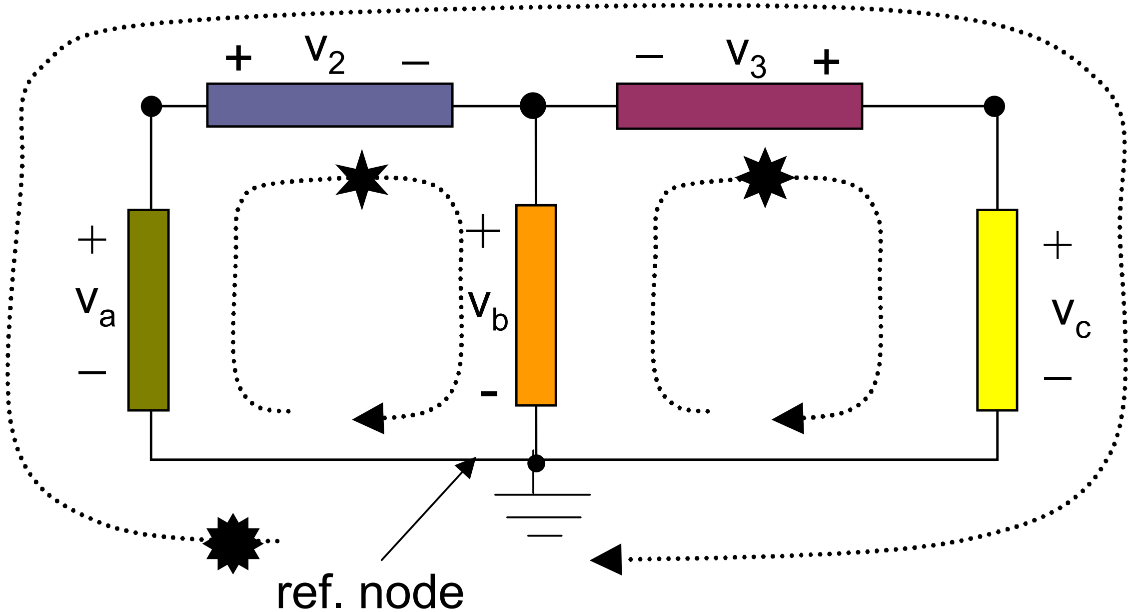


Closed loop: Path beginning and ending on the same node

KVL EXAMPLE

Examples of
Three closed paths:

✶, ✷, ✹



Note that:

$v_2 = v_a - v_b$

$v_3 = v_c - v_b$

Path 1:

$-v_a + v_2 + v_b = 0$

↑

$v_a - v_b$

YEP!

Path 2:

$-v_b - v_3 + v_c = 0$

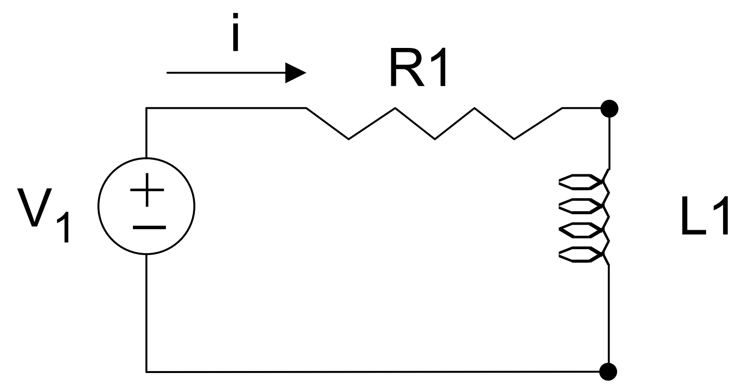
Path 3:

$-v_a + v_2 - v_3 + v_c = 0$

Example of the use of KVL

Voltage drop across R1 (left to right):

i R1



Voltage drop across L1 (top to bottom):

Ldi/dt

Starting at the bottom and walking clockwise, summing voltage drops:

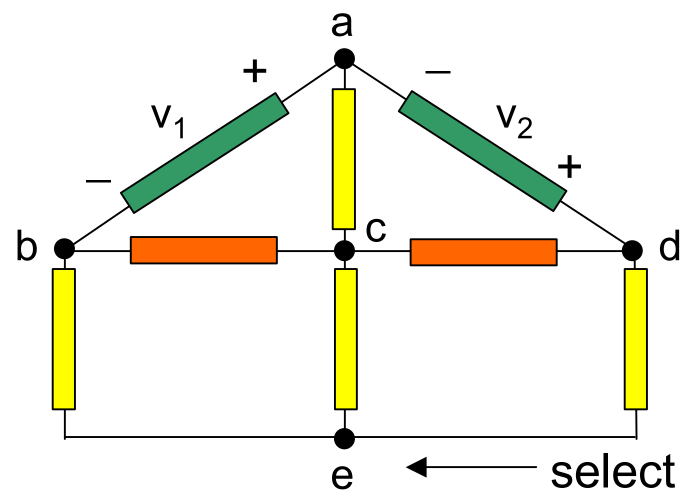
$-V_1 + i\,R1 + L\,di/dt = 0$

Given V_1, this differential equation can be solved for i(t).

