

## Lecture 13: October 15, 2001

# Circuit Analysis with Dependent Sources

**A) Node Equations**

**B) Equivalent Sources**

**C) Amplifier Parameters:**

**Gain,  $R_{IN}$ ,  $R_{OUT}$**

**D) Non-Ideal Op-Amp Model**

The following slides were derived  
from those prepared by Professor  
Oldham For EE 40 in Fall 01

**Reading:**

**Schwarz and Oldham 4.1 - 4.2**

# The 4 Basic Linear Dependent Sources

Constant of proportionality

Parameter being sensed

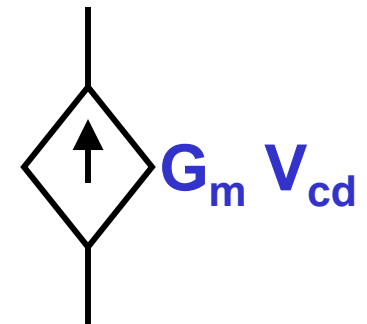
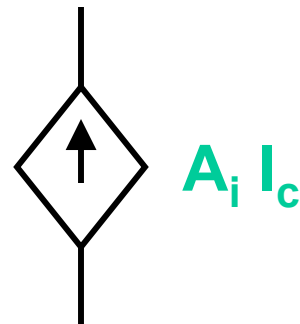
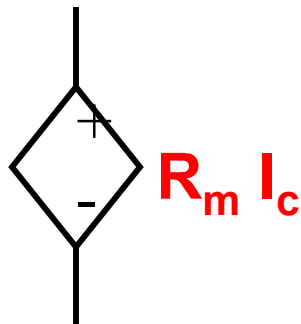
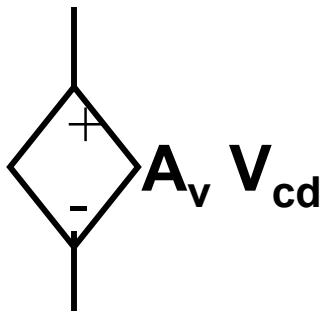
Output

Voltage-controlled voltage source ...  $V = A_v V_{cd}$

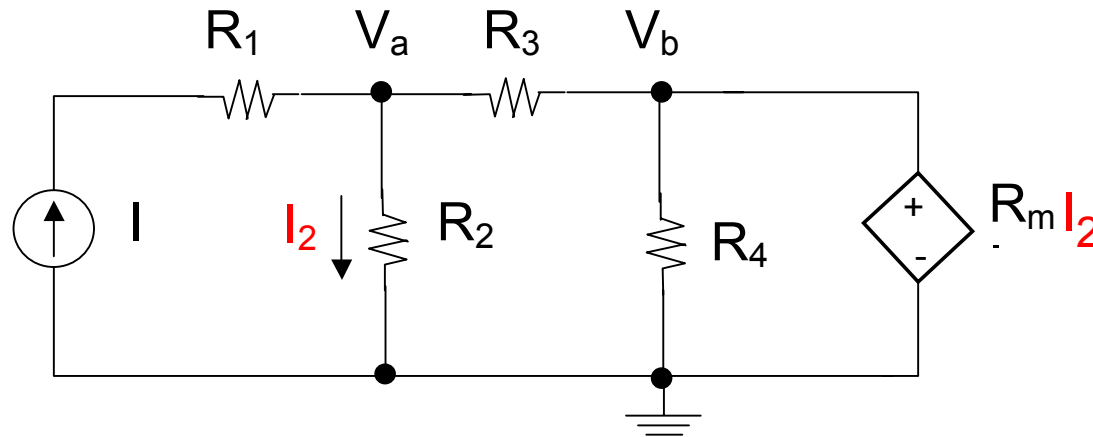
Current-controlled voltage source ...  $V = R_m I_c$

Current-controlled current source ...  $I = A_i I_c$

Voltage-controlled current source ...  $I = G_m V_{cd}$



## EXAMPLE OF NODAL ANALYSIS WITH DEPENDENT SOURCES



Standard technique, except an additional equation is needed if the dependent variable is an unknown current as here. Note  $V_b$  is redundant.

$$I = V_a / R_2 + (V_a - R_m I_2) / R_3 \quad \text{and} \quad I_2 = V_a / R_2$$

Solving: 
$$I = V_a (1/R_2 + 1/R_3 - R_m / R_2 R_3)$$

So 
$$V_a = I R_2 R_3 / (R_2 + R_3 - R_m)$$

## THEVENIN EQUIVALENT WITH DEPENDENT SOURCES

Method 1: Use  $V_{oc}$  and  $I_{sc}$  as usual to find  $V_T$  and  $R_T$  (and  $I_N$  as well)

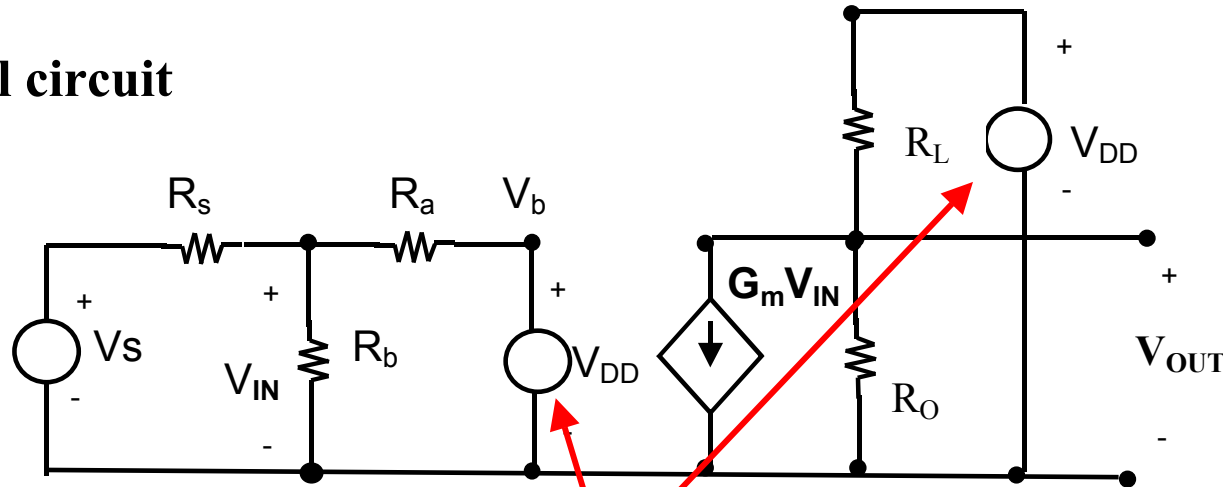
Method 2: To find  $R_T$  by the “ohmmeter method” turn off only the *independent* sources; let the dependent sources just do their thing.

See examples in text (such as Example 4.3).

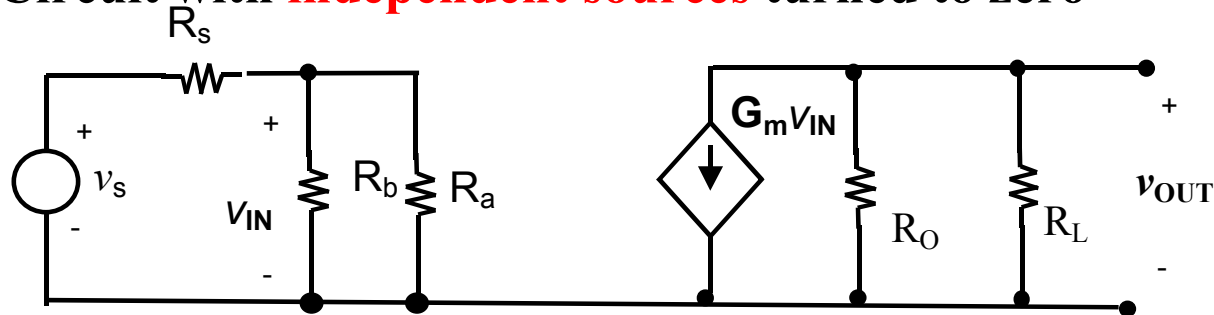
This method also works when computing **incremental signals** such as a change in the source  $V_S$  (given by  $\Delta V_S$  or  $v_S$ ) produces a change in  $V_{IN}$  or  $V_{OUT}$ , (given by  $\Delta V_{IN}$  or  $\Delta V_{OUT}$  also written  $v_{IN}$  and  $v_{OUT}$ ), and their ratio called the small-signal gain ( $\Delta V_{OUT} / \Delta V_S$ ) or ( $v_{OUT} / v_S$ )