HOW MANY NODE EQUATIONS DO WE NEED?

The circuit is completely described by the voltage of each node and the current through each branch. Thus the number of unknowns =

$(N_{NODES} - 1) + N_{BRANCHES}$



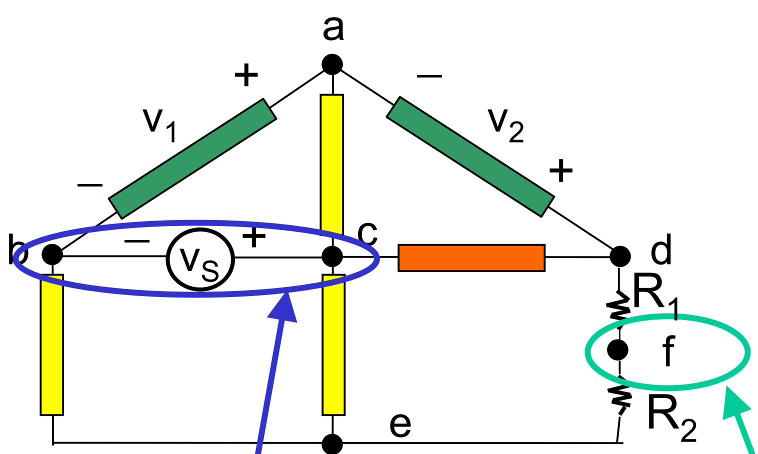
$v_e \equiv 0$ (since it's the reference)

select as ref. ⇒ "ground"

Since we already know the reference node voltage, we need $N_{NODES} - 1$ node equations.

Write a node equation at each of the nodes except the reference node!

SAVE TIME IN SPECIAL CASES!



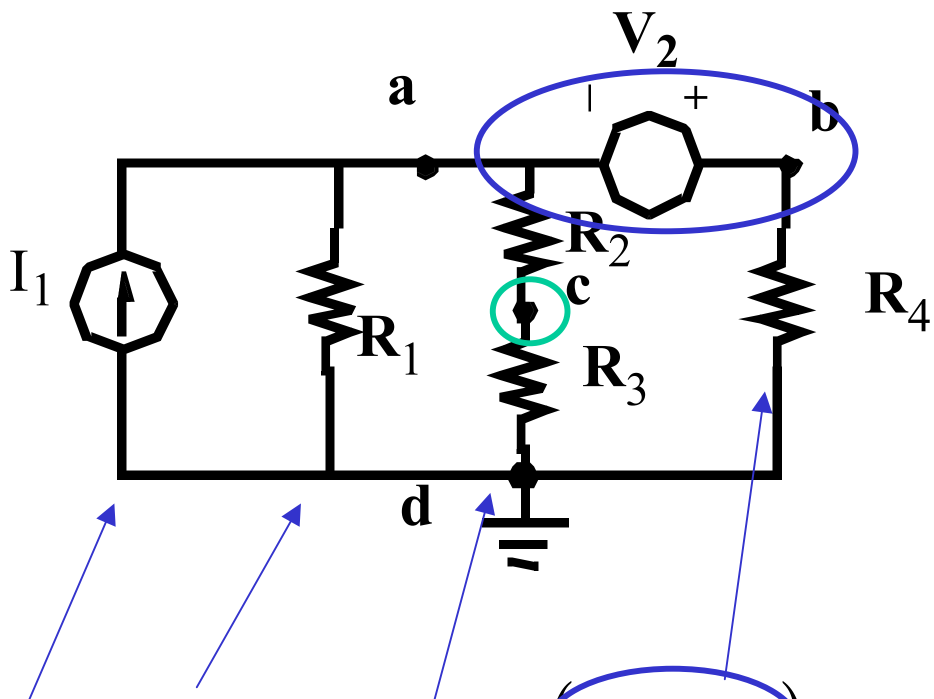
Eliminate either V_b or V_c by using a surface that encloses the voltage source and the nodes at is terminals.

This surfaces is a 'SUPERNODE.'