## EXAMPLE CIRCUIT WITH MULTIPLE SOURCES

Original circuit

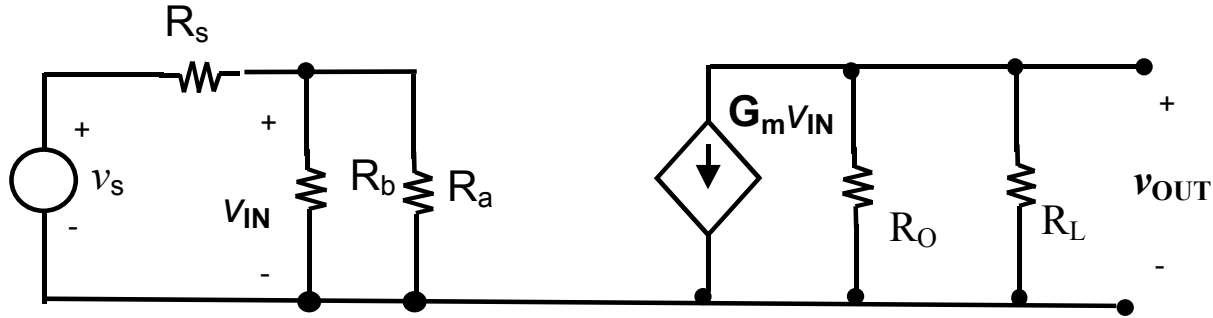


Circuit with **independent sources** turned to zero



Note  $R_L$  has been rotated down.

**EXAMPLE CIRCUIT: GAIN =  $(\Delta V_{OUT} / \Delta V_S) = (V_{OUT}/V_S)$**

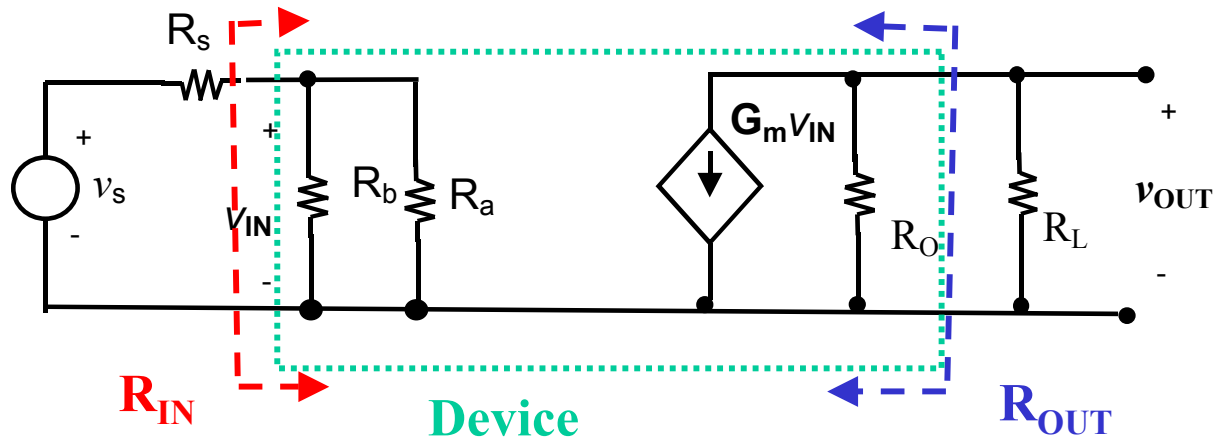


$$R_{IN} = \frac{R_a R_b}{R_a + R_b} \quad v_{IN} = \frac{R_{IN}}{R_S + R_{IN}} v_S$$

$$v_{OUT} = -G_m v_{IN} \frac{R_o R_L}{R_o + R_L} = -G_m \frac{R_{IN}}{R_S + R_{IN}} \frac{R_o R_L}{R_o + R_L} v_S$$

**Input voltage divider** and **output current divider**  
reduce the gain

## EXAMPLE CIRCUIT: INPUT/OUTPUT RESISTANCE



$$R_{IN} = \frac{R_a R_b}{R_a + R_b}$$

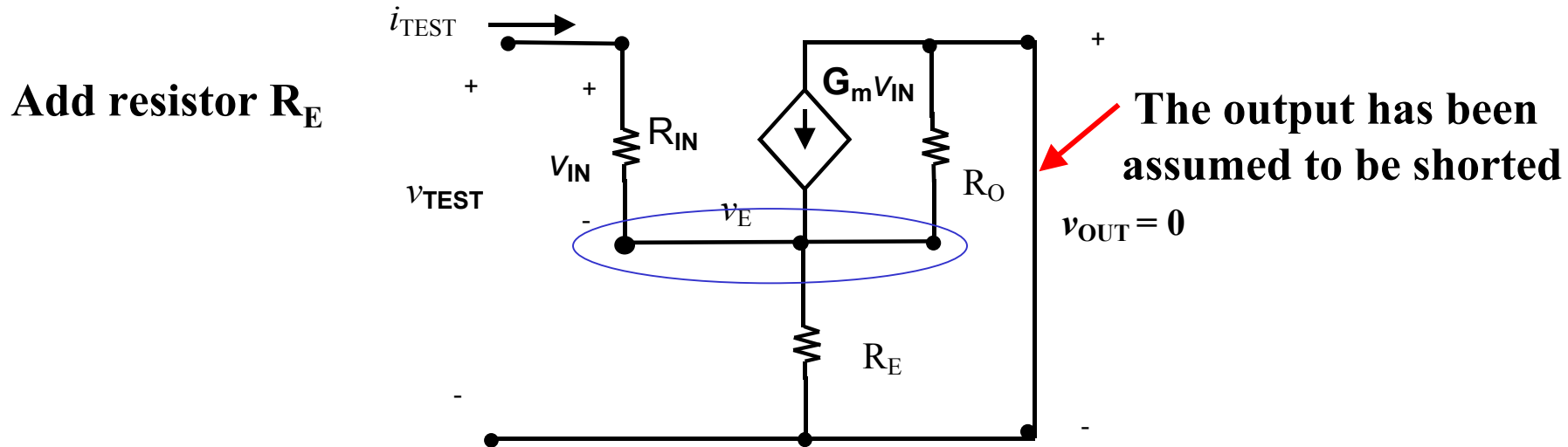
Output does not feed  
back to input

$$R_{OUT} = R_o$$

Assume  $v_s = 0 \Rightarrow v_{IN} = 0 \Rightarrow$  no  
current in dependent source

Can circuit design improve  $R_{IN}$  and  $R_{OUT}$  or do we need better devices?

## EXAMPLE CIRCUIT: INCREASED INPUT RESISTANCE



**Analysis: apply  $i_{TEST}$  and evaluate  $v_{TEST}$**

$$v_{IN} = R_{IN} i_{TEST} \quad v_{TEST} = R_{IN} i_{TEST} + v_E$$

**KCL** 
$$\frac{v_E}{R_E} + \frac{v_E}{R_O} - i_{TEST} - G_m R_{IN} i_{TEST} = 0$$

**Check for special case for  $R_O$  infinite**

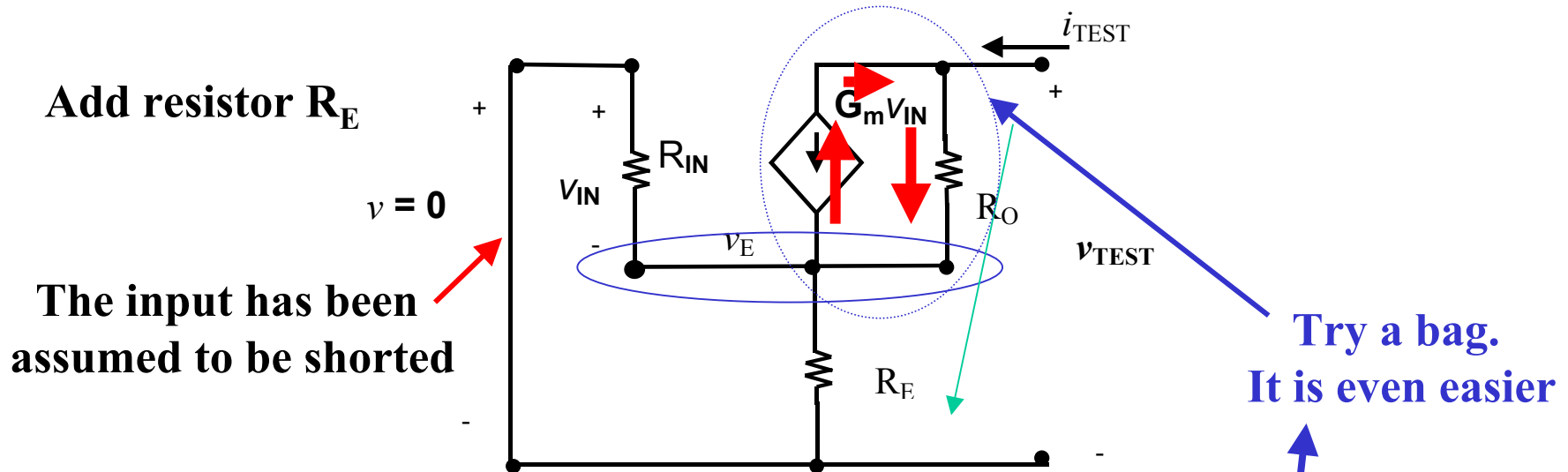
$$\frac{v_{TEST}}{i_{TEST}} = R_{IN} + (1 + G_m R_{IN}) R_E$$