Eliminate this trivial node by a single resistor

$$R_{EQ} = R_1 + R_2$$

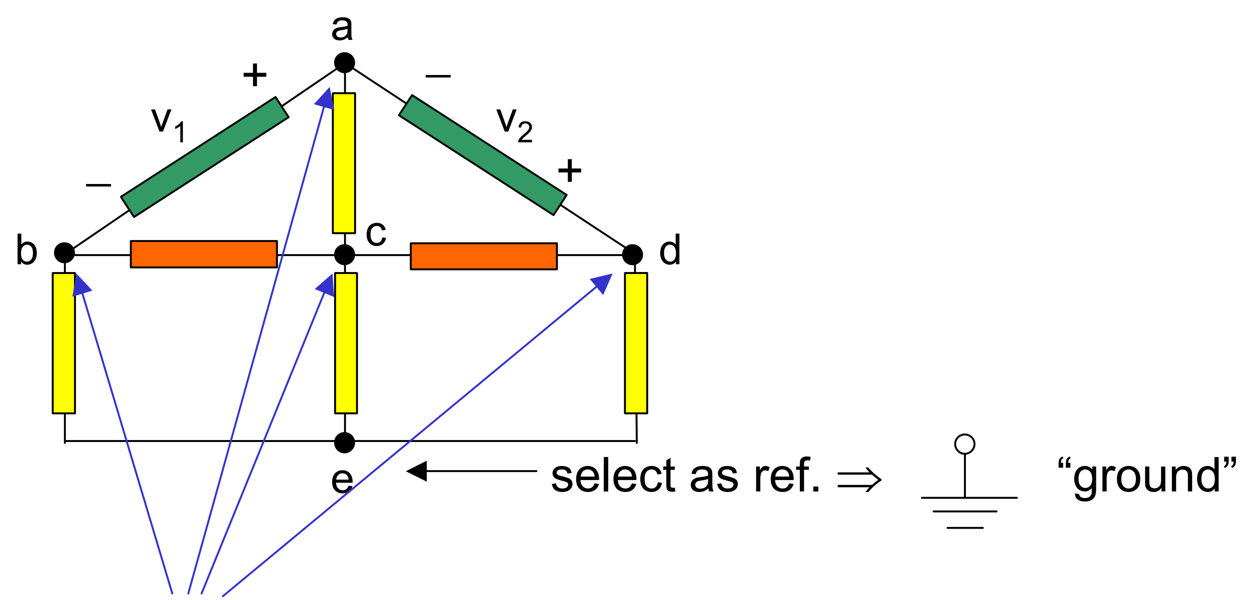
EXAMPLE WITH BOTH SPECIAL CASES



$$I_1 - \frac{V_a}{R_1} - \frac{V_a}{R_2 + R_3} - \frac{(V_a + V_2)}{R_4} = 0$$

ACCOUNTING FOR ALL UNKNOWNS

The number of unknowns = $(N_{NODES} - 1) + N_{BRANCHES}$



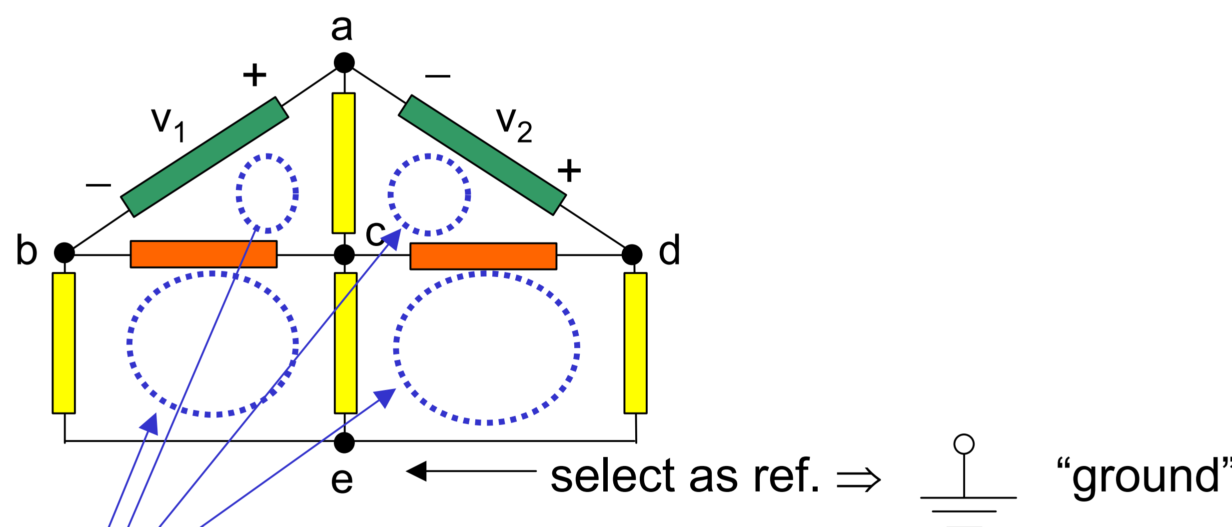
Node Method:

Use $(N_{NODES} - 1)$ node equations

Then one Branch Relationship for each branch

ACCOUNTING FOR ALL UNKNOWNS

The number of unknowns = $(N_{NODES} - 1) + N_{BRANCHES}$



Loop Method:

Use one loop equation for each window in the circuit

Or in 3D use [N_{BRANCH} - (N_{NODES} – 1)] Loop Equations

Then one Branch Relationship for each branch