**Finish this in the homework**

**Intuitive Explanation:**  
 $R_E$  puts  $R_{IN}$  on a node whose voltage increases in response to current in  $R_{IN}$ .



## EXAMPLE CIRCUIT: INCREASED OUTPUT RESISTANCE



Analysis: apply  $i_{\text{TEST}}$  and evaluate  $v_{\text{TEST}}$

Unknowns:  $i_{\text{TEST}}$ ,  
 $v_{\text{TEST}}$ ,  $v_{\text{IN}}$ ,  $v_{\text{E}}$

Need 3 equations  
 to find the ratio of  
 $i_{\text{TEST}} / v_{\text{TEST}}$

$v_{\text{IN}} = -v_{\text{E}}$  and is **not zero!**

KCL at  $v_{\text{E}}$   
 KVL at  $v_{\text{OUT}}$

Intuitive Explanation:  $G_m v_{\text{IN}}$  burps current  
 which has to also go through  $R_0$ . This raises  
 $v_{\text{TEST}}$  and the output impedance  $v_{\text{TEST}}/i_{\text{TEST}}$

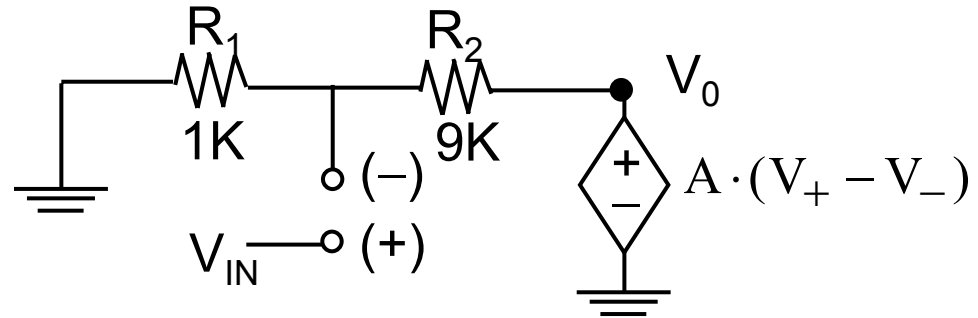
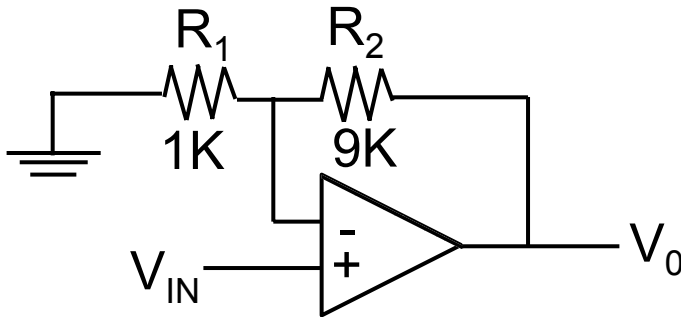
Finish this in  
 the homework

**NON-IDEAL OP-AMPS**

Version Date 10/16/01

JUST ANOTHER CASE OF ANALYSIS WITH DEPENDENT SOURCES

Example:

Circuit (assume  $R_{IN} \cong \infty$ )

Analysis:

**Outline your circuit analysis strategy here.**

**Hint: 1) Find  $V_-$  in terms of  $V_0$ , 2) plug into expression for  $V_0$  and then 3) solve for  $V_0$  which appears on both sides of the equation.**

**Answer:**

$$V_0 = V_{IN} \frac{A(R_1 + R_2)}{(A + 1)R_1 + R_2} \cong V_{IN} \frac{R_1 + R_2}{R_1} = 10V_{IN}$$

if  $A \rightarrow \infty$